PART I

INDEX

USAF RESERVE OFFICERS
ABDR TRAINING PROGRAM

PHASE I OUTLINE

LESSON 1

STRENGTH TERMINOLOGY

1.1 STRESS

In aircraft strength analysis the word stress refers to unit stress which is a measure of the intensity of the internal load or force acting at a point in a structural component. Stress is measured in units of force per unit area. The units used are pounds per square inch, abbreviated psi, or kips (1000 pounds) per square inch, abbreviated ksi.

The stress distribution acting on a cross section of a structural member may or may not be uniform, depending on the nature of the loading and the geometry of the cross section.

The stresses acting at a cross section of a structural member can be resolved into components normal to the plane of the cross section and parallel to the plane. Stresses perpendicular to a plane are called normal stresses and those parallel to the plane are called shear stresses. These are explained further in a subsequent paragraph.

In engineering practice, several different symbols, including s and the Greek letter sigma (σ), are used to denote stress. For aircraft strength analysis the symbols used are f for an actual stress and F for an allowable stress, both usually followed by a subscript denoting the type of stress. One exception is shear stress in a thin panel, for which the Greek letter tau (τ) is often used in place of f and F.

The maximum load expected in any structual member is called the design limit load or, simply, limit load. The stress in the member at limit load is called limit stress. The structure is designed to withstand, without failure, a higher stress called the design ultimate stress or, simply, ultimate stress. The ratio of ultimate stress to limit stress is called the ultimate factor. For nearly all aircraft design the ultimate factor is 1.5.

1.1.1 Types of Stress

The principal types of stress are normal stress and shear stress as described above. Normal stresses are either tension or compression stresses, depending on whether they are pulling or pushing on the section being considered. The symbols for tension and compression stress and f_t and f_c, respectively. The formula for tension or compression stress due to an axial load is:

$$f = P/A$$

where P is the force, or load, and A is the cross sectional area on which the force acts.

The symbol for shear stress is f_s or, in some cases, τ as noted above. The formula for shear stress on a compact section is $f_s = V/A$ where V is the shear force and A is the cross sectional area.

The other type of stress to consider is bearing stress which is the effective intensity of the shear load in a fastener acting on the surface of the fastener hole. The symbol for bearing stress is f_{br} and its formula is:

$$f_{br} = \frac{P}{A} = \frac{P}{Dt}$$

where P is the applied load, D is the fastener diameter and t is the thickness of the part.

1.1.2 Other Stress Terms

There are several terms of interest which relate to the effects on metal structure of various stress levels. Some of these are illustrated in the stress-strain diagram that follows.

- Proportional limit - This is the highest tension stress at which the strain (deflection) is proportional to the stress.

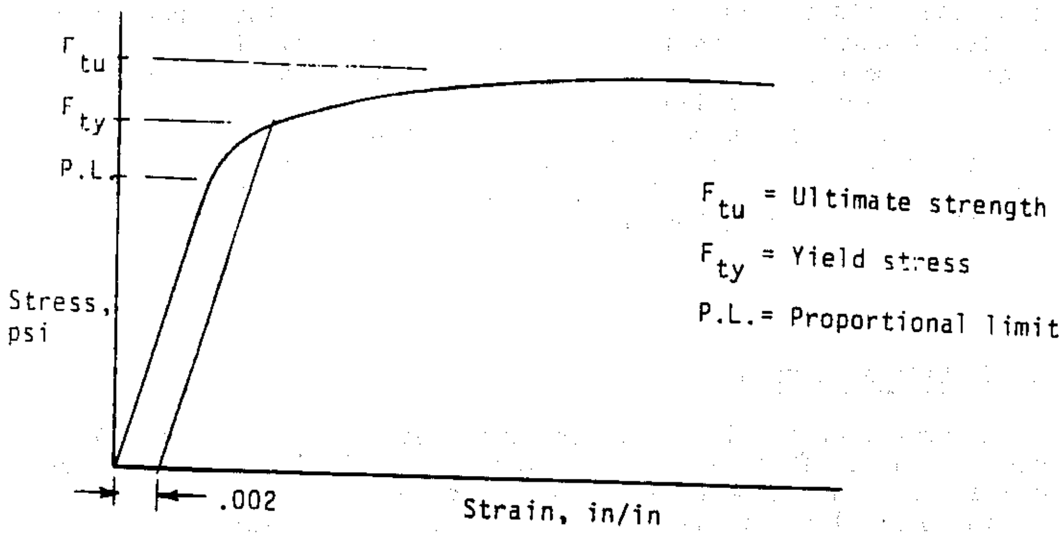
- Yield stress - The tension and compression yield stresses are the lowest tension and compression stresses at which the strain increases without an increase in stress. Because the exact stress is difficult to determine, the value used is the stress level that causes a permanent deformation or set of 0.002 inches per inch.

- Elastic limit - This is the lowest stress that will cause permanent set. For practical use it is the same as the yield stress.

- Permanent set - (also called set, permanent deformation, plastic strain or plastic deformation). This is any strain remaining after removing the stress.

- Ultimate strength - The ultimate strength (or ultimate allowable stress) in tension, compression shear or bearing is the maximum stress of the specified type that the material can sustain without rupture. Ultimate strength in compression is very rarely a design stress because structural members loaded in compression nearly always have a lower failing stress due to instability (buckling or crippling). Similarly, thin shear webs usually fail at a lower stress than the ultimate allowable shear stress due to instability (buckling that precipitates failure). Note that this ultimate stress is an allowable stress, whereas the ultimate stress discussed in paragraph 1.1 is an applied stress.

- Stress-strain diagram - A stress-strain diagram is a graph showing a plot of stress versus strain for a particular alloy and heat treat. The following graph depicts a typical shape for a tension stress-strain diagram for aluminum, titanium and high strength steel.



STRESS-STRAIN DIAGRAM

- Elastic stress - An elastic stress is a stress that is within the elastic limit. An elastic stress distribution is the stress distribution over a particular cross section of a structural member under a loading condition that does not cause the elastic limit to be exceeded. At all points on the section the stress is proportional to the strain.

- Plastic stress distribution - As the load on a structural member with an unequal stress distribution is increased, the stresses increase in proportion to the load until the maximum stress reaches the yield stress. As the load increases further the stress will increase everywhere except where it is at yield. Before the member will fail the stress over the entire section will be at or near the ultimate stress. Because of the permanent set inherent to this stress distribution it is known as a plastic distribution. The stress in this case is not proportional to the strain.

  Plastic stress distribution is used for design of structural components loaded in bending or torsion. It is not used for major components such as a fuselage or a bulkhead. This topic will be explained further in Lessons no. 12 and 14.

1.2 STRAIN

Strain is the term used to denote unit strain which is the elastic deformation per unit length caused by a stress. The symbol for strain is the letter e. The three types of strain are explained in the following paragraphs.

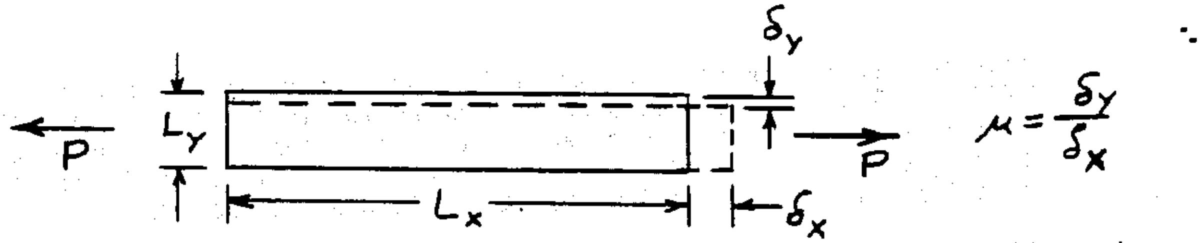
1.2.1 Normal strain

The strain of most interest is normal strain. This is the strain associated with a normal stress and takes place in the direction of that stress. A normal strain is also called an axial strain because it is the strain in the direction of the load in an axial member. A normal, or axial, strain is either a tensile strain caused by a tension stress or a compressive strain caused by a compression stress. A tensile strain is an elongation of the member and a compressive strain is a foreshortening. The Greek letter delta (δ) denotes the total deflection over a length L. Strain (e) is the deflection over a unit length. Therefore:

$$e = \frac{\delta}{L}$$

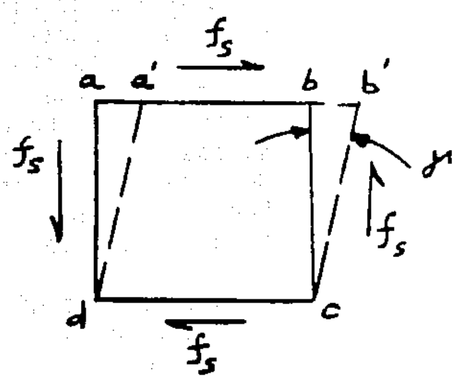
1.2.2 Lateral Strain

A normal, or axial, strain is always accompanied by lateral strains of the opposite sign. That is, in a tension member the lateral dimensions will decrease and in a compression member they will increase. The ratio of lateral strain to axial strain is approximately constant within the elastic range and is called Poisson's ratio, denoted by the Greek letter mu (μ). For the metals used in aircraft structures, Poisson's ratio is usually between .30 and .35. In some books, Poisson's ratio is denoted by the Greek letter nu (ν).



1.2.3 Shear Strain

A shear stress causes a displacement in the direction of the stress. The shear strain is obtained by dividing the displacement between two planes by the distance between them. Because the displacement is small, the shear strain is also equal to the change in angle (in radians) between two lines originally perpendicular to each other.



1.3 RELATIONSHIP OF STRESS AND STRAIN

For stresses within the proportional limit, strain is proportional to stress as explained in paragraph 1.1.2. The ratios of stress to strain in tension, compression and shear are measures of stiffness of the material. They are used to determine deflections and to predict buckling and crippling stresses.

1.3.1 Tension - The ratio of stress to strain in tension is the slope of the straight line portion of the stress strain diagram below the elastic limit. This ratio is called the modulus of elasticity, or Young's modulus, and is denoted by the letter E. Stated as an equation:

$$E = f/e$$

where f is the tension stress and e is the strain, or elongation, at that stress.

For a particular material, i.e. aluminum, the value of E varies only slightly from one alloy to another and does not vary with heat treatment. Because strain (e) has units of in./in. it can be seen from the equation that the units for modulus of elasticity (E) must be the same as for stress (psi). The values are approximately 10.5×10^6 psi for aluminum alloys, 29.0×10^6 psi for steel, 26.0×10^6 psi for corrosion resistant steel and 16.0×10^6 psi for titanium alloys.

1.3.2 Compression - The modulus of elasticity in compression, E_c, is essentically the same as in tension. The exact value is the same as the tension modulus for steel, about 2% higher than the tension modulus for aluminum, and about 3% higher than the tension modulus for titanium alloys. The tension modulus can be used without significant error.

1.3.3 Shear - The ratio of shear stress to shear strain is called the modulus of rigidity, or the modulus of elasticity in shear. It is designated by the letter G. The relationship between the modulus of rigidity and the modulus of elasticity is shown in the following equation:

$$G = \frac{E}{2(1+\mu)}$$

where μ is Poisson's ratio. The values of G are approximately 4.0×10^6 psi for aluminum alloys, 11.0×10^6 for steel and stainless steel and 6.2×10^6 psi for titanium alloys.

1.3.4 Modulii above the elastic limit - The modulii of elasticity described above are only valid below the elastic limit. Above that stress level the stress-strain curve is no longer a straight line and the slope of the curve varies with stress level. Stress analysis in the plastic range sometimes uses one of two modulii: the Tangent modulus, E_t, or the secant modulus, E_s. The tangent modulus for a particular stress is the slope of a tangent to the stress-strain curve at the point corresponding to that stress. The secant modulus is the slope of the secant to the curve that passes through the origin and the point on the curve corresponding to the stress level.

PROBLEMS

LESSON 1 STRENGTH TERMINOLOGY

1.1 What types of stress can act on a cross section of a structural member?

1.2 What is the stress in a rectangular aluminum bar with a cross section 1.50" x 2.0" subjected to a tension load of 186,000 lb?

1.3 If the bar in the above problem has a 132,000 lb shear load, what is the stress?

1.4 What is the limit bearing stress for a .250 inch diameter bolt in a .100 inch thick plate with a limit load of 2,000 lb?

1.5 What is the ultimate bearing stress in the above plate?

1.6 What would be the approximate strain in an aluminum stringer with a compression stress of 35 ksi?

1.7 If the stringer in the previous problem was initially 84 inches long, what would be the length with the load applied?

1.8 Determine the modulus of rigidity for a material with a modulus of elasticity of 10.5×10^6 psi and Poisson's ratio of 0.33.

1.9 At a stress level between the proportional limit and the yield stress:

a) Would the tangent modulus be greater or smaller than the modulus of elasticity of the material?

b) Would the secant modulus be greater or smaller than the tangent modulus?

LESSON 2

REVIEW OF STATICS

2.1 GENERAL

The first step in the strength analysis of any structural component is to check that all external loads are included and that they are in equilibrium. This involves the branch of engineering mechanics known as statics. This lesson will review the equilibrium of forces in a plane (two dimensional balance) and of forces in space (three dimensional balance).

A structural component for which all external forces (loads) are known and in equilibrium is called a free body because it can be analyzed by itself, free of the mating structure. The forces on this free body constitute a static balance.

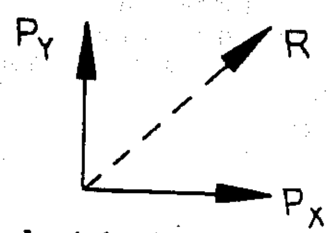
2.2 FORCES ACTING IN A PLANE

Many structural components lie essentially in a plane with all external loads applied in that plane. Examples are spars, ribs, bulkheads, rings, frames, floors and keel webs. The following paragraphs consider forces acting in a plane.

2.2.1 Resultant Loads

Loads that intersect at a point can be combined into a single resultant load. If the forces to be combined are mutually perpendicular the resultant load is found by using the equation:

$$R = (P_x^2 + P_y^2)^{1/2}$$

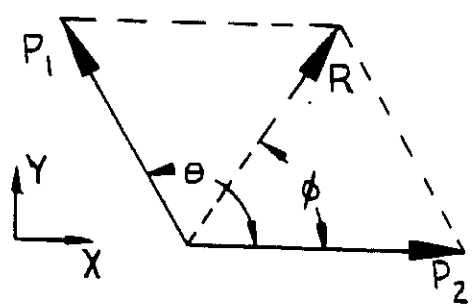


If the forces are not perpendicular, the resultant load is found from the law of cosines:

$$R = (P_1^2+P_2^2+2P_1P_2 \cos\theta)^{1/2}$$

$$\phi = \arctan\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \arctan\left(\frac{P_1 \sin\theta}{P_2+P_1 \cdot \cos\theta}\right)$$



Conversely, a force acting in any direction can be resolved into components in any two other directions by using trigonometric functions.

2.2.2 Equilibrium of Forces

For the forces in a plane to be in equilibrium, three equations must be satisfied. They are:

$$\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma M = 0$$

where x and y are two perpendicular axes, ΣF is the summation of forces parallel to one of the axes, and ΣM is the summation of moments about any point.

These equations of statics are used to determine the reactions to the loads on structural components. Because there are three equations, three reactions can be determined. In order to determine these reactions it is necessary to know their locations and the directions in which they may act.

In addition to being used to find external reactions, these equations of statics are also used to determine internal load distributions. An example of this is the example problem on a truss in paragraph 2.4.

2.3 FORCES IN SPACE (Three dimensional balances)

Structural components that have applied loads having components in three mutually perpendicular directions require a space balance. For equilibrium in three-dimensional space, six equations must be satisfied. They are:

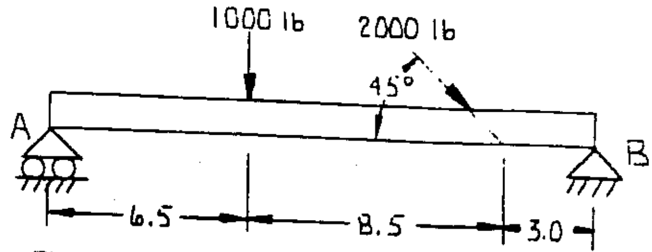
$$\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma F_z = 0$$
$$\Sigma M_x = 0, \ \Sigma M_y = 0, \ \Sigma M_z = 0$$

where the meanings of symbols are the same as in the preceding paragraph except that the summations of moment are with respect to the three axes, rather than a point.

These equations are used to determine the reactions to the loads on structural components which have loads that are not all coplanar. Because there are six equations, they can be used to solve for six reactions. Usually, the number of reactions is less than six so only a few of the equations are required. If there are more than six reactions the load balance is redundant and cannot be solved by the equations of statics alone.

2.4 EXAMPLE PROBLEMS

o Example 1 Find the reactions at points A and B for the beam shown in the sketch. Point A cannot have a horizontal reaction.



Solution:

Resolve the 2000# load into vertical and horizontal components.

$$P_h = P_v = 2000 \sin 45° = 1414\#$$

Point A cannot have a horizontal reaction. Therefore:

$$\Sigma F_h = 0 = R_{Bh} - P_h$$

$$R_{B_h} = P_h = 1414\#$$

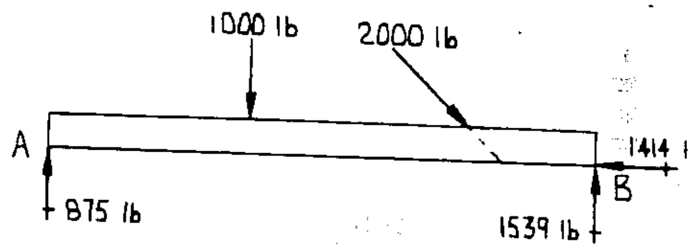
$$\Sigma M_A = 0 = 6.5 \times 1000 + 15.0 \times 1414 - 18.0\, R_{Bv}$$

$$R_{B_v} = 1539\#$$

$$\Sigma F_v = 0 = 1000 + 1414 - 1539 - R_A$$

$$R_A = 875\#$$

Note: The equation $\Sigma M_B = 0$ could have been used to solve for R_A instead of the equation $\Sigma F_v = 0$.



o Example 2 Find the reactions at points A and B for the truss shown in the sketch and find the internal loads in each of the three members intersecting at point A.

Solution:

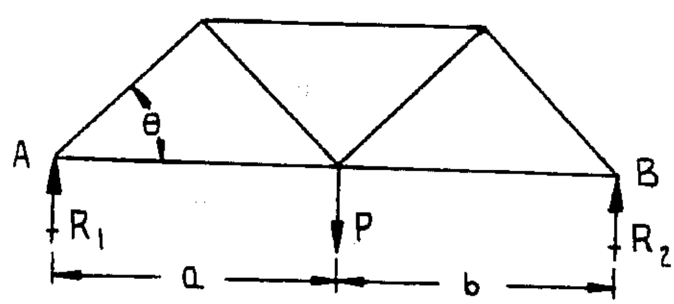
External balance:

$$\Sigma M_B = 0 = Pb - R_1\,(a+b)$$

$$R_1 = \frac{Pb}{a+b}$$

$$\Sigma F_v = 0 = P - R_1 - R_2$$

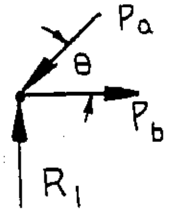
$$R_2 = P - R_1$$



Note that the equation $\Sigma F_h = 0$ was not used because there were no horizontal forces. The equation $\Sigma M_A = 0$ could have been used to find R_2 instead of $\Sigma F_v = 0$.

Internal loads at A:

At point A the vertical load R_1 is known. P_a and P_b must put this joint in equilibrium.



$$\Sigma F_v = 0 = P_a \sin\theta - R$$

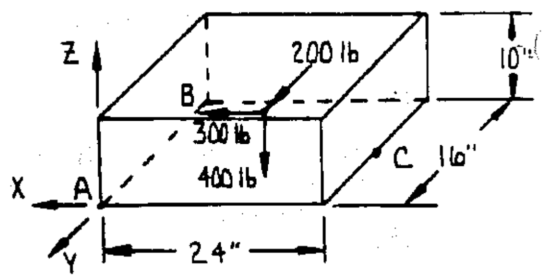
$$\underline{P_a = \frac{R_1}{\sin\theta}}$$

$$\Sigma F_h = 0 = P_b - P_a \cos\theta$$

$$\underline{P_b = P_a \cos\theta}$$

Note that the equation $\Sigma M = 0$ was not needed.

- Example 3 Find the reactions for the equipment shelf shown in the sketch. The three applied loads act at the center of the volume shown. Supports A and B cannot take reactions in the y direction and support C cannot take a reaction in the x direction.



Solution:

$$\Sigma M_y = 0 = 400 \times 12 - 300 \times 5 - 24 R_3$$

$$\underline{R_3 = 138\#}$$

$$\Sigma F_y = 0 = 200 - R_4$$

$$\underline{R_4 = 200\#}$$

$$\Sigma M_x = 0 = 400 \times 8 - 200 \times 5$$

$$- 8R_3 - 16R_2$$

$$R_2 = \frac{1}{16}\ 3200 - 1000 - 8 \times 138 = \underline{69\#}$$

$\Sigma F_z = 0 = 400 - R_1 - R_2 - R_3$

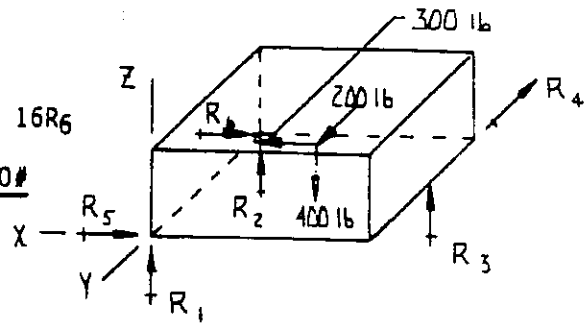
$R_1 = 400 - 69 - 138 = \underline{193\#}$

$\Sigma M_z = 0 = 200 \times 12 - 24R_4 - 300 \times 8 + 16R_6$

$\underline{R_6} = \frac{1}{16}\left(-2400 + 4800 + 2400\right) = \underline{300\#}$

$\Sigma F_x = 0 = 300 - R_6 - R_5$

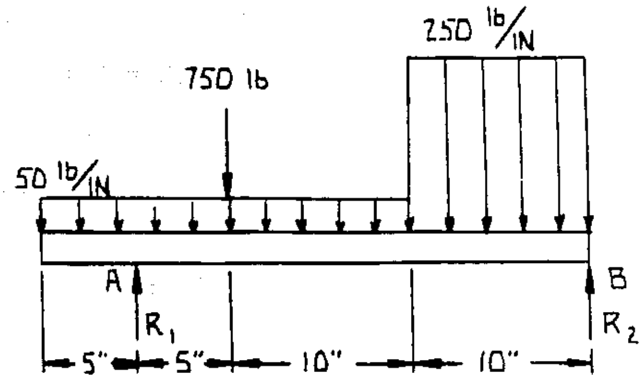
$\underline{R_5} = 300 - 300 = \underline{0}$



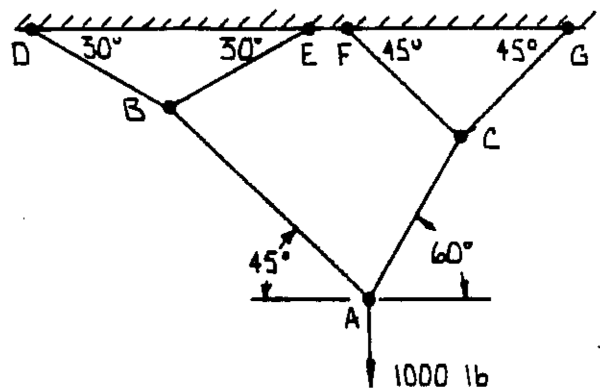
PROBLEMS

LESSON 2 REVIEW OF STATICS

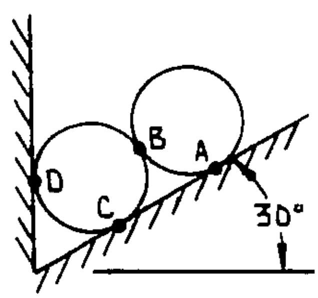
2.1 Find the reactions at the supports for the beam shown in the sketch.



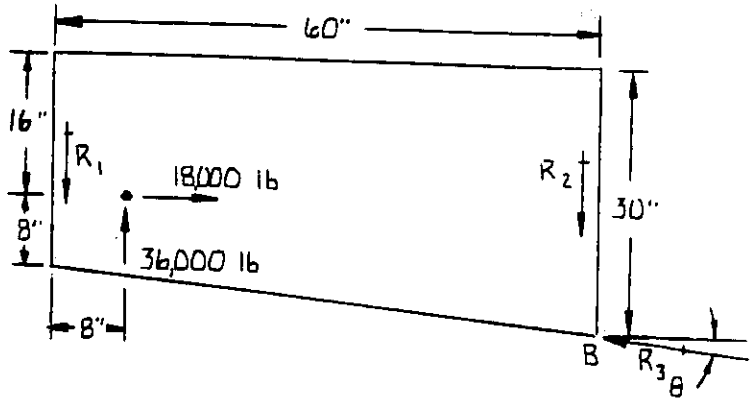
2.2 A 1000 lb weight is supported by the system of cables shown in the sketch. Find the tension in each cable.



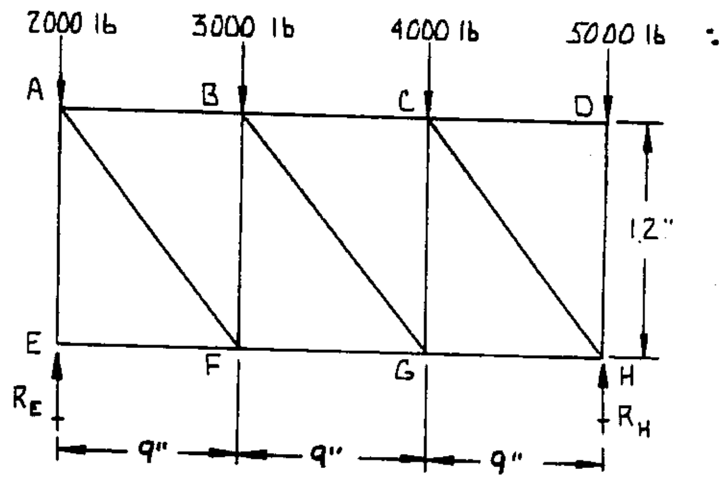
2.3 The two cylinders in the sketch are each 20 in. in diameter. The upper cylinder weighs 200 lb and the lower cylinder weighs 180 lb. Find the load at each point of contact: A, B, C and D.



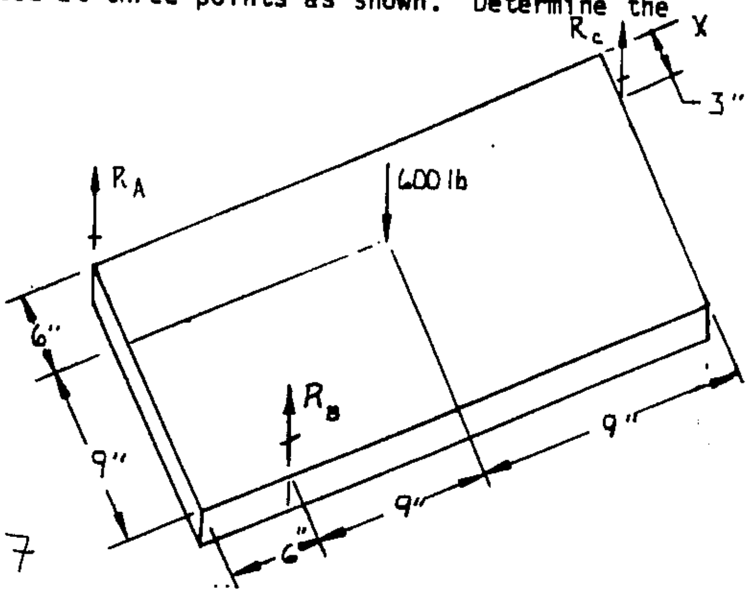
2.4 For the keel web shown in the sketch find the three reactions R_1, R_2, and R_3 for the nose landing gear loads shown.



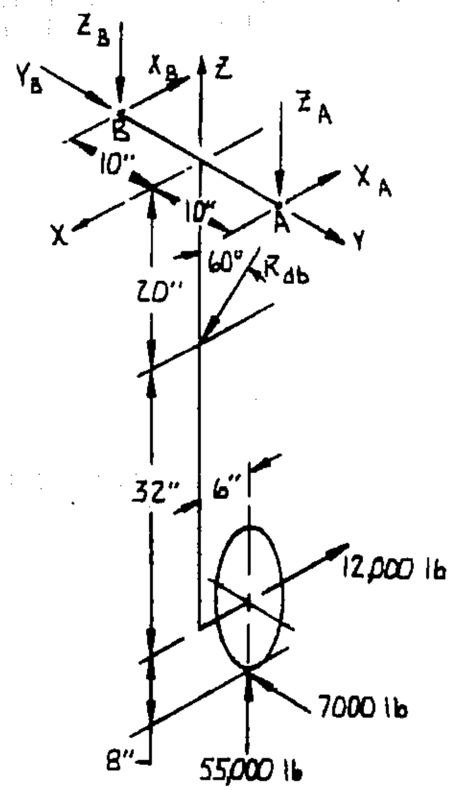
2.5 Find the reactions for the truss shown in the sketch and determine the internal loads in each member.



2.6 An equipment shelf is supported at three points as shown. Determine the reaction at each support for the load shown.



2.7 For the nose landing gear shown in the sketch, find the load in the drag brace and the vertical, drag and side reactions at each of the two trunnions (A & B). The side reaction is by bear-up so only acts on one of the two trunnions.



LESSON 3

REVIEW OF STRENGTH OF MATERIALS

3.1 GENERAL

The branch of engineering mechanics known as strength of materials is concerned with the relationship between the external forces acting on an elastic body and the internal forces and deformations caused by these forces. Some of the topics that are usually included in a course on strength of materials will be considered in detail in subsequent lessons so are not included in this lesson. These topics include columns, bending and shear stresses and torsion. Also omitted are statically indeterminate structures and deflections of beams which are beyond the scope of ABDR.

3.2 TENSION MEMBERS

It was shown in lesson 1, paragraph 1.2.1 that elongation $e = \delta/L$ within the elastic limit. And in paragraph 1.3.1 it was shown that the modulus of elasticity $E = f/e$. Equating these two expressions we have:

$$e = \delta/L = f/E$$

or

$$\delta = fL/E$$

Substituting P/A for the stress f, we have

$$\delta = \frac{PL}{AE}$$

where δ is the total deflection, P is the axial load, L is the member length that is subjected to the load, A is the area of the cross section and E is the modulus of elasticity. This equation is the standard equation for determining the total change in length of a tension (or compression) member due to an axial load.

Rearranging the equation to solve for P/A and substituting f for P/A we have:

$$f = \frac{\delta E}{L}$$

This indicates that for any given δ and L the stress, f, is proportional to E. This shows that if two or more members with different moduli of elasticity are attached together such that they will have the same deflection, the stress in each will be proportional to the moduli of elasticity. The sign convention foraxial loads and stresses is that tension is positive and compression is negative.

3.3 COMPRESSION MEMBERS

The equations and statements about tension members in the preceding paragraph 3.2 are equally valid for compression members. As for tension members, the upper limit for the stress level at which the equations are valid is the proportional limit. However, compression members frequently have a lower limit because of stability. A column may buckle and a thin section may cripple at stresses lower than the proportional limit. These modes of failure will be investigated in subsequent lessons.

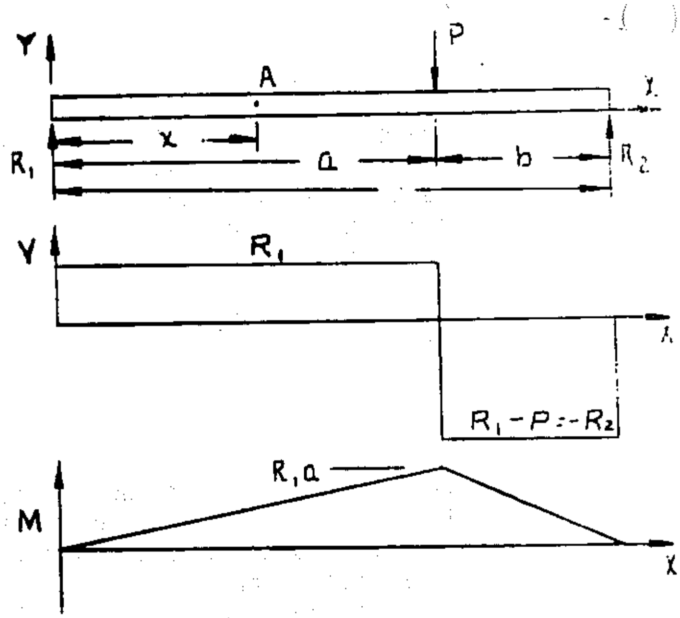
3.4 BENDING MEMBERS

The strength of bending members (beams) will be investigated in a later lesson. This lesson is concerned with determining internal loads in bending members. These internal loads include both shear and bending moment because most beams have both types of loads.

To find the critical section, or sections, of a beam, it is often helpful to draw a shear and bending moment diagram. These are plots, over the length of the beam, of the values of shear and bending moment at each point. The following subparagraphs will illustrate the derivation of shear and bending moment diagrams for various types of beams and loading. The sign convention for bending loads is that a moment that causes compression stresses in the upper portion is positive.

3.4.1 Simple Beam with Concentrated Loads

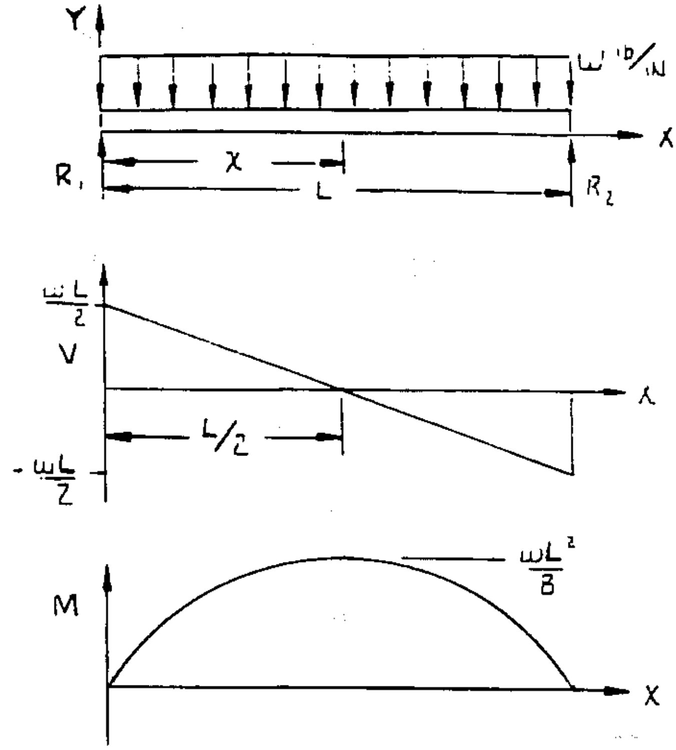
The beam in the sketch is first balanced, using the equations of statics to find the reactions. The value of shear at any section can be found by taking, as a free body, the portion of the beam between the section of interest and any point at which the shear is known. In this example, the shear at each end is equal to the reaction.



Summing the vertical forces between R_1 and point A, it is apparent that the shear at point A is equal to R_1. Summing moment about point A of the forces on this same portion of the beam we find that $M = R_1(x)$ at any section between $x = 0$ and $x = a$. This establishes the values of V and M for the length "a". The values for the length "b" are found either by balancing a portion of the beam with x greater than "a" or by considering the other end of the beam.

For a beam with two or more applied loads, the same procedure can be followed or the loads can be treated independently with the values for shear and bending moment added algebraically.

3.4.2 Simple Beam with Distributed Load



For a uniformly distributed load as shown in the sketch, the reactions are equal to $wL/2$. At any cross section, the shear and moment can be found as above. At a distance x from a support, the shear is:

$$V = R_1 - wx = \frac{wL}{2} - wx$$

and the bending moment is:

$$M = R_1(x) - wx\left(\frac{x}{2}\right) = \left(\frac{wL}{2}\right)x - \left(\frac{w}{2}\right)x^2$$

The shear and moment for a non-uniformly distributed load can be found by the same procedure. In the case of complex loading, it is helpful to know that the change in shear between two cross sections of a beam due to distributed loading is equal to the area of the load diagram between the two sections. Also, the change in bending moment between the two sections is equal to the area of the shear diagram between the two sections. Then:

$$V_2 - V_1 = \int_{x1}^{x2} wdx \quad = \text{area of load diagram between } x_1 \text{ and } x_2.$$

and

$$M_2 - M_1 = \int_{x1}^{x2} Vdx \quad = \text{area of shear diagram between } x_1 \text{ and } x_2.$$

Thus, the shear and moment at x = 6 in the sketch are found as follows:

$$V_1 = R_1 = \frac{300 \times 12}{2} = 1800\#$$

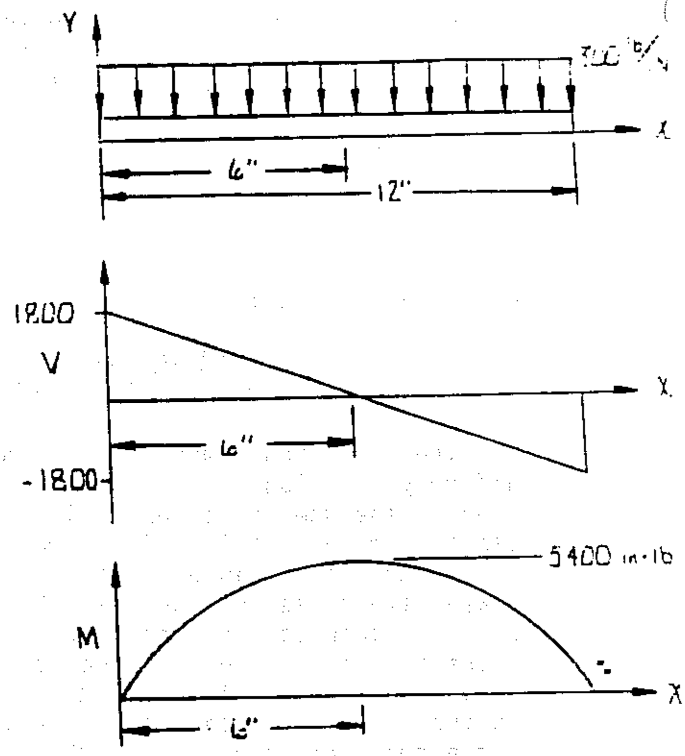
$$M_1 = 0$$

$$V_2 - V_1 = \int_{x_1}^{x_2} w\,dx = \int_0^6 300\,dx$$

$$= 300\,x \Big]_0^6 = 1800\#$$

$$V_2 - V_1 = \int_{x_1}^{x_2} w\,dx = \int_0^6 300\,dx$$

$$= 300\,x \Big]_0^6 = 1800\#$$



and

$$V_2 = V_1 - (V_2 - V_1) = 1800 - 1800 = 0$$

or, using the area A_{0-6} of the load diagram:

$$V_2 = V_1 - 300 \times 6 = 1800-1800 = 0$$

$$M_1 - M_2 = \int_{x1}^{x2} V\,dx = \int_0^6 (1800-300\,x)\,dx = \left[1800\,x - \frac{300x^2}{2}\right]_0^6$$

$$= 10{,}800-5400 = 5400"\#$$

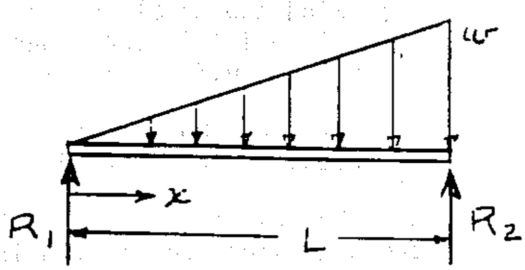
$$M_2 = -\left[M_1 - (M_1 - M_2)\right] = -(0-5400) = 5400"\#$$

or, using the area A_{0-6} of the shear diagram:

$$M_2 = \frac{1800 \times 6}{2} = 5400"\#$$

For a non-uniform distributed load as shown in the sketch, at any value of x between zero and L:

$$w_x = w\,(x/L)$$



Taking ΣF_y:

$$V_x = R_1 - \frac{wx}{L} \cdot \frac{x}{2} = R_1 - \frac{wx^2}{2L}$$

Taking ΣM_x:

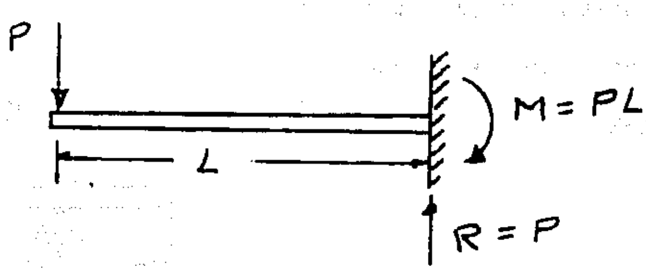
$$M_x = R_1 x - \frac{wx^2}{2L} \cdot \frac{x}{3} = R_1 x - \frac{wx^3}{6L}$$

Simple beams with concentrated loads, distributed loads or a combination of both have maximum bending moments at points of zero shear. The shear diagram has steps at applied concentrated loads and reactions and changes of slope at points of changes in the magnitude of distributed load. Bending moment diagrams have steps at applied moments and changes of slope at points where the shear changes. For distributed loads, the shear changes continuously; therefore the slope of the bending moment diagram also changes continuously, forming a curve.

3.4.3 Cantilever Beams

A cantilever beam is a beam which is "built-in" at one end (supported for shear and moment) and free (unsupported) at the other. Shear and bending moment diagrams can be drawn for cantilever beams using the same procedures as for simple beams. Note that the moment will be zero at the free end (unless a moment is applied at that point) but will not normally be zero at the built-in end.

For most loading conditions, cantilever beams have the maximum shear and bending moment at the support. Steps and changes of slope for shear and bending moment diagrams are the same as for simple beams.



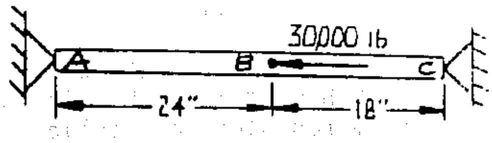
PROBLEMS

LESSON 3 REVIEW OF STRENGTH OF MATERIALS

3.1 Find the total change in length of an aluminum longeron that is 60 inches long, 1.25 in^2 in cross section and loaded with a 65,000 lb tension load. Assume $E = 10.5 \times 10^6$ psi.

3.2 Same as above except that the load is a compression load that causes a compression stress of 35,000 psi.

3.3 The bar in the sketch has a concentric load applied at point B. The cross section area is .18 in^2 and the material is titanium 6Al-4V with a modulus of elasticity of 16.0×10^6 psi. Find the reactions at points A and B and find the deflection at point B.



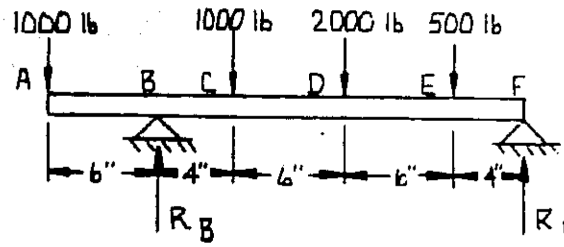
3.4 An aluminum spar cap and a steel strap work together to carry a 172,000 lb tension load. The length is 84 inches. Find the load and stress in each member and determine the margins of safety for each.

Member	Effective A	Net A*	E	F_{tu}
Spar Cap	1.50	1.35	10.4×10^6	88,000
Strap	.75	.65	29.0×10^6	160,000

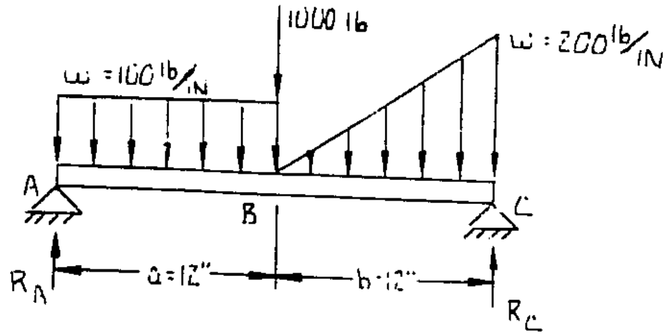
*Area through fastener holes.

3.5 Determine the required cross sectional area of a 6Al-4V titanium strap to have the same axial stiffness as a 180,000 psi heat treat 4130 alloy steel strap that is 3.75 inches wide and .156 inch thick. Assume steel $E = 29 \times 10^6$ psi and titanium $E = 16.0 \times 10^6$ psi.

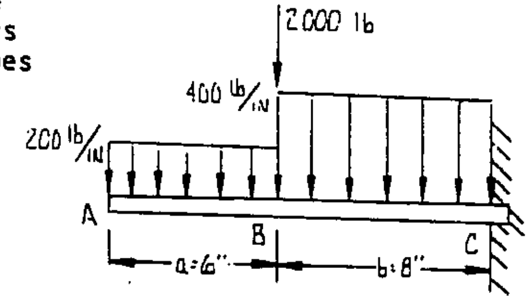
3.6 Construct shear and bending moment diagrams for the beam shown in the sketch.



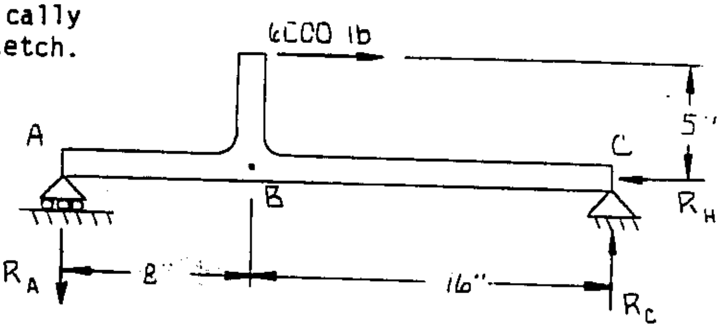
3.7 Construct shear and bending moment diagrams for the beam shown in the sketch. Find values for shear and bending moment at the centers of spans a and b. Also find shear and bending moment one inch from A and one inch from C.



3.8 Construct a shear and bending moment diagram for the cantilever beam in the sketch. Find bending moment at centers of spans a and b to establish the shapes of the curves.



3.9 Construct shear and bending moment diagrams for the eccentrically loaded simple beam ABC in the sketch.



LESSON 4

AIRCRAFT EXTERNAL LOADS

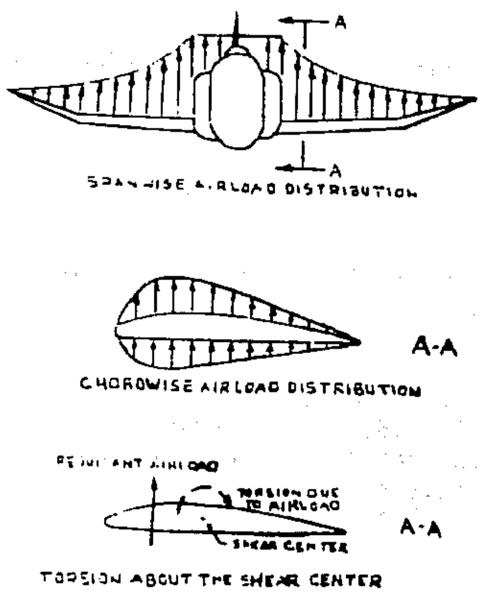
4.1 GENERAL

External loads are loads applied to the aircraft structure by outside forces. The principal types of external loads are aerodynamic and inertia loads. The other types of loads are grouped under the heading of miscellaneous loads. These three types of external loads are considered further in the following paragraphs.

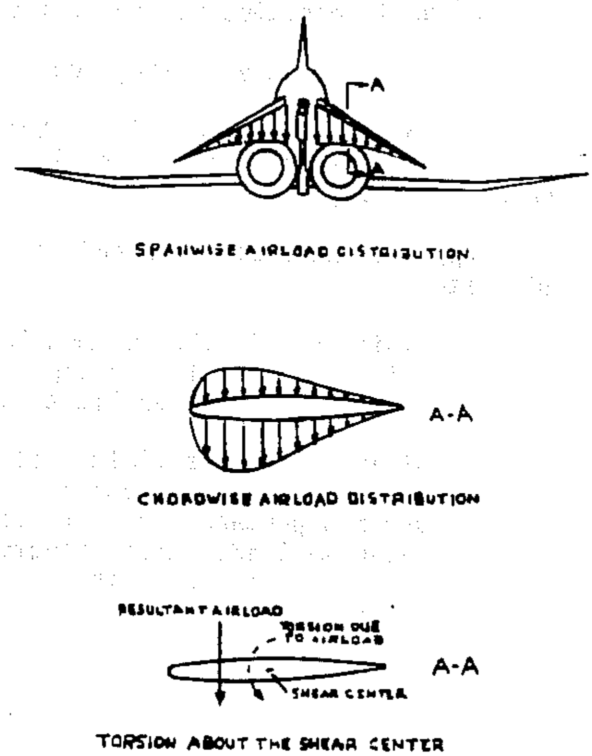
4.1.1 Aerodynamic Loads

Aerodynamic loads, often called airloads, are the loads applied to the external surface of the aircraft by the flow of air. The principal aerodynamic loads are lift and drag on lifting surfaces (wings and tail planes) and drag on the fuselage. Lift can be positive (acting upward) or negative (acting downward). The fuselage also has some lift.

The left-hand sketch below shows a typical positive airload distribution on an airplane wing. The right-hand sketch shows a typical negative airload distribution on a stabilator (horizontal tail control surface). Both spanwise and chordwise distributions are shown as well as the resulting torque.



TYPICAL AERODYNAMIC WING LOADS



TYPICAL AERODYNAMIC STABILATOR LOADS

Aerodynamic drag on the fuselage is of little concern structurally as it acts parallel to the plane of the skin. However, there are also aerodynamic pressures that act perpendicular to the skin surfaces. These vary in location, direction and magnitude depending on the attitude of the aircraft as well as speed and load factor; and they seldom exceed about 3.0 psi limit.

4.1.2 Inertia Loads

Inertia loads are loads caused by accelerations. The acceleration may be translational, caused by a constant rate of change in direction of motion, or rotational, caused by varying pitch, yaw or roll rates.

It is a basic law of physics that for each force acting on an object, there is an equal reaction in the opposite direction. When there is no reacting force available, the object will accelerate with an acceleration of $a = F/m$ where a is acceleration, F is the unbalanced force and m is the mass of the object. Similarly, unbalanced moments cause rotational accelerations.

For an airplane in flight, accelerations may be caused by maneuvering, gusts, or changes in flight speed. On the ground, they may be caused by landing, taxiing, field arrestments, etc. In any case, these accelerations cause inertia loads that balance the otherwise unbalanced load. The ratio of these accelerations to the acceleration due to gravity is called a load factor. Load factors are given in aircraft coordinates and may be positive or negative.

- o N_x is positive acting forward.
- o N_y is positive acting to the left.
- o N_z is positive acting down.

Load factors are non-dimensional but are frequently referred to as a number of g's, where g is the gravitational force. Thus, a load factor of $N_z = 6.5$ can be called "6.5g down".

Applied external loads and load factors may be specified as limit or ultimate.

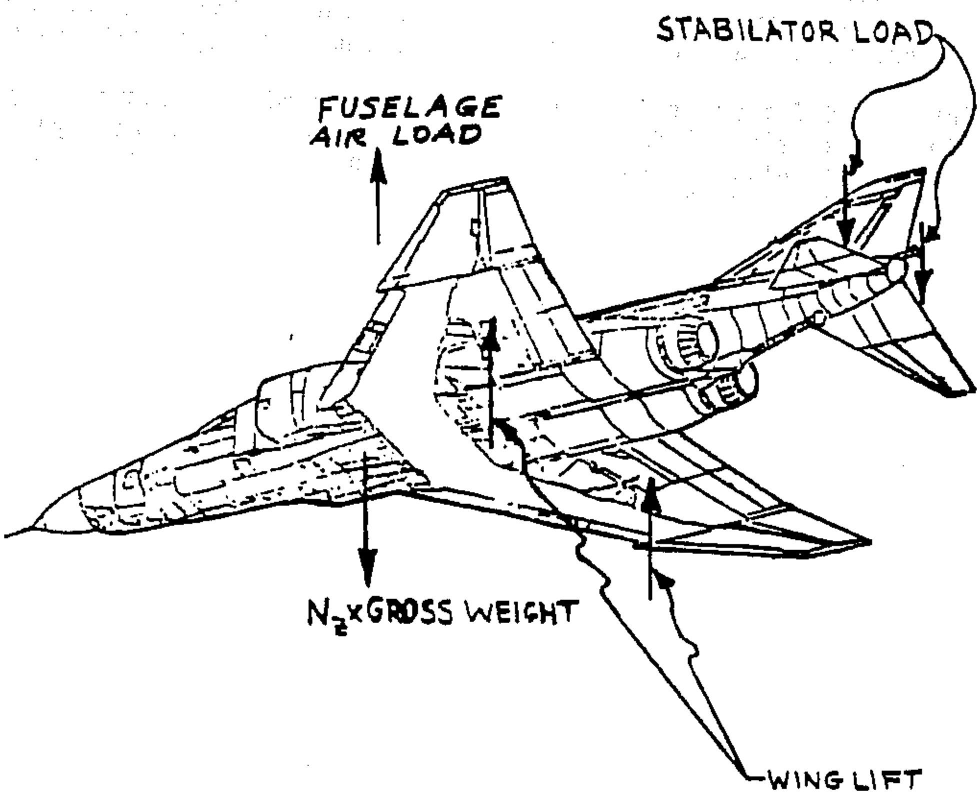
- o Limit load is the maximum load the airplane (or load application point) is expected to experience. The structure must not sustain permanent set under limit load.
- o Ultimate load is limit load times a safety factor. For most aircraft, including the F-4, the safety factor is 1.5. The structure may sustain permanent set if limit load is exceeded but must not fail at ultimate load.. Most structural materials have a yield strength greater than 2/3 of the ultimate strength so that the ultimate strength limitation usually prevails.

4.1.3 Miscellaneous Loads

In addition to aerodynamic and inertia loads, there are several other sources of loads applied to the aircraft structure. These include, but are not limited to, pressure, pilot effort, engine thrust, actuator loads, canopy and seat ejection, and crash. (The latter is actually a special case of inertia load.) Some of these miscellaneous loads will be discussed further in subsequent paragraphs.

4.1.4 Aircraft Load Balance

At any time, the forces acting on an airplane must be in static equilibrium. Therefore, the aerodynamic loads (and/or ground loads) and engine thrust must be balanced by inertia loads. A typical balance is shown in the sketch.



4.2 DESIGN CONDITIONS

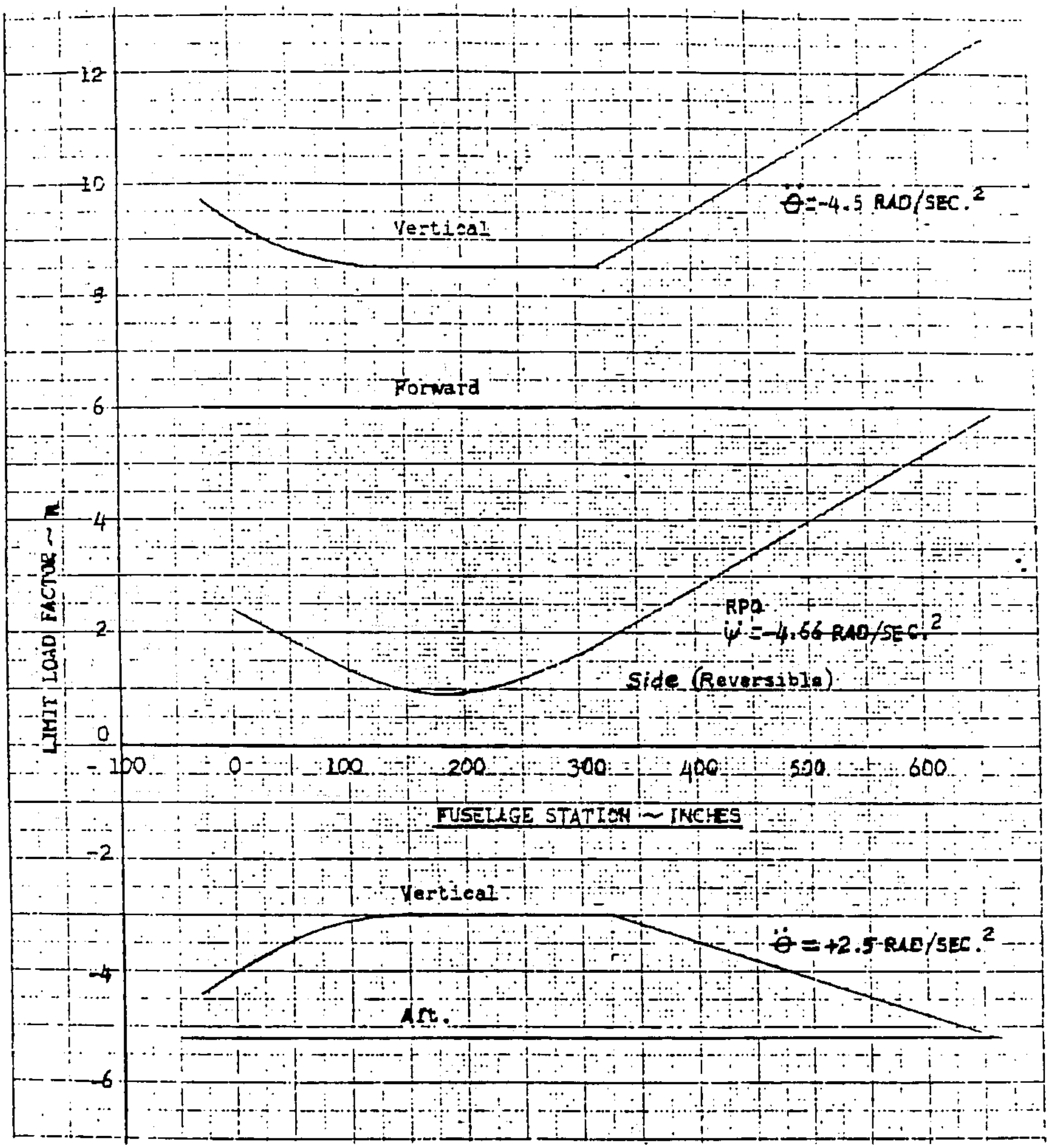
A design condition is a set of circumstances that produce a critical load on some part of the aircraft. For flight conditions, these circumstances include speed, altitude, weight, weight distribution, load factor and rotational acceleration. Ground conditions include catapulting, arrested landing, field landing, taxiing, towing, hoisting, jacking and crash. For these conditions, the circumstances include weight and weight distribution. Other circumstances are catapult tow load for catapulting, sink speed for landing, field roughness for taxiing and towing and load factor for hoisting, jacking and crash.

4.2.1 Load Factors - When an airplane is being designed, the maximum load factors are specified by the licensing agency. For the F-4, the limit load factors are:

- N_x = +6.0 (forward); -5.2 (aft)
- N_y = $\pm$0.9 (side)
- N_z = +8.5 (down); -3.0 (up)

These load factors are the maximum for the airplane as a whole. However, when a design condition includes rotational acceleration, the linear acceleration increases with distance from the axis of rotation. This results in higher load factors at the airplane extremities as shown on pages 4-5 and 4-6.

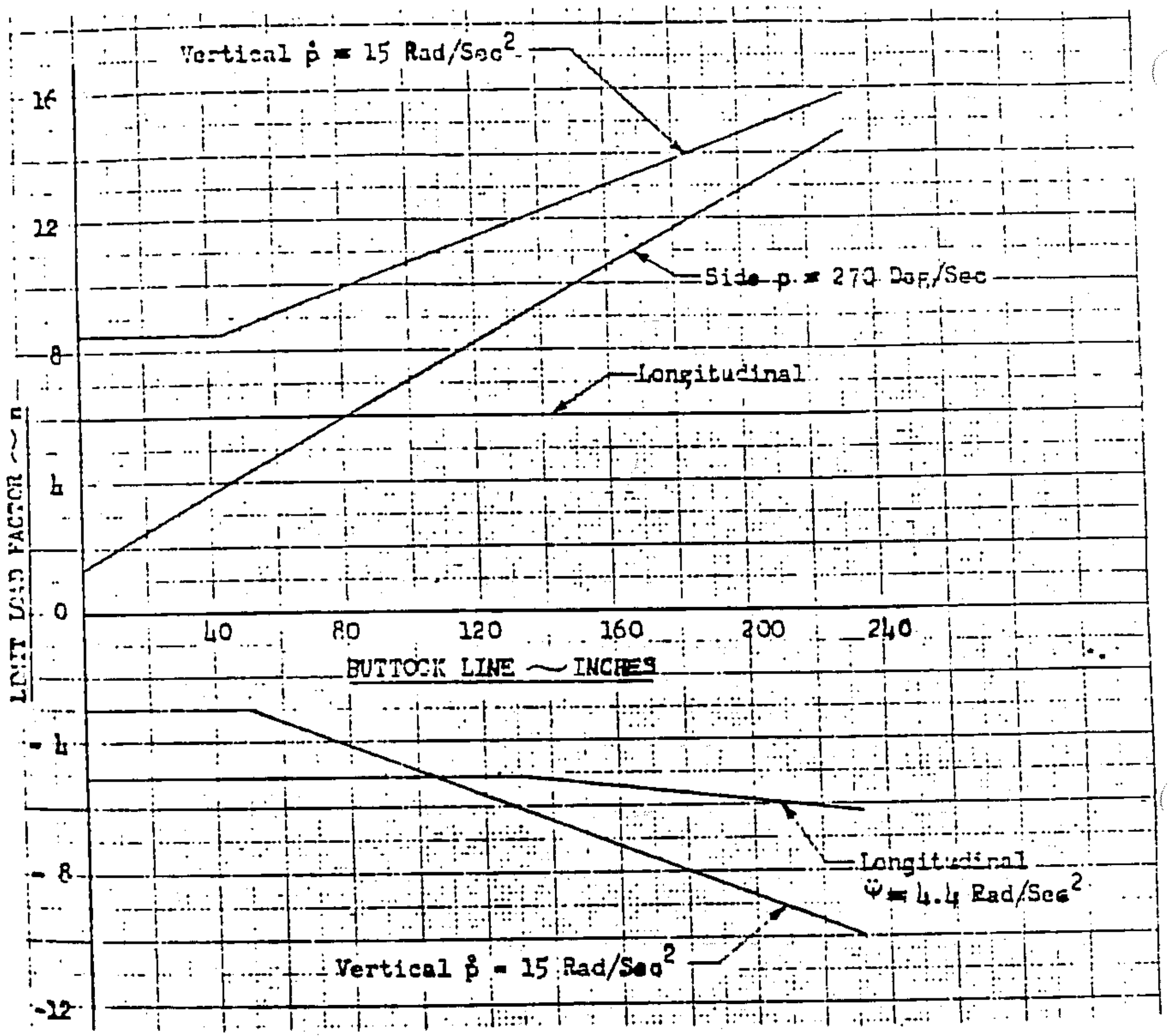
Crash load factors for the F-4 are 40g forward, 40g acting at any angle up to 20° either side of forward and 20g down (not simultaneously). These load factors are ultimate and apply only to the crew seat and items in and directly behind the cockpit.



LIMIT LOAD FACTORS FOR MASS ITEMS IN THE FUSELAGE

$\ddot{\theta}$ is pitch acceleration

$\ddot{\psi}$ is yaw acceleration



LIMIT LOAD FACTORS FOR MASS ITEMS IN THE WING

p is roll velocity

$\dot{p}$ is roll acceleration

$\ddot{\psi}$ is yaw acceleration

4.2.2 Pressures - The entire external surface of the airplane is subject to aerodynamic pressures in flight. These vary from about 3 psi limit (4.5 psi ultimate) on portions of the fuselage skin to significantly higher pressures on lifting surfaces. Most of these pressures are actually aerodynamic loads.

Less obvious are the internal pressures which act on other portions of the structure. These include:

- The cockpit area floors, bulkheads and skins are designed for cockpit pressurization of 11.0 psi ultimate.
- The engine air duct skins are loaded by a pressure as high as 32 psi limit (48 psi ultimate) during engine stall.
- The engine compartment has an internal pressure of 20 psi ultimate.
- Fuel cell floors, liners and bulkheads have a pressure of 3.75 psi due to fuel cell pressurization plus additional pressure due to load factors acting on the fuel. The total pressure exceeds 20 psi ultimate in some areas.

4.2.3 Pilot Effort Loads

All control system parts that are loaded by pilot effort are designed to the loads shown in the table on page 4-8. Except for rudder controls, the parts are designed to the loads of Air Force Specification MIL-A-008865 shown in the third column. Rudder system components are designed to the more conservative load in the first column labeled MCAIR.

No.	Limit Loads					Operating Load Conditions	
	Control	MCAIR	FAA (Transports)	Air Force Specification MIL-A-008865A	Navy ASG Specification MIL-A-8865A	Maximum Operating Load (Not a Limit Load)	Allowable Breakout Forces MIL-F-878
1	Aileron Stick — Load		100 lb Max 40 lb Min	100 lb (4)	100 lb	25 lb (3)	½ - 4 lb
2	Elevator Stick — Load		250 lb Max 100 lb Min	250 lb (4)	200 lb	50 lb (3)	½ - 5 lb
3	Control Column (Wheel Type) Side Load — Load Applied Independently			100 lb (4)	100 lb	40 lb (3)	½ - 6 lb
4	Control Column (Wheel Type) Fore & Aft — Load		300 lb Max 100 lb Min	300 lb (4)	200 lb	75 lb (3)	½ - 7 lb
5	Wheel (Torque) — D — Load		80 D in.-lb Max 40 D in.-lb Min	180 D (4)	180 D	50 R in.-lb (3)	
6	Rudder Pedal — Load	450 lb	300 lb Max 130 lb Min	300 lb on Each Pedal Simultaneously or Separately	300 lb on Each Pedal Simultaneously or Separately	175 lb (3)	1-14 lb
7	Simultaneous Operation (Dual Controls)		75% Load (1) from Pilot & Co-Pilot	75% Load from Pilot & Co-Pilot	75% Load from Pilot & Co-Pilot		
8	Foot Brakes (Toe Type) — Load		Max Load Pilot is Likely to Apply (Use Mil. Reqmt)	300 lb on Each Pedal Simultaneously or Separately	300 lb on Each Pedal Simultaneously or Separately		
9	Flap, Trim, Stab., Spoiler, Landing Gear & Nose Wheel (At ℄ of Knob)		$F\ 50\left(\frac{1+R}{3}\right)$, But Not Less Than 50 lb Nor More Than 150 lb (2)	$F = 50\left(\frac{1+R}{3}\right)$, But Not Less Than 50 lb Nor More Than 150 lb (2)	$F = 50\left(\frac{1+R}{3}\right)$, But Not Less Than 50 lb Nor More Than 150 lb (2)	25 R	
10	Engine & Accessory Control Levers (At ℄ of Knob)	150 lb to Stop and Control System (2)				7.5 lb at Knob 30 lb Max for Simultaneous Operation of 4 Levers	
11	Twist Controls (At ℄ of Knob)		133 in.-lb	133 in.-lb	133 in.-lb		
12	Small Knob Push or Pull			100 lb	100 lb		

(1) When pilots are working in opposition: –75% load from pilot and co-pilot, but not less than minimum pilot and co-pilot load.

(2) Applicable to any angle within 20° of planes of control.

(3) MIL-F-8785.

(4) Combine lateral and longitudinal stick loads in most critical direction except the total resultant shall not exceed 300 lb and the lateral component shall not exceed 150 lb.

CONTROL SYSTEM PILOT EFFORT LOADS

PROBLEMS

LESSON #4 AIRCRAFT EXTERNAL LOADS

4.1 In which direction does the applied load act on a mass item for each of the following load factors:

a.) $+N_z$

b.) $+N_x$

c.) $+N_y$

4.2 Determine the sign and magnitude of the load factor N_z for a manuevering airplane with a weight of 37,500 lbs., an airload on the wings of 252,000 lbs. acting upwards, an airload on the stabilator of 40,000 lbs. acting downwards and an airload on the fuselage of 31,750 lbs. acting upward.

4.3 What is the maximum limit design load factor not including crash safety, in each of the six directions for mass items in the following locations in an F-4 aircraft.

a) On the forward cockpit console at F.S. 180.

b) In the nose fuselage at F.S.0.

c) In the aft fuselage at F.S.650.

d) In the wing at B.L.60.

e) In the wing at B.L.220

4.4 For which of the locations listed in problem 4.3 would the crash load factors apply and what would the magnitudes and directions of the ultimate crash load factors be?

4.5 What is the highest design ultimate pressure for any portion of the F-4 structure, what causes this high pressure, and on what portion of the structure does it act?

4.6 What are the design limit and ultimate pilot effort loads for the F-4 on each of the following controls.

a) A rudder pedal.

b) A foot brake (on the rudder pedal).

c) The control stick (forward acting load).

d) The control stick (side load).

LESSON 5

INTERNAL LOADS

5.1 DEFINITIONS

Internal loads are the loads in structural members as they carry externally applied loads to their external reactions or inertia balance. These internal loads are the loads of concern to an engineer designing structural repairs.

The internal loads in a structural member may be one or more of the following:

- o Bending moment in beams
- o Torsion in torque boxes
- o Tension in axial members
- o Compression in axial members
- o Shear in beams and shear webs

5.1.1 Beams

A beam is a structural member that is loaded primarily in bending (and usually also in shear) as it carries its applied loads to one or more reactions. Most structural members that carry internal loads are some form of beam:

- o Wing and empennage spars are cantilever beams. The skins are part of these beams because they act as part of the beam caps.
- o Ribs that are between two spars are simple beams while leading edge and trailing edge ribs are cantilever beams.
- o The fuselage itself is a beam with wing and tail loads being reacted by a distributed inertia load.
- o Rings and frames are curved beams.
- o Most bulkheads are beams.
- o Cockpit sills are continuous beams on several supports.
- o Many shear webs and intercostals are actually beams.

5.1.2 Torque Boxes

Most major structural assemblies are also "torque boxes", i.e., they are (non-circular) cylindrical members carrying torsion as shear flow in the skins and webs.

- o Wings and empennage carry torsion as shear flow in the skins and spars.

- o A fuselage is a torque box carrying torsion as shear flow in the moldline skins, floors, shear webs and sometimes duct skins. Usually, where the structural skins are interrupted by cockpit closures and nose landing gear doors, there is no torque box.

5.1.3 Axial Members

Axial members are structural members which carry primarily tension and/or compression. Most axial members are actually caps of bending beams:

- o Spar caps carry the axial loads caused by bending moment in the spar.
- o Fuselage longerons carry axial loads due to bending moment on the fuselage.
- o Ribs, frames, rings and bulkheads have caps that carry axial loads due to bending.

Axial members that are not caps of beams are usually members that pick-up a concentrated load and distribute it into a shear web. Examples include:

- o Stringers that carry engine thrust.
- o Stiffeners that back-up hydraulic actuators.
- o Members that distribute longitudinal loads from ejection seat tracks into shear webs.
- o Keel web members that carry nose landing gear loads.

5.1.4 Shear Webs

Panels that are primarily loaded by shear are called shear webs. These includes:

- o Spar and rib webs
- o Cockpit and fuel cell floors
- o Keel webs
- o Frame, ring and bulkhead panels
- o External skins

The shear carried by shear webs can be due to applied shear loads on a beam or torsion on a torque box. Frequently it is a combination of the two because much of the structure acts as both a beam and a torque box.

In thick walled sections, the shear stress is not uniform over the height of the section but when the shear web is thin, in comparison to other section dimensions, the shear stress is, for practical purposes, uniform over the distance between centroids of edge members. Essentially all F-4 structure is in the latter category. For convenience in determining internal loads the shear force is converted to shear flow with units of pounds per inch. Shear flow is denoted by the letter q.

$$q = v/h$$

where q is shear flow, V is total shear load and h is the height of the beam or shear panel between cap centroids. Shear flow due to torsion will be explained in Lesson 14 "Torsion".

5.2 INTERNIAL LOADS IN SPECIFIC STRUCTURAL ITEMS OF THE F-4

Each item of structure has one or more specific functions and must be able to carry the internal loads incurred in performing that function. The first requirement for designing a structural repair is to learn the function of, and internal loads carried by, the damaged part.

5.2.1 Center Section Wing Main Box

This box is the structure inboard of the wing fold between the front (14.50%) and main (40.475%) spars. It carries the wing primary shear, bending and torque. The major componets are:

- o Upper and lower skins carry most of the compression and tension due to wing bending. They carry shear due to wing torsion and drag. They also carry normal loads due to aerodynamic and fuel pressures on the skins to the spars and ribs.
- o Front and main spars carry bending due to wing bending and shear due to wing shear and torque. The main spar also picks up loads from the main landing gear trunnion.
- o The intermediate (29.70%) spar carries shear due to wing shear and torque and acts as a fuel baffle. It also carries tension perpendicular to the skins due to fuel pressure on the skins and stabilizes the compression skin against buckling.
- o Post beam posts at 22.75% and 35.60% stabilize the compression skin and carry tension due to fuel pressure on the skins.
- o The BL 0.00 rib caps splice the upper and lower skins and the rib web carries shear due to vertical kick loads from skin axial loads. It also acts as a fuel baffle.
- o The torque rib near the fuselage side wall is loaded in shear due to kick loads caused by the change in sweep angle of the skin axial loads. It also acts as a fuel baffle.
- o The landing gear actuator rib is a sheet metal rib loaded only in shear. It also acts as a fuel baffle.
- o The landing gear rib carries shear and bending, mainly due to loads from the outboard external store pylon.
- o The fold rib at BL 160.00 transfers the outer wing shear, bending moment and torque from the outer wing fold rib into the main box spars and skins and the rear spar. It also carries loads from the wing aft structure into the main box and reacts fuel pressure. It is loaded in shear and bending.

5.2.2 Center Section Wing Aft Structure

The portion between the main and rear spars is open on the bottom, inboard of about BL 117, to allow retraction of the main landing gear. In this region, the landing gear actuator rib at BL 82.4 supports both the landing gear side brace actuator and the uplatch. It is loaded in shear, bending and torsion. The upper skin in this region carries aerodynamic loads and shear. At the outboard end of this bay is the landing gear rib. It picks up the landing gear drag load from the MLG and most of the aft MLG trunnion vertical load from the rear spar and carries them forward to the main box. The rib is loaded in shear, bending and axial load.

Outboard of BL 117 in this same bay the structure is a closed torque box. It carries loads from the trailing edge structure forward to the main box. The skin is loaded by aerodynamic pressure and by shear. The ribs are loaded in shear and bending.

The portion from the rear spar aft is a closed torque box. It carries aerodynamic loads, including those on the T.E. flap and aileron to the main box and fuselage. The rear spar carries loads from the wingfold rib, the aft landing gear trunnion and the trailing edge structure to the ribs and fuselage. The rear spar and aft beam react torque in the aft box as couple loads at FS 359 and FS 400. The ribs carry aerodynamic and control surface loads to the rear spar and aft beam and are loaded in shear and bending moment. The skins are loaded in shear and by aerodynamic pressures.

5.2.3 Outer Wing Structure

The outer wing structure consists of a main box from 12.00% to 41.70% chord, an aft secondary box from 41.70% to 70.70% and a honeycomb trailing edge. The only loads on the outer wing are aerodynamic and inertia loads, and on slatted airplanes, L.E. slat loads.

The main box skins carry tension and compression due to wing bending, shear due to torsion, and aerodynamic pressures. The spars and ribs carry shear and bending and also stabilize the compression skin against buckling.

The aft secondary box skin carries only shear and aerodynamic pressure. The skin stringers stabilize the skins against buckling in shear and cracking due to buffet. The spar at 60% carries shear and bending to the fold rib. The ribs are loaded in shear and bending due to airloads on this box and the honeycomb.

The honeycomb trailing edge panel carries both shear and bending due to aerodynamic pressure.

5.2.4 Fuselage

The fuselage is a beam, torque box and axial member carrying airloads, inertia loads and empennage loads to the wing and to the inertia balancing loads.

Longerons and, to a lesser extent stringers, carry tension and compression loads due to fuselage bending and most large forward or aft acting external loads on the fuselage. Longerons loaded in compression are generally not critical as columns because attached skins, floors, keel webs and frames preclude column buckling. Some longerons also are loaded in bending and shear. The upper longeron in the cockpit area is the outer cap of the cockpit sill beam which has lateral bending and shear due to cockpit pressurization.

Stringers serve a variety of purposes. They sometimes carry axial tension and compression like longerons, serve as sills for doors, frame around non-structural doors, distribute locally applied loads and pressures and divide the skins into smaller panels, permitting them to carry higher shear and compression stresses without buckling. In addition to axial loads they frequently carry small bending moments and shears due to pressure and local loads.

Bulkheads have three basic purposes and any one bulkhead may serve from one to all three. The three purposes are:

- o Distribute large applied loads into the shear and bending structure of the fuselage. Typical applied loads are loads from the nose landing gear, crew seats, wing attachment, arresting hook and empennage.

- o Redistribute loads where there is a change in the fuselage shear or torque load paths.

- o Distribute loads that are applied perpendicular to the plane of the bulkhead to the fore and aft load carrying members. These loads are frequently due to pressure such as cockpit or fuel pressure.

Bulkhead webs are loaded in shear and sometimes by pressure loads. Caps and stiffeners carry tension and compression and stiffeners sometimes also carry bending moment and shear due to normal loads.

Frames, sometimes called formers (and frequently called rings, if they are continuous around a section of approximately circular cross section), serve several purposes. Their main function is to distribute applied loads into the fuselage bending and shear structure like a bulkhead. Frames are used instead of bulkheads when the loads are small or internal requirements preclude using a bulkhead. Applied loads on frames can come from any of many sources including equipment supports, actuators, engine mounts, external or internal pressure and control system supports. They also divide skins into smaller panels and stringers into shorter lengths, increasing the buckling strength of both. Most frames are loaded in bending moment and shear. When loaded by pressure, they also sometimes carry tension.

External skins carry shear due to fuselage shear and torque. They frequently also carry tension and compression stresses due to fuselage bending. Skins also carry pressure loads to the frames and stringers.

Floor webs carry shear loads, particularly those due to side shear on the fuselage. They also carry locally applied shear loads such as those from equipment inertia, NLG drag brace, seat rails, etc., to the axial members. Floor webs also frequently carry normal loads and pressures to the floor stiffeners which carry them to the fuselage vertical shear structure.

Internal webs, such as keel webs, carry shear due to fuselage shear and torque. They also distribute locally applied loads from the nose landing gear, engine mounts, arresting gear, actuators, etc., and sometimes, like floors, react normal loads and pressures.

Intercostals are local shear webs or beams between two other structural members, such as frames and bulkheads, that usually also attach to the skin or a stringer. Intercostals carry shear and, sometimes, bending moment.

5.2.5 Empennage

The empennage consists of the horizontal and vertical tail planes. These are made up of three distinctly different structures: the horizontal stabilator, vertical fin and rudder.

The horizontal stabilator is built very much like the center section wing. The main box spars and thick skins carry the spanwise shear and bending and most of the torsion, as for the wing. The leading edge structure carries aerodynamic loads to the main box by shear and bending moment in the ribs and spanwise members and by shear (due to torsion) in the skins. The trailing edge structure aft of the main box is honeycomb which carries applied air loads forward to the main box as shear in the honeycomb core and tension and compression in the skins due to the resulting bending moment. The skins also carry shear due to torsion. The inboard portion of the stabilator is made of steel and titanium because of elevated structural temperatures from the engine exhaust.

The vertical fin is of conventional thin-skin construction. The spars and ribs are loaded in bending and shear due to air loads on the fin and rudder. The skin is loaded by shear due to torque and by pressure loads. The skin is also loaded in tension due to fin bending but carries very little compression because the buckling stress of the thin skins is low.

The rudder structure carries the rudder aerodynamic loads to the hinges and actuator. The honeycomb trailing edge panel carries shear, bending moment and torsional shear like the stabilator honeycomb. The ribs are loaded in shear and bending. The forward spar is also loaded in shear and bending, as it carries the applied loads to the hinges, and shear due to torque. A torque tube carries the torsion from the lower rib to the actuator. It also carries shear and bending.

PROBLEMS

LESSON 5 INTERNAL LOADS

5.1 List the types of load that might be carried by each of the following types of structure:

a) Beams

b) Torque boxes

c) Axial members

d) Shear webs

5.2 In which one or more of the above structure types does each of the following F-4 components belong?

a) Center section wing main box

b) Fuselage

c) Vertical fin

d) Bulkhead

e) Fuselage longeron

f) Cockpit floor

g) Wing spar

h) Cockpit sill

i) Wing main box upper skin

5.3 For each of the following F-4 structural components tell what types of external loads are applied and what types of internal loads these produce:

a) Inner wing front spar

b) Fold rib at B.L.160.00

c) Wing main box lower skin

d) Leading edge wing ribs

e) Outer wing honeycomb trailing edge panels

f) Cockpit frames between the cockpit floor and sill

g) Engine air duct rings

h) Bulkhead at F.S.77.00

i) Center fuselage frames

j) Rudder ribs

k) Rudder trailing edge honeycomb panel

l) Stabilator main box lower skin

LESSON 6

STRESS AND MARGIN OF SAFETY

6.1 DESIGN STRESS

In Lesson 5, "Aircraft Internal Loads" we discussed the various types of internal loads. Invariably, they were bending moment, tension, compression, shear and torsion. Each internal load produces stress in the member carrying the load. Tension, compression and shear loads cause tension, compression and shear stresses respectively. Bending moment is carried as a couple, consisting of a tension load and a compression load, so it produces tension and compression stresses. Torsion produces shear stresses.

For all three kinds of stress the magnitude of the stress is equal to the magnitude of the load divided by the effective area $\left(f = \frac{P}{A}\right)$. For individual pieces of structure loaded in shear, bending and/or torsion there are many other formulas for calculating stress levels. These formulas take into account the section properties of the member carrying the load. Some of these formulas will be considered in later lessons.

The fourth kind of stress of interest is bearing stress. This is the stress or pressure acting on the cylindrical surface of a fastener hole due to load transfer from the fastener shank. For analysis purposes, the stress is considered to be equal to the load divided by the projected bearing area, where the projected bearing area is the diameter times the thickness $\left(f = \frac{P}{Dt}\right)$.

6.2 LIMIT AND ULTIMATE STRESS

The terms "limit" and "ultimate" have the same meaning for stresses that was given for loads and load factors in Lessons 1 and 4. Limit stress is the actual stress acting on an item of structure. Design limit stress is the maximum limit stress expected at a given point. The limit stress is not allowed to exceed the yield strength of the material.

Ultimate stress is equal to the limit stress times an "ultimate factor", sometimes called a safety factor. The factor for the F-4 is 1.50. The ultimate stress is not allowed to exceed the failure strength of the material. Stress analysis is performed using ultimate loads and stresses except for special cases such as fatigue analysis or deflection calculations.

6.3 ALLOWABLE STRESS

An allowable stress, as the name implies, is the maximum stress that can be safely carried. Exceeding that stress level could cause rupture, permanent set, collapse or permanent buckles depending on the kind of stress and the type of structure. Allowable stresses fall into two general groups: mechanical properties and allowable stresses that depend on geometry.

6.3.1 Mechanical Properties

The mechanical properties of a material depend only on the alloy, heat treatment and to some extent on the form by which the material was produced (rolled, forged, cast, extruded, etc.). These properties include the yield and ultimate strengths in tension, compression, shear and bearing and the modulus of elasticity. These values are not affected by the geometry of the member and are found in MIL-HDBK-5 "Military Standardization Handbook - Metallic Materials and Elements for Aerospace Vehicle Structures". For elevated temperatures this handbook includes plots of factors to apply to the tabulated room temperature mechanical properties to obtain the properties at elevated temperatures.

MIL-HDBK-5 has tables of mechanical properties for a number of aluminum, steel, stainless steel and titanium alloys, as well as some other metals not used for F4 structure. For some of the metals, particularly aluminum, the mechanical properties vary somewhat for different types of stock (sheet, plate, extrusion, bar, forging, hand forging, etc.) and for different thicknesses. This, along with other information in MIL-HDBK-5 that is not applicable to ABDR, make that handbook too bulky and time consuming to be used for ABDR. For use in this course, and (until better data is made available) for use in designing ABDR repairs, the table at the end of this lesson can be used to find mechanical properties.

The properties given in the table are taken from MIL-HDBK-5 for the following materials:

- 2024 sheet and plate: 2024-T4 alclad, t less than .063.
- 2024 extrusion: 2024-T4, T3510, T3511, t less than .250.
- 2024 tube: 2024-T3, all thicknesses.
- 7178 sheet and plate: 7178-T6 or T62, t .045 to .249.
- 7178 extrusion: 7178-T6, T6510 or T6511, t .062 to .249.
- Ti-6Al-4V: annealed, t .045 to .249.
- Alloy steel: AISI 4130, 4340, 8630, 8740, etc., all thicknesses.
- Corrosion resistant steel: AISI 301 stainless steel sheet, all thicknesses.

6.3.2 Allowable Stresses That Depend on Geometry

Allowable stresses can never be higher than the mechanical properties but they can be lower due to instability. Instability includes panel buckling in shear or compression, column buckling and crippling. Methods for determining these allowable stresses are found in company stress manuals and some engineering textbooks.

At McDonnell Aircraft Company the manuals used are Report 339 "Structures Handbook" and Report 338 "Structural Analysis Bulletins". These manuals will be referred to extensively in subsequent lessons and can be used in working the homework problems.

6.4 ALLOWABLE LOADS

For some types of structure it is more convenient to work with allowable loads than with allowable stresses. Generally this is the case for structural items with stress distributions that are complex or nonuniform and/or difficult to determine but are common enough to require a fast, accurate method of analysis. This includes mechanical fasteners, tension clips, lugs, columns, and torsion members. Methods of determining these allowable loads are found in the same sources as are allowable stresses that depend on geometry. Some of these will be considered in later lessons.

6.5 MARGIN OF SAFETY

In Paragraph 1.1 it was explained that the 1.5 factor applied to limit loads and stresses to obtain ultimate loads and stresses is a safety factor although it is more commonly called an "ultimate factor". The internal loads to which the aircraft structure is designed and tested include this factor.

Any strength capacity above ultimate load produces a margin of safety. This margin, abbreviated M.S., is defined as:

$$M.S. = \frac{\text{allowable load or stress}}{\text{ultimate load or stress}} - 1.0$$

Thus, $M.S. = \frac{P_{all_{ult}}}{P_{ult}} - 1.0$ or $M.S. = \frac{F_{ult}}{f_{ult}} - 1.0$

The static strength is adequate if the margin of safety is equal to zero or greater.

MECHANICAL PROPERTIES

	2024 Aluminum			7178 Aluminum (2)		Ti-6Al-4V	Alloy Steel All Wrought Forms		Corr. Res. Steel AISI 301 Sheet	
	Sheet & Plate	Extrusion (1)	Tube	Sheet & Plate	Extrusion (3)	Sheet (4)	125 ksi	200 ksi	1/2 Hard	Full Hard
F_{tu}	59	61	66	86	88	139	125	200	151	185
F_{ty}	38	47	45	76	80	131	103	176	105	142
F_{cy}	38	38	45	80	78	138	113	181	72	98
F_{su}	37	31	40	52	44	81	75	120	82	100
$F_{bru}(\frac{e}{d}=1.5)$	97	90	99	129	126	204	194	272	-	-
$F_{bru}(\frac{e}{d}=2)$	121	114	126	163	157	261	251	355	304	372
E ($X10^{-6}$ psi)	10.5	10.8	10.5	10.3	10.4	16.0	29.0	29.0	26.0	260
G ($X10^{-6}$ psi)	4.0	4.1	4.0	4.0	4.0	6.2	11.0	11.0	11.5	11.0
	.33	.33	.33	.33	.33	.31	.32	.32	-	-

(1) Use for all 2024 wrought forms not shown on the chart.

(2) Can be used to conservatively determine strength of 7075, which is not quite as strong.

(3) Use for all 7178 wrought forms not shown on the chart.

(4) Use for all titanium alloys and wrought forms.

5 All stresses are ln ksi

PROBLEMS

LESSON 6 STRESS AND MARGIN OF SAFETY

6.1 List the principal types of stress carried by aircraft structure.

6.2 What types of stress would there be in a simple beam loaded in shear and bending?

6.3 Is permanent set allowed at limit load? Is it allowed at ultimate load?

6.4 Calculate the margin of safety of a part with a design limit load of 1000 lb. that fails at 1500 lb.

6.5 List the allowable stresses, modulii and ratios that are included in mechanical properties.

6.6 Give the name of a document where values can be found for mechanical properties of aircraft structural materials.

6.7 Tell which, if any, types of stress sometimes have allowable stresses of lower values than the mechanical properties and the reason or reasons for each.

6.8 Tell which, if any, types of stress sometimes have allowable stresses of higher values than the mechanical properties and the reason or reasons.

6.9 List several sources for allowable stresses that differ from the mechanical properties.

6.10 Using the allowable stresses given in the table of mechanical properties in this lesson, find the margin of safety (M.S.) of the following: All loads are ultimate.

a) A 2024 extruded tee section with a net cross sectional area of .188 square inches carrying a tension load of 9500 lb.

b) A strap of 7178-T6 plate with a net area of .312 square inches and a tension load of 25,000 lb.

c) A flange of a 2024-T4 forging with a .25 square inch cross section and a shear load of 6500 lb.

d) A 5/16 inch diameter high strength bolt bearing on a hole in a sheet of .050 thick titanium alloy Ti-8Mn with a load of 3500 lb. Edge distance (e/D) = 2.

6.11 A .125 inch thick beef-up plate of AISI 301-1/2 hard corrosion resistant steel is to be replaced with a plate of 125 ksi 4130 steel with the same width. Find the thickness required to have at least as much strength as the original plate in tension and fastener bearing.

LESSON 7

SECTION PROPERTIES

7.1 DEFINITIONS

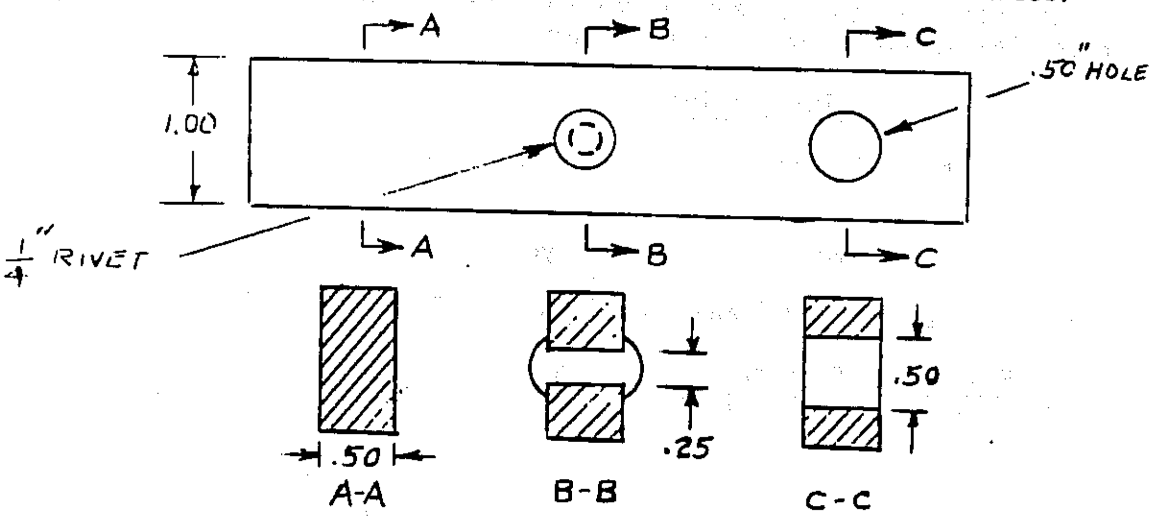
Section properties are those geometric characteristics of the cross section of a structural member that affect the magnitude and distribution of stress over the section for a given loading. They are area, centroid, moment of inertia, static moment of area and radius of gyration. The use for each section property and methods of determining its value are presented in the following paragraphs.

7.2 AREA

The area of a cross section is used to determine the magnitude of tension, compression and shear stress. For tension and compression the area is measured normal to the load direction and for shear it is measured parallel to the load.

For a member with holes, the gross area is the total area without subtracting the area removed by the holes. The net area is the gross area minus the area removed by the holes.

The effective area is the area that can carry load. For tension and shear, the effective area is equal to the net area. For compression, the effective area is equal to the gross area minus the areas of any holes that are not filled by fasteners. If a hole is filled by a fastener, a compression load can be carried through the fastener shank by bear-up. The following sketch illustrates the areas effective for the three kinds of stress.



A_t = effective area in tension
A_c = effective area in compression
A_s = effective area in shear

Section A-A: $A_t = A_c = A_s$ = gross area = 1.0 x .5 = .5 in^2

$$f_t = \frac{Pt}{.5} \qquad f_c = \frac{Pc}{.5} \qquad f_s = \frac{V}{.5}$$

Section B-B: $A_t = A_s$ = net area = .5 - .5 x .25 = .375 in^2

Ac = gross area = .5 in^2

$$f_t = \frac{Pt}{.375} \qquad f_c = \frac{Pc}{.5} \qquad f_s = \frac{V}{.375}$$

Section C-C: A_t = Ac = As = net area = .5 - .5 x .5 = .25 in^2

$$f_t = \frac{Pt}{.25} \qquad f_c = \frac{Pc}{.25} \qquad f_s = \frac{V}{.25}$$

7.3 CENTROID

The centroid of an area is that point at which the area could be concentrated and have the same moment with respect to an axis as the actual section. It can also be defined as the point through which any axis would have a static moment of area equal to zero.

Axes that pass through the centroid are called centroidal axes. In analyzing sections, two mutually perpendicular centroidal axes are generally used. The centroidal axes of a section are the neutral axes in bending. Usually, bending moment is about one axis only so that we are only interested in that one centroidal axis. For columns, we are only interested in the centroidal axis about which the column will buckle, which is the axis about which the moment of inertia is minimum. For these cases where only one centroidal axis is of interest, that axis is usually called the centroid.

Centroids of simple geometrical sections can be found by integration and, for complex sections, by a tabular summation.

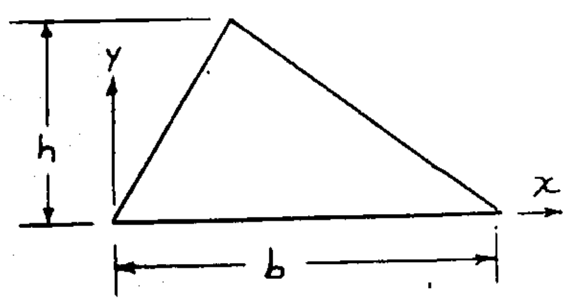
7.3.1 Centroids by Integration

The centroidal axes of a section can be found from the equations:

$$A\bar{x} = \int x\,da \quad \text{and} \quad A\bar{y} = \int y\,da$$

where A is the area and $\bar{x}$ and $\bar{y}$ are centroidal distances from reference axes.

o Example 1 Find the location of the centroidal axis that is parallel to the base of the triangle in the sketch.

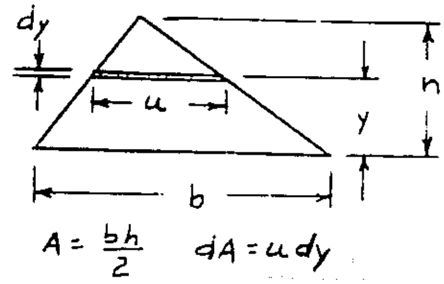


Solution:

Select a differential area dA having all points equally distant from the X axis and take moments about that axis.

$A\,\overline{y} = \int y\,da$

$= \int y\,u\,dy$



By similar triangles:

$\frac{u}{b} = \frac{h-y}{h}$

$u = \frac{b}{h}(h-y)$

Then $A\overline{y} = \left(\frac{bh}{2}\right)\overline{y} = \int_0^h \left(\frac{b}{h}\right)(h-y)y\,dy$

$= \frac{b}{h}\int_0^h (h\,y\,dy - y^2 dy)$

$= \frac{b}{h}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$

$= \frac{b}{h}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{bh^2}{6}$

$\overline{y} = \left(\frac{bh^2}{6}\right)\left(\frac{2}{bh}\right)$

$\overline{y} = \frac{h}{3}$

7.3.2 Centroids by Tabular Summation

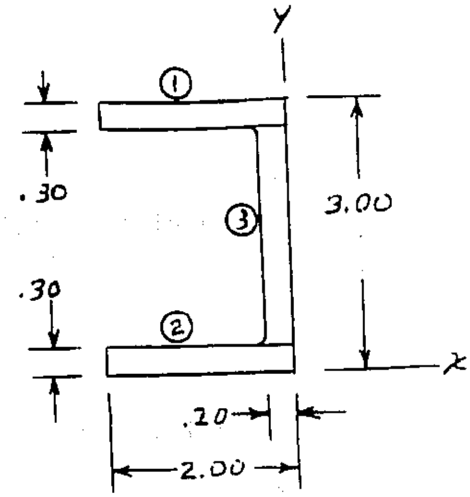
The equations for this method are a variation of the equations for the integration method:

$(\Sigma A_n)\,\overline{y} = \Sigma(A_n y)$ or $\overline{y} = \frac{\Sigma(Ay)}{\Sigma A}$ and

$(\Sigma A_n)\,\overline{x} = \Sigma(A_n x)$ or $\overline{x} = \frac{\Sigma(Ax)}{\Sigma A}$

where A_n, usually simply called A, is the area of an element of the section and x and y are the distances from reference axes to the centroid of the element.

o <u>Example 2</u>. Find the centroidal axes of the section in the sketch.



<u>Solution</u>

Divide the section into elements for which the centroid is known, choose arbitrary reference axes and calculate ΣA, ΣAx and ΣAy. Then find $\bar{x}$ and $\bar{y}$ from these terms.

Element	A	y	x	Ay	Ax
1	.60	2.85	1.0	1.71	.60
2	.60	.15	1.0	.09	.60
3	.48	1.50	.10	.72	.048
	1.68			2.52	1.248

$$\bar{x} = \frac{\Sigma(Ax)}{\Sigma A} = \frac{1.248}{1.68} = .743 \text{ in.}$$

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{2.52}{1.68} = 1.50 \text{ in.}$$

7.4 MOMENT OF INERTIA

The moment of inertia (I) of an area about an axis is the sum of the products obtained by multiplying each element (dA) of the area by the square of its distance (y) from the axis. Thus, $I = \int (y^2 dA)$ or $I = \Sigma (A_n y^2)$. For structural cross sections, the moments of inertia of interest are those about the centroidal axes.

Moments of inertia are used in determining stiffness and bending stresses in beams and the buckling loads of columns. For beams, the moment of inertia of interest is the one about the bending axis. For columns, it is the minimum moment of inertia.

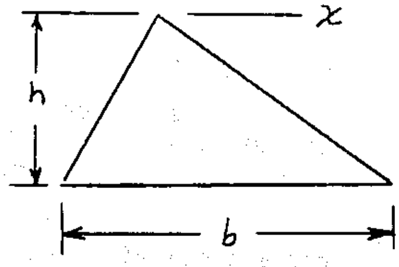
7.4.1 Parallel Axis Theorem

This theorem, also known as the transfer formula, states that the difference between the moments of inertia about a centroidal axis and a parallel axis is equal to the quantity Ad^2 where A is the cross section area and d is the distance between the axes. This theorem is only valid if one of the axes is centroidal. To find the moment of inertia about a non-centroidal axis from that about another non-centroidal axis, it is necessary to transfer from the first axis to the centroidal axis and then transfer from there to the second axis. The quantity Ad^2 is added to the moment of inertia about the centroidal axis to obtain the moment of inertia about the other axis. To transfer from a non-centroidal to a centroidal axis, the quantity is subtracted. Example problems in succeeding paragraphs illustrate the theorem.

7.4.2 Moment of Inertia by Integration

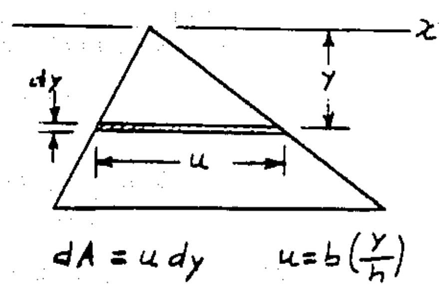
The moment of inertia of an area with respect to any axis can be found from the equations $I_x = \int y^2 dA$ and $I_y = \int x^2 dA$.

o Example 3. Find the moment of inertia of the triangle shown in the sketch about a horizontal axis through its apex (the x axis). Then find the moment of inertia about the horizontal centroidal axis and about the base using the parallel axis theorem.



Solution:

As for the centroid derivation, select a differential area dA having all points equally distant from the x axis.



$$I_x = \int y^2 dA = \int y^2 u\, dy$$

$$= \int y^2 \left(\frac{b}{h}\right) y dy$$

$$= \frac{b}{h} \int_0^h y^3 dy$$

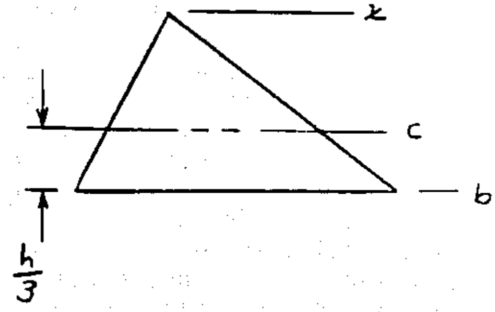
$$= \frac{b}{h} \left[\frac{y^4}{4}\right]_0^h = \frac{b}{h}\left(\frac{h^4}{4}\right)$$

$$I_x = \frac{bh^3}{4}$$

From the previous example:

$$\bar{y} = h - \frac{h}{3} = \frac{2h}{3}$$

And from the parallel axis theorem:



$$I_c = I_x - Ad^2$$

$$= \frac{bh^3}{4} - \frac{bh}{2}\left(\frac{2h}{3}\right)^2$$

$$I_c = \frac{bh^3}{36}$$

Similarly:

$$I_b = I_c + Ad^2$$

$$= \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{h}{3}\right)^2$$

$$I_b = \frac{bh^3}{12}$$

7.4.3 Moment of Inertia by Tabular Summation

Most structural member cross sections are not suitable for determination of moment of inertia by integration because of complexity of the geometry. These moments of inertia are determined as follows:

1) Divide the cross section into elements of geometrical shape (usually rectangles) with easily determined area, centroid location and elemental moment of inertia.

2) Select a reference axis parallel to the centroidal axis of interest. This axis can be anywhere but it is usually most convenient to assume it at the lower or upper extremity of the section so that all elements have the same sign for y.

3) Find the moment of inertia of each element about its own centroid (I_x and/or I_y).

4) Find the moment of inertia of each element about the reference axis by using the parallel axis theorem ($\Delta I = Ay^2$ and/or Ax^2).

5) Find the centroid of the section from the equations:

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} \text{ and } \bar{x} = \frac{\Sigma(Ax)}{\Sigma A}$$

6) Find the moment of inertia of the entire section about its centroid by using the parallel axis theorem again.

A convenient way to accomplish the above calculations is to use a table as shown in the following example. If the location of the section centroid is not known, the calculation for its location can be combined into the same table.

Example 4. Find the moments of inertia of the section in example 2 about the centroidal axes parallel to axes x and y. Note: The moment of inertia of a rectangle about its centroid is $I = bh^3/12$.

Solution:

Element	A	y	x	Ay^2	Ax^2	I_h	I_v
1	.60	2.85	1.00	4.8735	.60	.0045	.200
2	.60	.15	1.00	.0135	.60	.0045	.200
3	.48	1.50	.10	1.0800	.0048	.2304	.002
	1.68			5.967	1.2048	.2394	.402

$\bar{x}$ = .743 in. and $\bar{y}$ = 1.50 in. (from Example 2, page 7-4)

$$Ix = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y}^2) = .239 + 5.967 - 1.68\ (1.50)^2 = 2.426\ in^4$$

$$Iy = \Sigma I_v + \Sigma(Ax^2 - A(\bar{x}^2) = .402 + 1.205 - 1.68\ (.743)^2 = .679\ in^4$$

Note that the above equations for Ix and Iy include both axis transfers.

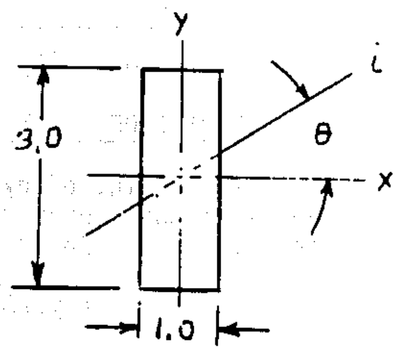
7.4.4 Moments of Inertia About Inclined Axes

Except for special cases, the moment of inertia of a section is not the same with respect to all axes passing through a given point. The mutually perpendicular centroidal axes about which the moments of inertia are maximum and minimum are called principal axes. For a section with at least one axis of symmetry, that axis and the one perpendicular to it are the principal axes. By use of Mohr's Circle, it can be shown that the moment of inertia of a symmetrical section about an axis inclined relative to the principal axes can be found from the equation:

$$I_i = I_x \cos^2 \theta + I_y \sin^2 \theta$$

where I_i is the moment of inertia about the inclined centroidal axis, I_x is the maximum moment of inertia, Iy is the minimum moment of inertia and θ is the angle between the inclined axis and the principal axis about which the moment of inertia is maximum.

Example 5 - For the rectangle shown in the sketch, find the moment of inertia about each principal axis and about the i axis for θ = 30° and for θ = 60°.



Solution:

The x and y axes are axes of symmetry and are, therefore, the principal axes.

$$I_x = \frac{bh^3}{12} = \frac{1.0(3.0)^3}{12} = 2.25\ in^4$$

$$I_y = \frac{3.0(1.0)^3}{12} = 0.25\ in^4$$

For θ = 30°:

$$I_i = I_x \cos^2\theta + I_y \sin^2\theta$$
$$= 2.25 \cos^2 30° + .25 \sin^2 30° = 1.6875 + .0625$$
$$= 1.75 \text{ in}^4$$

For θ = 60°:

$$I_i = I_x \cos^2\theta + I_y \sin^2\theta$$
$$= 2.25 \cos^2 60° + .25 \sin^2 60° = .5625 + .1875$$
$$= 0.75 \text{ in}^4$$

7.5 STATIC MOMENT OF AREA

The static moment of area (Q) about an axis is the sum of the products obtained by multiplying each element (dA) of the area by its distance (y) from the axis. Thus, $Q = \int y\,dA$, or, by dividing the area into elements (A), $Q = \Sigma Ay$.

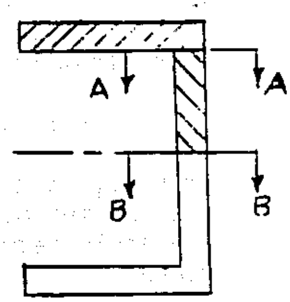
The static moment of area is used to find the shear stress distribution over a cross section of a shear - carrying member, as will be shown in a subsequent lesson.

Example 6. For the section used in Examples 2 and 4, find the static moment of area about the horizontal centroidal axis of the area above section A-A and of the area above section B-B.

Solution:

Above section A-A the area is a rectangular element.

$$Q = Ay = (2.0 \times .30)\ 1.35 = 0.81 \text{ in}^3$$



Above section B-B the area must be divided into two elements.

Element	A	y	Ay
1	.60	1.35	.81
2	.24	.60	.144
			.954

$$Q = \Sigma Ay = .954 \text{ in}^3$$

7.6 RADIUS OF GYRATION

The radius of gyration (ρ) of an area with respect to a given axis is equal to the square root of the quantity obtained by dividing the moment of inertia with respect to that axis by the area. Thus:

$$\rho = \sqrt{I/A}$$

The radius of gyration with respect to the centroidal axis is used to determine the buckling stress of columns.

Example 7. For the section of Example 4, find the radius of gyration about each of the two centroidal axes.

Solution:

$$\rho_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2.426}{1.68}} = 1.202 \text{ in}$$

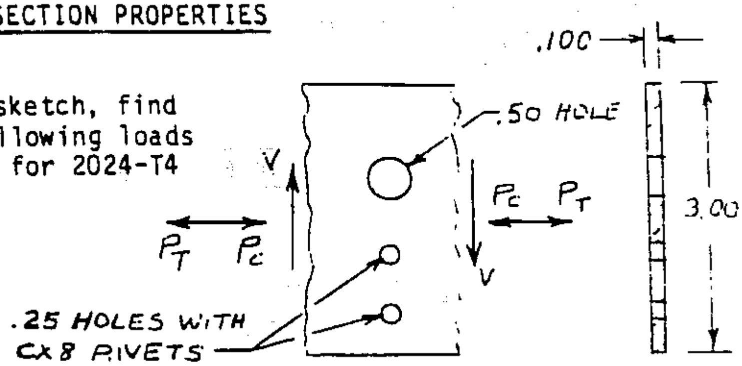
$$\rho_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{.679}{1.68}} = 0.636 \text{ in}$$

PROBLEMS

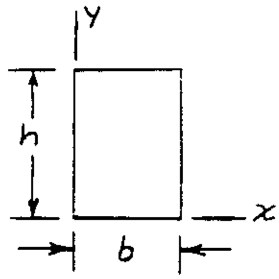
LESSON 7 SECTION PROPERTIES

7.1 For the section shown in the sketch, find the stress for each of the following loads and find the margin of safety for 2024-T4 plate.

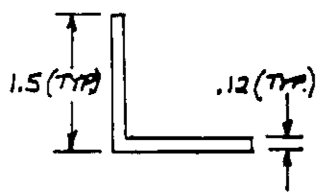
a) P_t = 10,500 lbs.
b) P_c = 10,500 lbs.
c) V = 7,200 lbs.



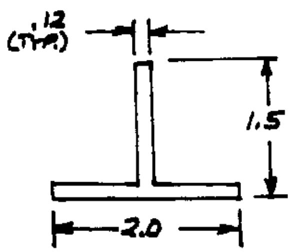
7.2 By use of integration, find the centroid and moment of inertia of a rectangle about the centroidal axis parallel to its base.



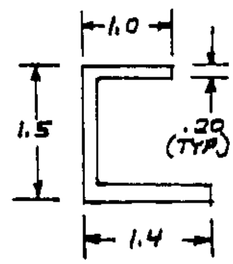
7.3 Find the area, centroid, moments of inertia about both centroidal axes and radii of gyration about both axes for the cross sections shown in the sketches. Use tabular method of Example 2 and Example 4.



a) angle

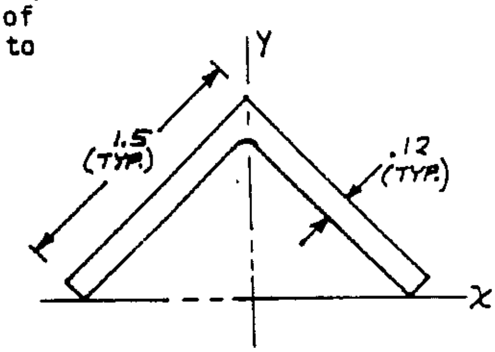


b) Tee

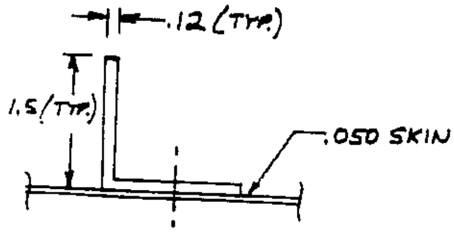


c) channel

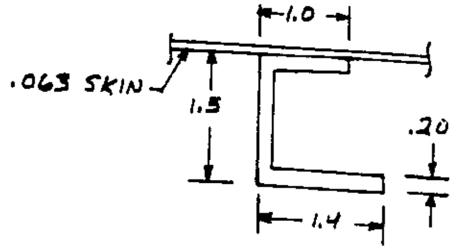
7.4 Find the minimum moment of inertia and radius of gyration for the angle in problem 7.3a). (Note: It could be shown by using Mohr's Circle that the minimum moment of inertia is about a centroidal axis parallel to the x axis in the sketch).



7.5 Find the area, centroid, and moment of inertia about the horizontal centroidal axis for each of the following sections. (Note: A reasonable approximation of the width of effective skin is 30 times the thickness).

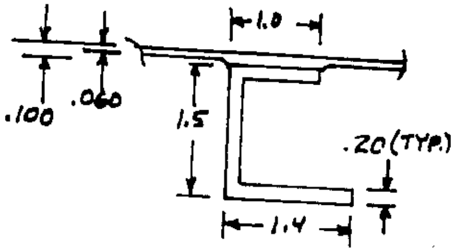


a) angle



b) channel

7.6 For the section shown, find the area, centroid, and moment of inertia about the horizontal centroidal axis. Note: when an attached skin is chem-milled, as in this case, the effective skin is the thick land portion plus a width of about 30 times the thickness of the thinner part.



7.7 For the section shown find the following section properties with respect to a horizontal axis. Assume We = 15t = 0.90in.

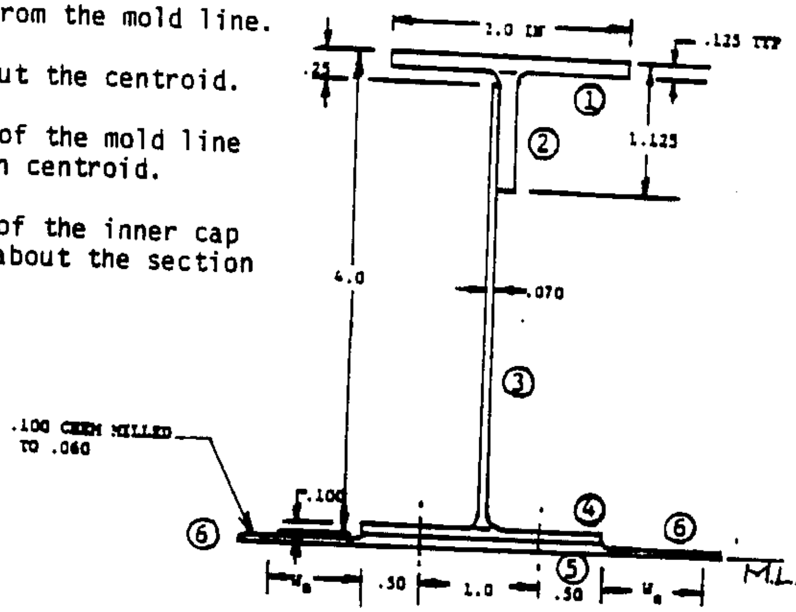
a) Area.

b) Centroidal distance from the mold line.

c) Moment of inertia about the centroid.

d) First moment of area of the mold line skin about the section centroid.

e) First moment of area of the inner cap (the .125 thick tee) about the section centroid.



LESSON 8

FASTENERS & JOINTS

8.1 GENERAL

Mechanical fasteners, usually simply called fasteners, include rivets, bolts and nuts, hi-shear rivets, hi-loks and many special-purpose fasteners such as blind rivets, blind bolts, quick-release fasteners, etc. Bolts with flush heads are commonly called screws. Almost all kinds of fasteners are available with either protruding heads or flush heads. Flush heads are used on the moldline of the airplane for aerodynamic reasons and sometimes on interior structure to avoid interference with another part. Fasteners are used to transfer loads of all kinds from one structural member to another. Rivets, bolts, screws, hi-shears and hi-loks are the most widely used aircraft fasteners because they are reliable, economical and convenient to install, inspect and service.

For ABDR the fastener most commonly used is the protruding head Jo-bolt. It is a high strength, blind fastener which does not require access to the back side of the structure being repaired and can be installed quickly. The protruding head is used on moldline repairs as well as interior repairs because the requirement for fast repairs overrides the aerodynamic considerations.

The fastener strength criteria, design policies and ultimate allowable fastener strengths used by McDonnell Aircraft Co. in the design of the F-4 are presented on pages 1.00 through 1.83 of McDonnell Report 339. These pages are appropriate for use in repair design and for determining the strength of existing structure. Some of these pages have different allowable loads for two different ratios of ultimate factor of safety to yield factor of safety. The Air Force uses a ratio of 1.5.

8.2 SINGLE FASTENERS LOADED IN SHEAR

Most aircraft fasteners are loaded in shear rather than tension. The failure modes and sources of allowable loads for protruding head and flush head fasteners are shown in the following paragraphs. Fasteners smaller than 1/8 inch diameter are not used for structural applications.

8.2.1 Protruding Head Fasteners

The failure modes for protruding head fasteners loaded in shear are shear failure of the fastener and bearing failure of the attached parts. These are treated separately in the following paragraphs. McDonnell Report 339 (hereinafter called MAC 339) has tables of failure loads for several of the more common fasteners bearing in various thicknesses of various alloys.

- Shear failure - single shear - In this mode the fastener shank is sheared through. The sketch shows the start of a failure.

  $P_{all} = P_{s_{all}}$ from MAC 339

  or

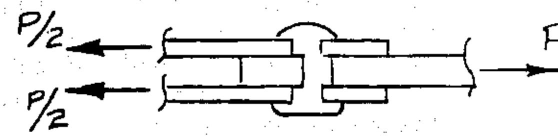
  $P_{all} = F_{su} A$ where F_{su} is the allowable shear stress of the fastener and A is the cross sectional area of the fastener.

- Shear failure - double shear - In this case the rivet must fail along two planes as shown..

  $P_{all} = 2P_{s_{all}}$ from MAC 339

  or

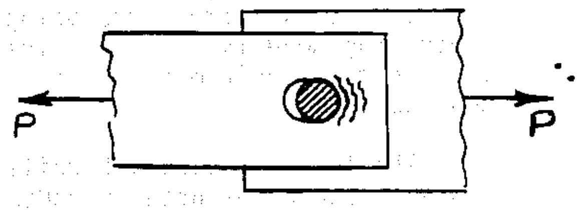
  $P_{all} = 2\ F_{su}\ A$



- Bearing failure - In this failure mode the material being joined fails as shown in the sketch or tears out.

  $P_{all} = F_{bru}\ Dt$

  where F_{bru} = the ultimate allowable bearing stress.
  D = fastener diameter
  t = material thickness

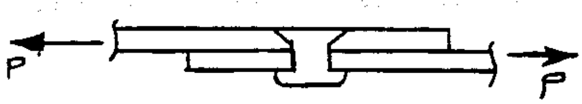


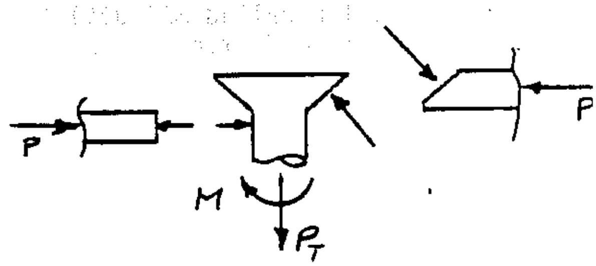
For edge distance of 2.0 or 1.5 times diameter, F_{bru} can be obtained from MIL-HDBK-5. For values between 2.0 and 1.5 a straight line variation between the two allowable stresses can be used. For values less than 1.5 times the diameter, F_{bru} can be obtained from MAC 339 page 12.12.

8.2.2 Flush Head Fasteners

Fasteners with flush heads are used with either machine countersunk or dimpled holes.

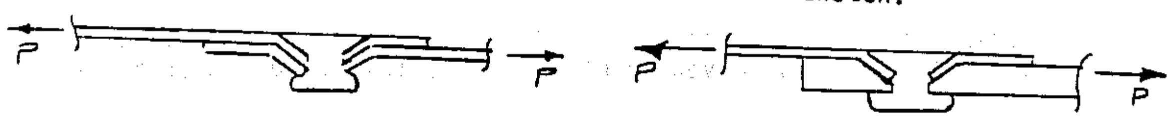
- Countersunk fasteners - In almost all cases only one of the pieces being joined is countersunk. Therefore, countersunk joints can fail in either shear or bearing as for protruding head fasteners. However, they also have an additional failure mode which is critical in many cases. The countersunk portion of the fastener hole acts as a wedge and tends to lift one side of the fastener head as shown in the sketch.





This puts both a tension load and a bending moment in the fastener, in addition to the shear, and can lead to a complex failure at a smaller load than for a protruding head fastener. Because of the difficulty of analyzing this complex failure, the allowable loads for countersunk fasteners are tabulated from test values. MAC 339 has allowable ultimate loads for many types of countersunk fasteners.

- <u>Dimpled fasteners</u> - Dimpled fastener joints can be either double dimpled (both sheets dimpled) or dimple-countersunk (upper sheet dimpled, lower sheet countersunk) as shown in the sketch.



Double Dimpled   Dimple Countersunk

In either configuration it can be seen from the sketch that as the upper sheet starts to move relative to the lower sheet it acts as a wedge on the fastener head as for the countersunk fastener. However, in this case the sloped flange of the upper sheet bears against the slope of the lower dimple or countersink. Although, as with the countersunk fastener, this puts tension and bending loads in the fastener it also helps transfer the shear load. Thus the allowable loads for dimpled fasteners are larger than for countersunk and in some cases larger than for protruding head fasteners.

The failure load of a dimpled fastener is even more complex than for a countersunk fastener. Therefore the allowable loads are based on test values and are available in MAC 339.

8.2.3 <u>Blind Fasteners</u>

Fasteners that can be installed from one side of the fastener holes without access to the other side are called blind fasteners. These include Jo-bolts made of aluminum, steel and A-286 (a heat-resistant alloy) and pull stem rivets made of aluminum, monel and A-286. These fasteners and their designations and usages are shown on pg. 1.41 of MAC 339 which also includes strength reduction factors. These factors, applied to the allowable loads presented on pages 1.42 through 1.46, depend on the amount of vibration and the inspectability of the blind side of the fastener.

The diameters of the three smaller sizes of jo-bolts are slightly larger than other fasteners of the same nominal diameter. These diameters, which are to be used in calculating bearing strengths, are .164 for 5/32 inch, .200 for 3/16 inch, and .260 for 1/4 inch, which are designated -08L, -3L and -4L respectively. The larger sizes, -5L and -6L (5/16 and 3/8 inch) have nominal diameters of .312 and .375 respectively.

8.2.4 Temperature Effects

Elevated temperatures reduce the strength of all fasteners whether shear critical or bearing critical. Reduction factors for allowable loads in shear and in bearing for several rivet alloys and sheet materials are given on pages 1.81 through 1.84 of MAC 339. Rivet alloys for all common fasteners are given on page 1.05 and 1.06 of MAC 339. The procedure and an example problem are shown on page 1.80 of that report. That page refers to MIL-HDBK-5 for bearing strength reduction factors; however, most of the factors needed for ABDR purposes are given on the above pages of MAC 339. For both shear and bearing, the 1/2 hour exposure time factors are to be used except in areas of known long time exposure such as the engine compartment.

Because temperature factors for the bearing strength of alloy steel and stainless steel are not given in MAC 339, the following tables are presented.

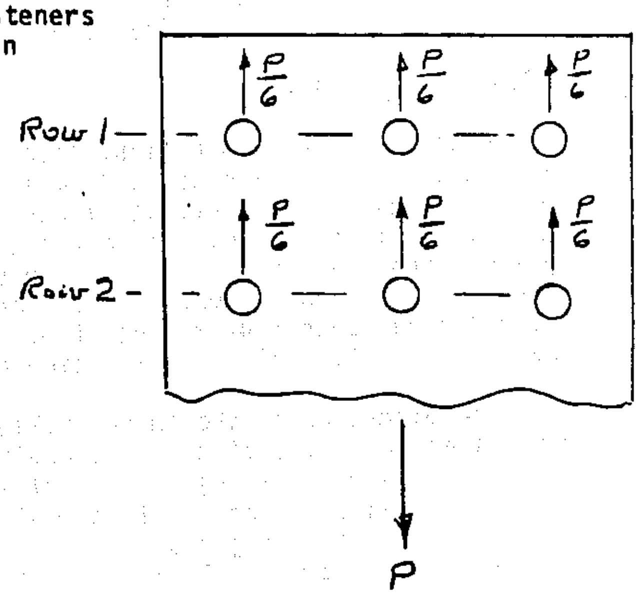
Temp. / Matl.	Temperature factor for F_{bru}, 1/2 hr. exposure					
	200	400	600	800	1000	1200
Alloy steel	.97	.96	.92	.74	.57	.38
301 st. 1/2 hard.	.87	.74	.67	.59	.52	.44

For temperatures below or between the temperatures shown, use straight line interpolation between values shown. All temperatures are measured in °F.

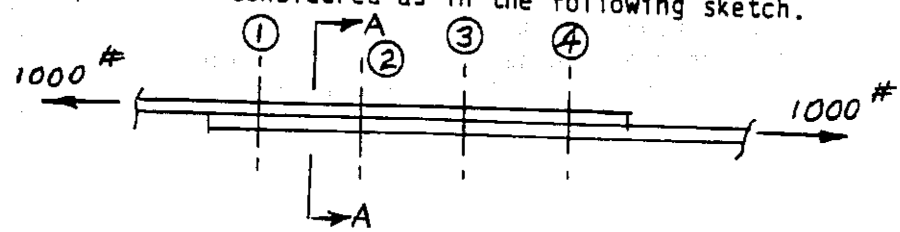
8.3 MULTIPLE FASTENERS LOADED IN SHEAR

8.3.1 Symmetrical Loading

With one or two rows of fasteners the load is evenly distributed if the fasteners are the same size. Otherwise it is in proportion to the fastener's cross-sectional area (πr^2).



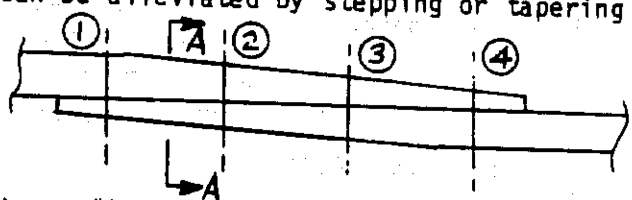
For three or more fasteners in a line (parallel to the load) the load peaks at the first and last fasteners. The reason for this is clear when strains in the two (or more) parts are considered as in the following sketch. If the load was evenly distributed (25% in each of the four fasteners) the load at section A-A would be 750# in the upper member and 250# in the lower. For the same area and modulus of elasticity (A and E) the elongation between fasteners 1 and 2 would be 3 times as great in the upper member as in the lower because elongation $\delta = \frac{PL}{AE}$. But the elongation must be approximately the same in both members and, because of this, fastener #1 has to transfer more than its share of the load. The same is true of fastener #4.



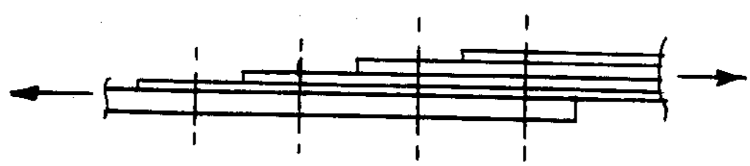
If the fasteners were infinitely rigid all of the load would transfer at fasteners 1 and 4. If they were totally flexible the load would be equally divided between the four fasteners. The actual load distribution is between these two extremes and is dependent on the spring constants of the fasteners. Usually, the distribution is fairly close to being equally divided. However, when the fastener spring constant is unknown, a safer assumption on load distribution is that shown on the following table. This distribution is based on a high but realistic spring constant.

No. of Fasteners	End Fastener Load in % of P
3	40
4	37.5
5	37

This unequal load distribution can be alleviated by stepping or tapering the members. For the tapered configuration shown, the cross sectional area of the upper member at section A-A can be approximately 3 times the area of the lower member. Therefore, for equal elongation, fastener #1 transfers one fourth of the load.

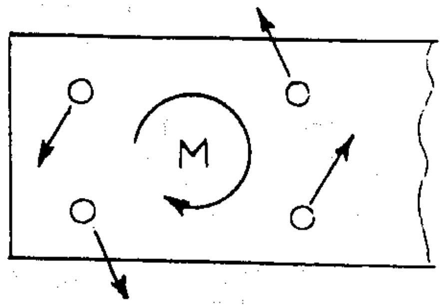


Another way of obtaining uniform load distribution is to have the fasteners that tend to overload be bearing critical; that is: weaker in bearing than in shear. Thus, when a fastener starts to overload, the hole can elongate in the direction of the load and yield slightly, allowing the next fastener to load up fully. End fasteners can be made bearing critical by tapering or stepping the thickness, counterboring the fastener hole, using a larger fastener that has a high shear strength, or by using multiple thin layers of material, each being thin enough for the fasteners to be bearing critical. This last method is usually the only one suitable for ABDR repairs.



8.3.2 Antisymmetrical Loading

An antisymmetrical load (a moment) on a fastener pattern is reacted by loads that are proportional to the fastener cross sectional area and act normal to lines drawn from the fasteners to the pattern centroid as shown in the sketch. This is an elastic load distribution.



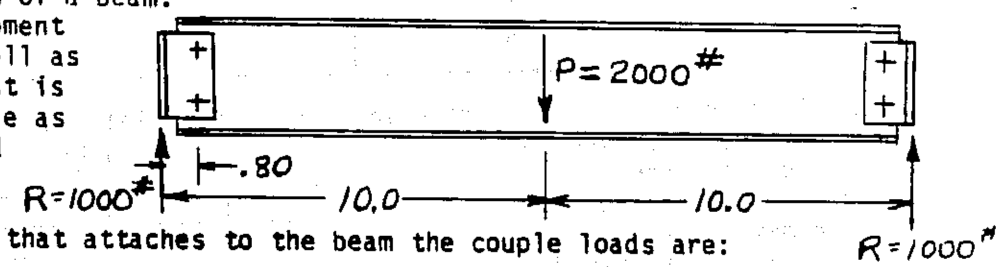
The centroid of a fastener pattern is found by the method used in section properties. The area used for each element (fastener) is the cross sectional area of the fastener or a number proportional to it, such as diameter squared. If the fasteners are all the same size, a unit number such as 1.0 can be used for the area. The load on each fastener is found from the equation:

$$P = \frac{MeA}{\Sigma (e^2 A)}$$

where P = fastener load, e is the distance from the fastener to the fastener pattern centroid and A is the rivet cross sectional area or number proprotional to it.

8.3.4 Shear Clips

When two pieces of structure are joined by a shear clip, the internal loads are balanced with the load transfer at the apex of the clip angle as shown in the sketch of a beam. Thus, there is a moment at the rivets as well as a shear. The moment is balanced by a couple as shown in the second sketch.



On the flange that attaches to the beam the couple loads are:

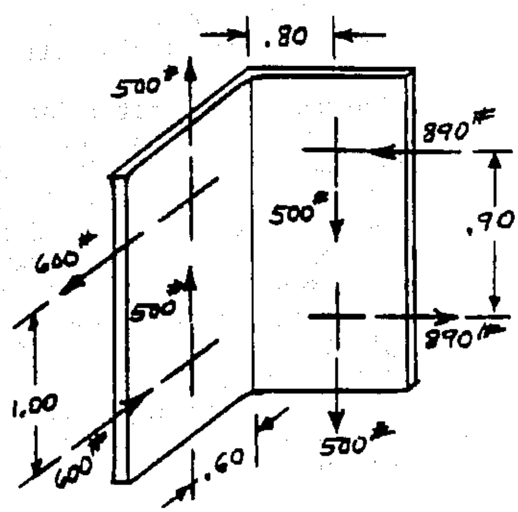
$$P_m = 1000\left(\frac{.80}{.90}\right) = 890\#$$

and the resultant fastener load is:

$$R = (890^2 + 500^2)^{1/2} = 1020\#$$

On the other flange the couple loads are:

$$P_m = 1000\left(\frac{.60}{1.0}\right) = 600\#$$



and the resultant fastener load is:

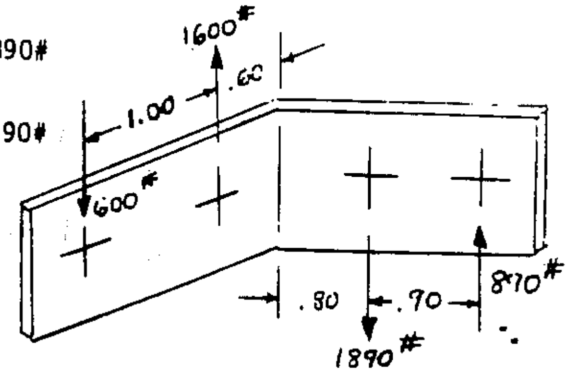
$(600^2 + 500^2)^{1/2} = 780\#$

Note that in the first instance, although two rivets carry a 1000 lb. load, the load on each rivet is greater that 1000 lb. Three-rivet clips are more efficient but often unnecessary. It should now be obvious why one-rivet clips are unacceptable.

A less desirable, but sometimes both necessary and adequate, type of shear clip has rivet lines at right angles to the load direction as shown in the third sketch. This clip is balanced by the laws of statics as above.

$\Sigma M = 0$ $\qquad R_1 = 1000\left(\frac{.80}{.90}\right) = 890\#$

$\Sigma F_v = 0$ $\qquad R_2 = 1000 + R_1 = 1890\#$



Similarly for the other flange:

$$R_1 = 1000\left(\frac{.60}{1.00}\right) = 600\#$$

$$R_2 = 1000 + 600 = 1600\#$$

Note that the 1890# is considerably greater than the 1020# maximum rivet load on the first shear clip.

For a clip with more than five fasteners in a row, the eccentricity of the load can usually be neglected because it would have little effect on the resultant fastener loads.

8.4 FASTENERS LOADED IN TENSION

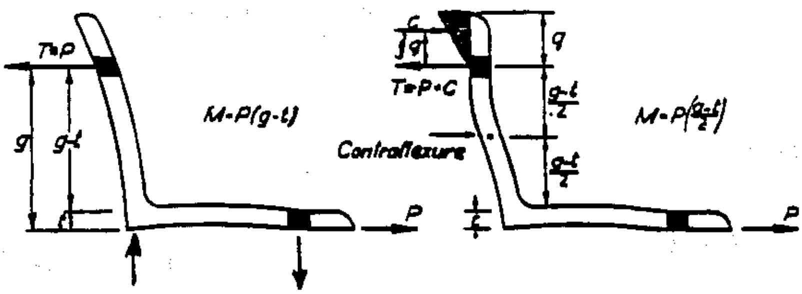
Rivets and blind fasteners are not used in joints where the primary load is tension. They do frequently carry small secondary tension loads. The allowable tension loads, or methods of determining them, for a variety of fasteners are given in MAC 339 on the pages listed in the following table. Note that in many cases there is a prying action which causes the fastener load to be higher than the applied load. Examples of this are shown in following paragraphs.

Fastener	Page
MS 20470 (BJ & CX)	1.11
MS 20426 (BB & CY)	1.11
Steel hi-shear rivets	1.20
Lock bolts	1.26
Bolt-nut combinations	1.32
Flush screws	1.35 - 1.39

8.4.1 Tension Clips

Angle and tee tension clips are frequently used to transfer tension loads from one member to another. McDonnell Report 339 pages 12.00 through 12.02.03 present criteria and curves of allowable limit or yield loads for sheet metal tension clips and allowable ultimate load for extruded (or machined) tension clips. The limit or yield allowables must be multipled by 1.5 to obtain the allowable ultimate load.

Tension clips are difficult to free body because the loading is complex and statically indeterminant. The following sketch gives an idea of the external balance and deflected shape. Nearly always a tension clip mates with a piece of structure that causes toe-up loads as shown in the right hand sketch. The bolt tension load is equal to the applied tension load plus this toe-up load.



The deflection is also an important consideration. Because the bolted flanges of tension clips are cantilever beams of low bending stiffness they have to deflect in order to carry a tension load. Therefore, a tension clip will be ineffective in tension if it is working with more rigid continuous structure that prevents deflection of the clip. Because of this, if a joint that includes a tension clip is overlaid with a repair strap the strap must be capable of carrying the entire load.

A bathtub fitting is a special form of tension clip in which the bolted flange is supported on two or three sides. These are somewhat stronger and more rigid than other tension clips.

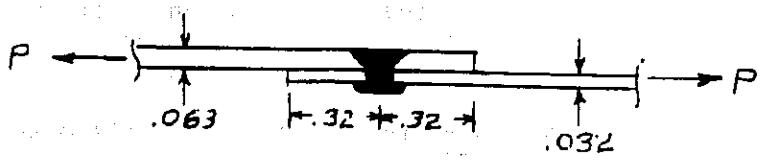
8.4.2 Lugs

The tension lug analysis methods used at McDonnell are presented on pages 12.10 through 12.16 of MAC 339. The allowable ultimate bearing stress is found on pg 12.12, the permissible clampup on pg 12.14 and the allowable loads for double shear lugs on pg 12.15. Page 12.16 gives correction factors for lateral lug loads. As was mentioned in section 8.2.1, pg. 12.12 of MAC 339 is also used to determine the allowable bearing stress when edge distance is less than 1.5 D.

For a double shear lug joint, the bolt may have a tension nut torqued to the proper tension nut torque of page 1.31 of MAC 339, a nut torqued to the proper shear nut torque, or it may have a nut bolt combination with no applied torque. In the latter case, the bolt might be replaced with some other type of pin. The amount of torque determines the amount of clamp up, which affects the allowable load.

8.5 EXAMPLE PROBLEMS:

o Example 1 - Find the allowable ultimate load for the joint shown in the sketch. Both sheets are 7075-T6 and the rivet is an MS20426 AD5 (BB5).



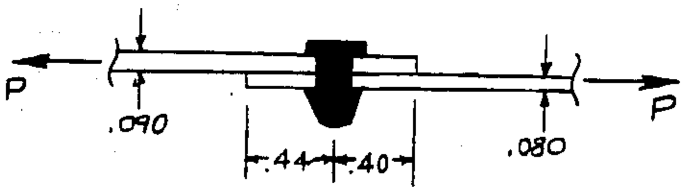
Solution:

In .063: P_{all} = 470# from MAC 339 pg. 1.14 for e/D = 2
In .032: P_{all} = 551# from MAC 339 pg. 1.13 for e/D = 2

Therefore the .063 sheet is critical.

$$P_{all} = 470\#$$

o Example 2 - Find the allowable ultimate load for the joint shown. The .090 sheet is 2024-T3 alclad, the .080 is a 7178-T6 extrusion and the fastener is a 3/16" diameter NAS 1054 hi-shear rivet.



Solution:

In both sheets e/D is greater than 2 so use allowable bearing stresses for e/D = 2. For 2024 sheet F_{bru} = 121,000 psi and for 7178 extrusion F_{bru} = 157,000 psi (from Lesson #6).

Upper (.090) sheet $P_{br_{all}} = F_{bru} A$ = 121,000 (.090 x .189) = 2060#

Lower (.080) sheet $P_{br_{all}} = F_{bru} A$ = 157,000 (.080 x .189) = 2370#

The fastener is steel with F_{tu} = 160 ksi as shown on page 1.05 of MAC 339. Fastener strength is:

$P_{s_{all}}$ = 2690# (from MAC 339 pg. 1.33)

The joint strength is the lowest of these three different allowable loads. Therefore, the upper (.090 2024-T3) sheet is critical and:

$$P_{all} = 2060\#$$

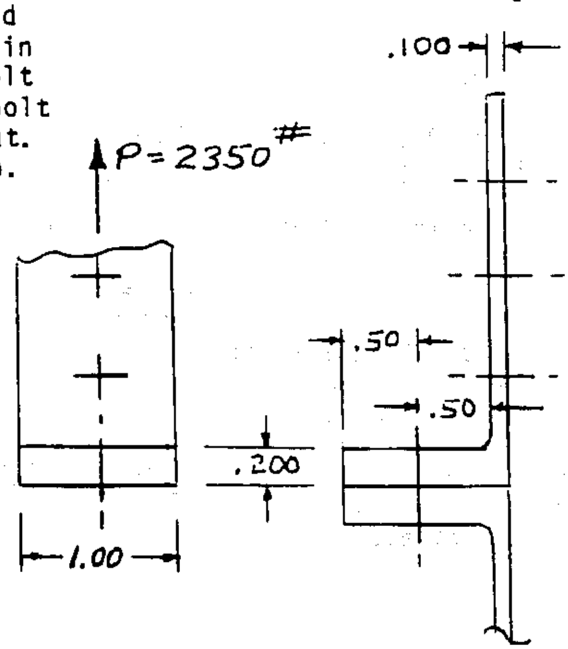
Note that for a double shear joint the joint strength is the lower of:

a) The allowable bearing strength of the center member ($P_{br_{all}} = t\ D\ F_{br}$).

or

b) The lesser of the allowable bearing strength of the upper member or the single shear strength of the fastener; plus the lesser of the allowable bearing strength of the lower member or the single shear strength of the fastener.

o <u>Example 3</u> - Find the allowable ultimate load and margin of safety for the 7075-T6 extruded tension clip shown in the sketch. The bolt is a 1/4" NAS 464 bolt with an NAS 686A nut. The load is 2350 lb.



<u>Solution</u>:

From MAC 339 pg. 1.32 it can be seen that the nut and bolt are tension fasteners and have an allowable tension load of 4580#.

From MAC 339 pg. 12.02: T_1 = .200, T_2 = .100, D = .250, e = .50, and W = 1.00. Using the graph, the allowable load is 3170#. However, the chart requires that e = 1.75 D = 1.75 x .25 = .4375 and the actual e = .50. From MAC 339 pg. 12.00:

m = e/D = .50/.25 = 2.0

K_2 = 1.75/m = 1.75/2.0 = .875

The chart also requires that W = 4D = 4 x .25 = 1.0. The actual W is 1.00 so this requirement is met. From MAC 339 pg. 12.00:

$n = W/D = 1.00/.25 = 4$

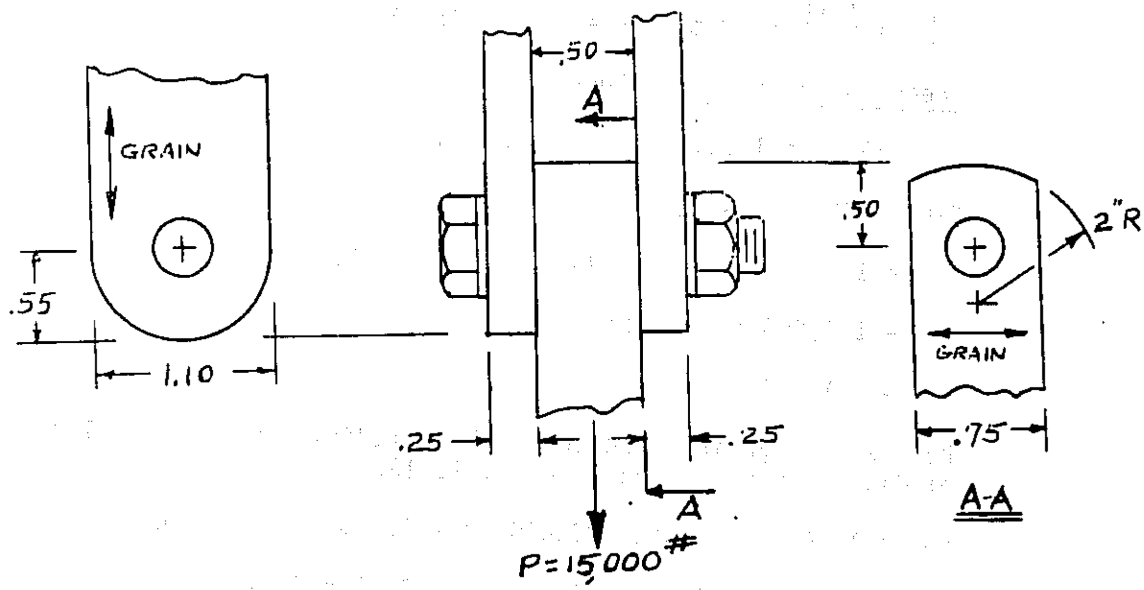
$K_1 = (n-1)/3 = (4-1)/3 = 1.0$

$\underline{P_{all}} = P_{corrected} = P_{chart} \times K_1 \times K_2$

$= 3170 \times 1.0 \times .875 = \underline{2770\#}$

$\underline{M.S.} = \frac{P_{all}}{P} - 1 = \frac{2770}{2350} - 1 = \underline{+.18}$

o Example 4 - Determine the proper bolt torque and find the allowable ultimate load and margin of safety for each lug and for the bolt in the following sketch. Also determine the allowable joint load and margin of safety. The material of all lugs is 7178-T6 extrusion with grain direction as shown. The fastener is a 3/8" NAS 464P bolt with an NAS 1022A nut.



Solution:

Find the required torque:

The nut is a shear nut as shown on pg. 1.32 of MAC 339. The proper torque for a 3/8" shear nut is shown on pg. 1.34 of MAC 339 as:

T = 95 - 110 in lb.

Check the center lug (lug #2):

R = .50, W = .75, D = .375, t = .50

Note that R is the edge distance of the bolt, and is not necessarily the machined radius of the part.

$R/D = .50/.375 = 1.33$

$F_{br} = 1.16\ F_{tu}$ per MAC 339 pg. 12.12

$F_{tu} = 88{,}000$ psi per Lesson #6

$F_{br} = 1.16\ (88{,}000) = 102{,}100$ psi based on R/D

$W/D = .75/.375 = 2.0$

Grain is long transverse per figure on pg. 12.12 of MAC 339. Therefore use curve #2 on that page per the table on pg. 12.13 of MAC 339.

$F_{br} = .97\ F_{tu} = .97 \times 88{,}000 = 85{,}400$ psi based on W/D

Using the lower of the two values of F_{br}:

$\underline{P_{all}} = F_{br}\ A = 85{,}400\ (.375 \times .50) = \underline{16{,}000\#}$ for lug #2

$\underline{M.S.} = \frac{P_{all}}{P} = \frac{16{,}000}{15{,}000} - 1 = \underline{+.07}$ for lug #2

<u>Check the outer lugs (lugs #1)</u>:

$R = .55,\ W = 1.10,\ D = .375,\ t = .25$

$R/D = .55/.375 = 1.47$

$F_{br} = 1.29\ (88{,}000)$ per MAC 339 pg. 12.12

$= 112{,}600$ psi based on R/D

$W/D = 1.10/.375 = 2.93$

Because the grain direction is longitudinal, use curve #1 on pg. 12.12 of MAC 339 per the table on pg. 12.13.

$F_{br} = 1.86\ F_{tu} = 1.86\ (88{,}000) = 163{,}700$ psi based on W/D

$\underline{P_{all}} = F_{br}\ A = 112{,}600\ (2 \times .25 \times .375) = \underline{21{,}100\#}$ for lugs #1

$\underline{M.S.} = \frac{P_{all}}{P} = \frac{21{,}100}{15{,}000} - 1 = \underline{+.41}$

It can be seen from the curves on pg. 12.12 of MAC 339 that whenever W = 2R (or greater), W/D is less critical than R/D.

Check the bolt - per MAC 339 pg. 12.15, F_{br2} = 85.4 ksi, F_{br1} = 112.6 ksi, e = 0.0 ∴ e/D = 0. Following the graph from F_{br2} to F_{br1} to the 160 ksi curve for a clamped shear nut to e/D = 0 to the clamped shear nut curve to K:

K = 1.64

V_{all} is the allowable ultimate single shear load.

V_{all} = 10,500# per MAC 339 pg. 1.33

$P_{all} = K\ V_{all} = 1.64 \times 10{,}500 = \underline{17{,}200\#}$

$\underline{M.S.} = \frac{P_{all}}{P} = \frac{17{,}200}{15{,}000} - 1 = \underline{+.15}$ for the bolt

Determine the joint strength:

The joint strength is the strength of the weakest component. Therefore:

Joint P_{all} = 16,000# (center lug is critical)

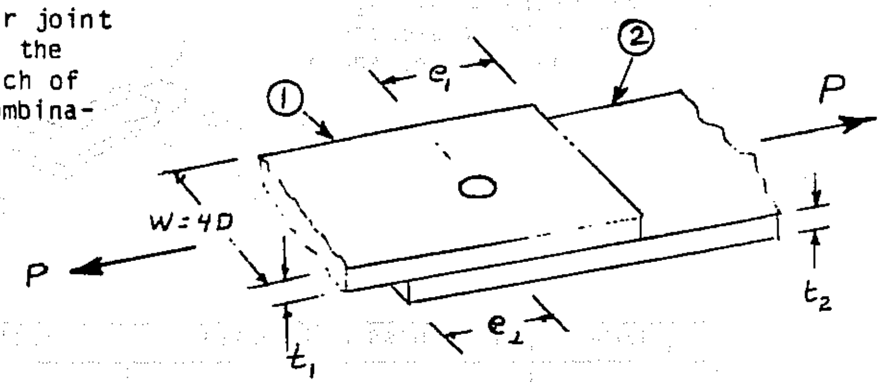
$\underline{\underline{M.S.}} = \frac{P_{all}}{P} = \frac{16{,}000}{15{,}000} - 1 = \underline{\underline{+.07}}$

PROBLEMS

LESSON #8 FASTENERS AND JOINTS

Before working the following problems read MAC 339 pages 1.00 to 1.02, 1.10, 1.20, 1.30, 1.30.01, 1.40, 1.40.01, 1.50, 1.80, 12.00, 12.00.01, 12.10 and 12.11. Also become familiar with the contents of the other pages of Section 1 and the other pages of Section 12 through page 12.16.

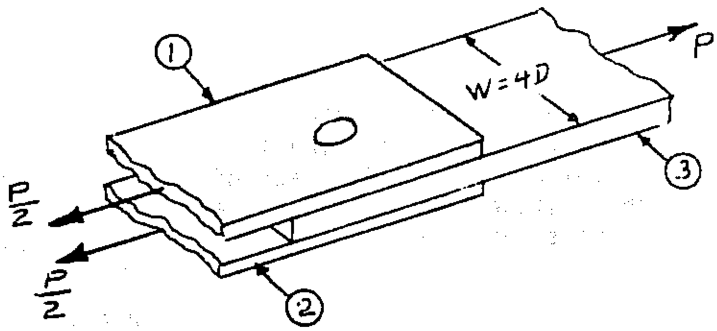
8.1 For the single shear joint shown in the sketch find the ultimate strength for each of the fastener-material combinations shown below.



	Member	Thickness & Material	Fastener	Head Type	E/D
a	1 2	.040 7178-T6 sheet .040 7178-T6 sheet	MS20470 DD6 (CX6)	prot. prot.	2 1.5
b	1 2	.063 7178-T6 sheet .032 7178-T6 sheet	MS20426 AD5 (BB5)	csk. prot.	2 2
c	1 2	.050 7178-T6 sheet .080 7178-T6 sheet	MS20426 AD5 (BB5)	dimp. csk.	2 2
d	1 2	.063 7178-T6 sheet .040 7178-T6 sheet	NAS 1097 AD5 (LZ5)	csk. prot.	2 2
e	1 2	.160 7178-T6 sheet .090 7178-T6 sheet	NAS 1055-6 3/16" hi-shear	csk. prot.	2 2
f	1 2	.100 Ti-6Al-4V sheet .050 Ti-6Al-4V sheet	NAS 333C 3/16" bolt	csk. prot.	2 2
g	1 2	.063 7178-T6 sheet .063 7178-T6 extrusion	3M249A5 5/32" stl. hi-lok	csk. prot.	2 1.5
h	1 2	.125 7178-T6 sheet .063 7178-T6 sheet	NAS 1670-3L 3/16" stl. jo-bolt	csk. prot.	2 2
i	1 2	.125 7178-T6 sheet .063 7178-T6 sheet	NAS 1674-3L 3/16" al. jo-bolt	csk. prot.	2 2
j	1 2	.190 2024 T4 sheet .190 2024 extrusion	NAS 1669-6L 3/8" stl. jo-bolt	prot. prot.	2 2

Note: Prot. is an abbreviation for protruding.
Csk. is an abbreviation for countersunk.
Dimp. is an abbreviation for dimpled.

8.2 For the double shear joint shown in the sketch find the ultimate strength for each of the fastener-material combinations shown below.



	Member	Thickness & Material	Fastener	Head Type	E/D
a	1	.040 2024-T3 sheet	MS20470 DD6	prot.	2
	2	.040 7178-T6 sheet	(CX6)	prot.	2
	3	.080 2024-T3 sheet			2
b	1	.050 Ti-6Al-4V sheet	NAS 1054-5	prot.	2
	2	.050 Ti-6Al-4V sheet	5/32" hi-shear	prot.	1.5
	3	.100 Ti-GAl-4V sheet			1.2
c	1	.125 2024-T4 sheet	3M248A-4	prot.	2
	2	.190 2024-T4 sheet	1/4" hi-lok	prot.	2
	3	.250 7178-T6 extrusion			2

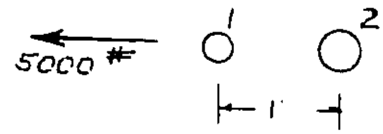
8.3 For each of the following joints, find the ultimate allowable load at room temperature and at the given temperature exposure. For all members e/D = 2.0 or greater. See the sketch of problem 8.1 for the single shear joints a) and b) and the sketch of problem 8.2 for the double shear joint c).

	Member	Thickness & Material	Fastener	Head	Temp.	Time
a	1	.063 2024-T81 sheet	MS20426 AD5	csk.	300°	1/2 hr.
	2	.040 7178-T6 sheet	(BJ5)	prot.		
b	1	Same as 1 above	same as above	csk.	300°	1000 hr
	2	Same as 2 above		prot.		
c	1	.100 Ti-6Al-4V sheet	ST3M415C4	csk.	650°	1/2 hr.
	2	.063 301 St. St. 1/2 hard.	(1/4" A286 hi-lok)	prot.		
	3	.160 125 ksi. steel				

8.4 In problems a) and b) determine the load on each fastener.

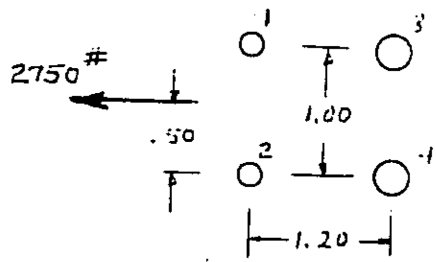
a)

Fastener no.	Fastener
1	NAS 1054-6 hi-shear
2	NAS 334 bolt

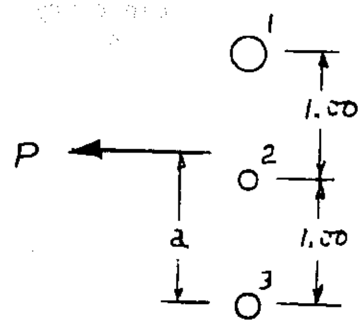


b)

Fastener no.	Fastener
1	MS20470 AD5 (BJ5)
2	MS20470 AD5 (BJ5)
3	MS20470 DD6 (CX6)
4	MS20470 DD6 (CX6)



c) In the fastener pattern shown find the value of dimension "a" that will allow the load P to load the three fasteners concentrically.

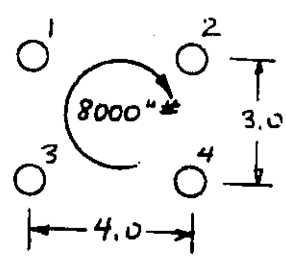


Fastener no.	Fastener
1	NAS 1669-4L
2	NAS 1669-08L
3	NAS 1669-3L

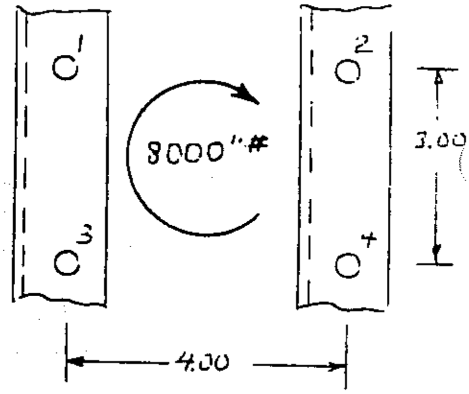
d) Find the allowable load P for the configuration shown in problem c) above and find the load this would produce on each fastener.

8.5 For the load and fastener configuration in each of the following problems, find the load on each fastener. In each problem, all fasteners are of the same type and size.

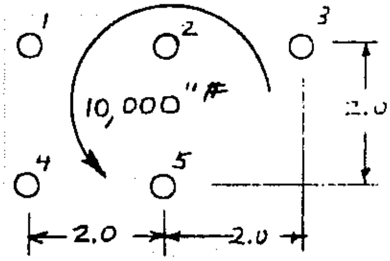
a) In this case the fasteners can carry load in any direction.



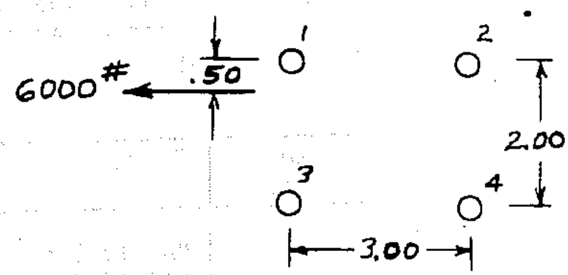
b) Same as a) above except that the rivets pick up two axial members that can carry significant loads in the axial direction only.



c) In this problem the fasteners can carry load in any direction.

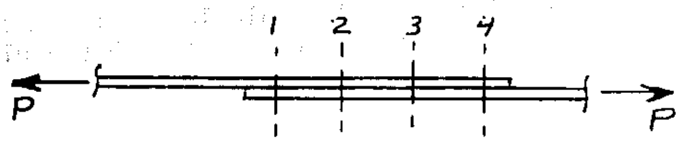


d) In this problem the fasteners can carry load in any direction.



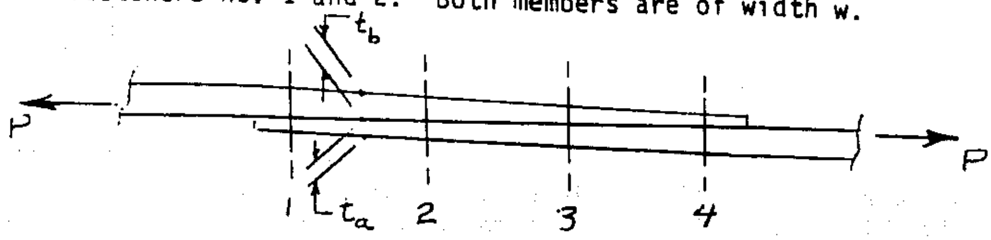
8.6 For the splice in the sketch determine the distribution of load among the four fasteners. The two straps are of the same material and thickness.

a) Assume the fasteners have infinite stiffness.



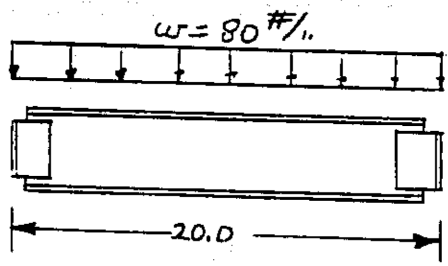
b) Assume the fasteners have the more realistic, but still conservative, stiffness that will produce the load distribution of the table in paragraph 8.3.1.

c) For the tapered splice illustrated, determine the thickness ratio t_a/t_b necessary for 20% of the load to be transferred by each of the end fasteners and 30% by each of the others, assuming infinite fastener stiffness. t_a and t_b are average thicknesses between fasteners no. 1 and 2. Both members are of width w.

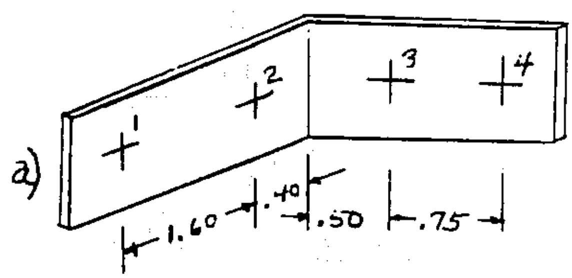


d) If the fasteners in problem a) above are bearing-critical in the upper strap, and the load increases to failure, determine the load distribution just before failure.

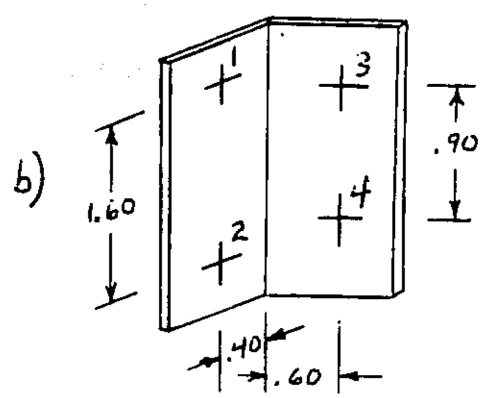
8.7 For the shear clips in problems a) and b) find the load in each rivet due to the beam loads shown in the sketch. Determine the smallest size MS20470 rivet that can be used in each location and find the margin of safety of each. Note: rivets smaller than 1/8 inch are not used in structural applications. The beam is .040 7178-T6 sheet metal.



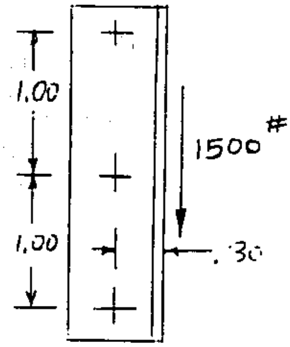
a) The clip is .063 2024-T3 sheet metal. Rivets no. 1 and 2 attach to .125 7178-T6 structure.



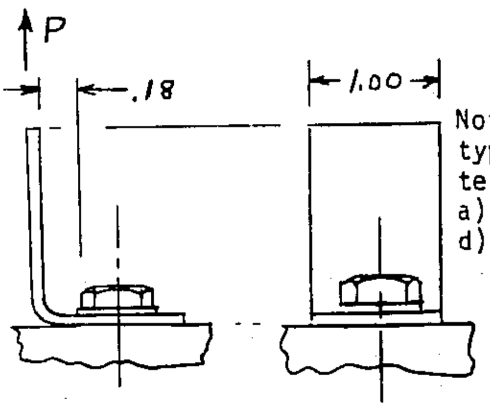
b) The clip is .050 2024-T3. Except for this and the clip geometry, everything is the same as in problem a).



c) Find the load on each of the three fasteners in the shear clip flange shown. All fasteners are the same type and size.

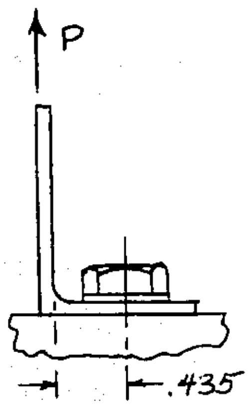


8.8 For the four tension clip configurations shown in the sketches, find the ultimate allowable tension load. All materials are .090 inch thick 2024-T3. All bolts are 1/4 inch tension bolts with tension type nuts.

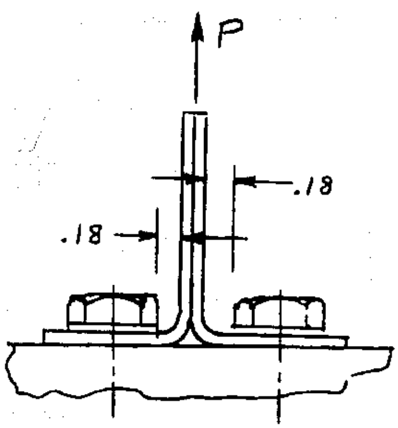


Note: This view typical for tension clips a), b), c), and d).

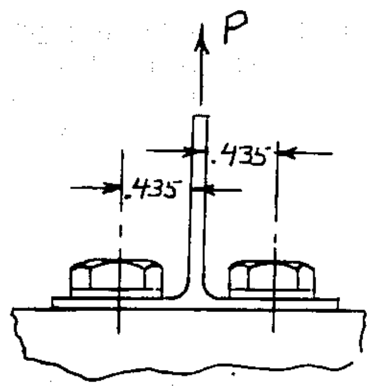
a) Sheet metal



b) Extrusion

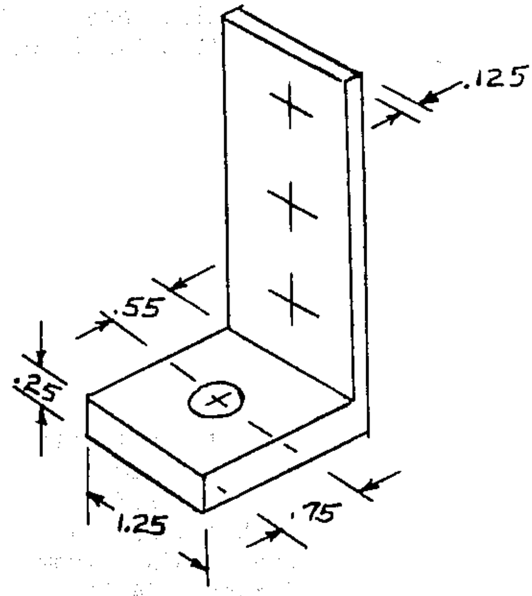


c) Sheet metal



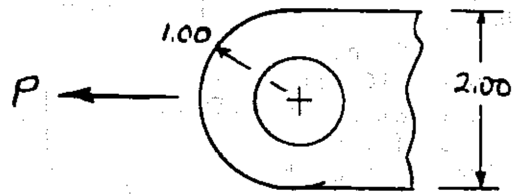
d) Extrusion

e) For the angle tension clip in the sketch, find the ultimate allowable tension load. The clip material is 7178-T6 extrusion with thicknesses as shown. The bolt is an NAS 1306 with an NAS 1021A nut.



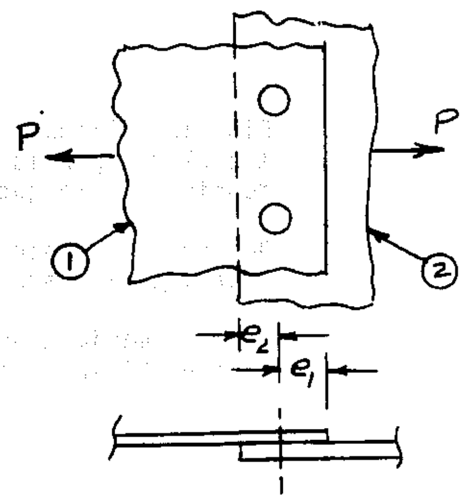
8.9

a) Find the ultimate allowable load for the .750 inch thick 2024-T3 forged lug in the sketch. The bolt diameter is 1.00 inch.

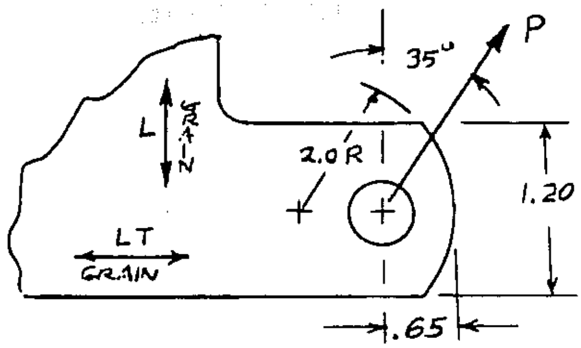


b) Determine the ultimate allowable load per fastener for the joint in the sketch. The fasteners are NAS 334 bolts.

Member	t	Material	e
1	.100	7178-T6 sheet	.35
2	.160	2024-T3 sheet	.30

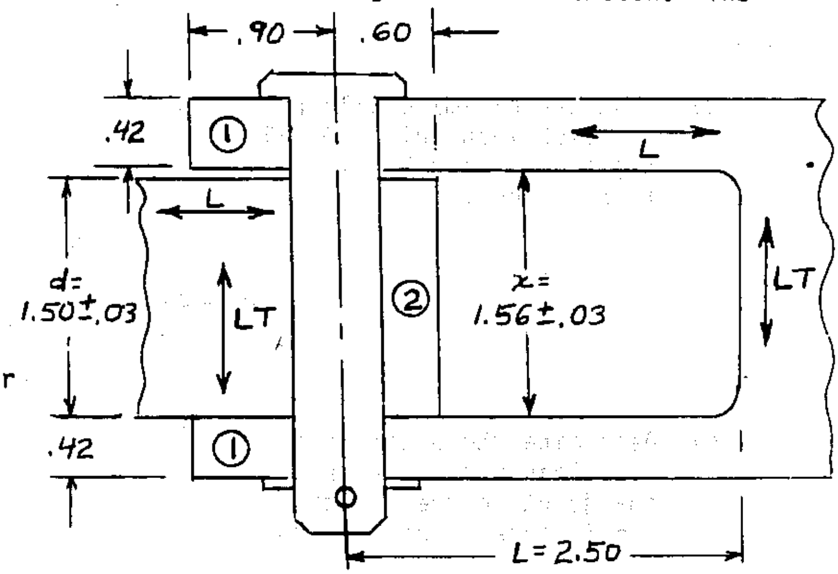


c) Determine the ultimate allowable load on the actuator support fitting shown in the sketch. The fitting is a 7178-T6 extrusion and has a thickness of 0.375 inch. The longitudinal (L) and long transverse (LT) grain directions are as shown. The bolt diameter is 0.500 inch.



8.10

a) Find the ultimate allowable load for the joint in the sketch. The male and female lugs are 1.80 inches wide and are machined from 3 inch 2024-T4 plate with grain directions as shown. The pin is a .500 inch diameter 160 ksi heat treat steel clevis pin, retained by a cotter pin and washer.



b) Find the ultimate allowable load for the above double shear joint if the clevis pin is replaced with a 160 ksi steel tension bolt with a tension nut torqued to 480-690 in lb.

c) Determine whether the outer (female) lugs in the previous problem can withstand the stresses due to clamp-up in torqueing the nut and bolt.

d) For the maximum gap shown in the sketch, find the minimum female lug length (L) that would allow clamp-up.

LESSON 9

CRIPPLING

9.1 COMPRESSION FAILURE MODES

Axial members carrying compression can fail in one of three modes:

- Block compression - A member with a very short length and a compact cross section can fail at a compression stress of about F_{tu}. This is very uncommon in aircraft structure.

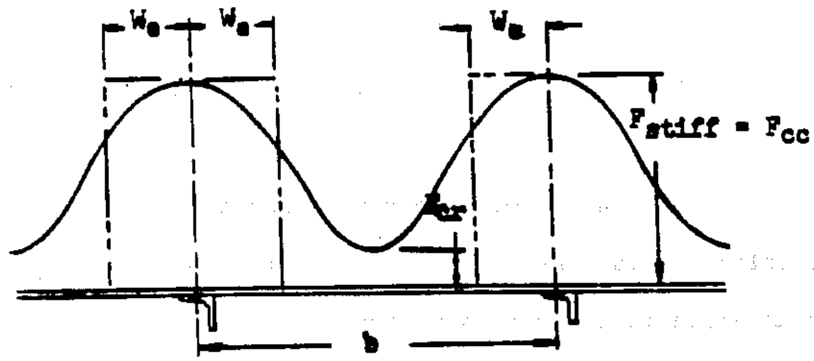
- Crippling - A member with thin flanges or members can fail at a low stress level by local crippling of one or more flanges. This mode is generally the critical one.

- Column - A long, slender member that is not restrained laterally can buckle at a low stress level. This is not very common because most members are restrained.

9.2 DEFINITION OF CRIPPLING

Such a failure is characterized by a local distortion of cross-sectional shape. The beginning of such distortion usually occurs at a stress appreciably less than the failing stress. The more stable portions of the cross-section continue to take additional load while supporting the already buckled portions until collapse occurs.

For example, consider a web with stiffeners as shown below. The buckling stress of the skin is much less than the crippling stress of the stiffener. Initially, upon application of a compression loading and up to the critical buckling load for the skin, the direct compressive stress is uniformly distributed.



Beyond this point, the center portion of the skin will buckle, and will be unable to carry additional load. The edges of the plate however, which are restrained by the stiffeners, can and do carry an increasing amount of load. The stress distribution is as shown in the previous sketch.

The load carried by the total panel width, (which causes a sinusoidal stress distribution to develop), can be approximated by the use of an "effective width" of skin which will be assumed to act at the stiffener stress level.

9.2.1 Parameters

Most compression members are composed of flanges which have different allowable crippling stresses. The ultimate or allowable crippling stress, F_{cc}, of a stiffener is computed from the formula:

$$Fcc = \frac{\Sigma\, b_n t_n\, Fcc_n}{\Sigma\, b_n t_n}$$

where: b_n is the width of an element

t_n is the thickness of an element

Fcc_n is the allowable crippling stress of an element, which is a function of F_{cy}, E_c and the ratio of flange width to thickness (b/t).

For stresses above the proportional limit, E_c is replaced by the secant modulus. Section 16 of McDonnell Report 339 contains crippling curves for several specific alloys and material forms and a non-dimensional curve that is applicable to any alloy at room or elevated temperature. For crippling stresses at elevated temperature, values of both F_{cy} and E at that temperature must be used.

9.2.2 Effects of Cladding

For alclad sheet metal, the cladding is assumed to carry no load. Therefore, the effective thickness is decreased by the total thickness of cladding. Also, the secondary compressive modulus, E_{c2} is used in place of E_c. Values of the secondary compressive modulus can be found in MIL-HDBK-5D or in the crippling section of MAC 339.

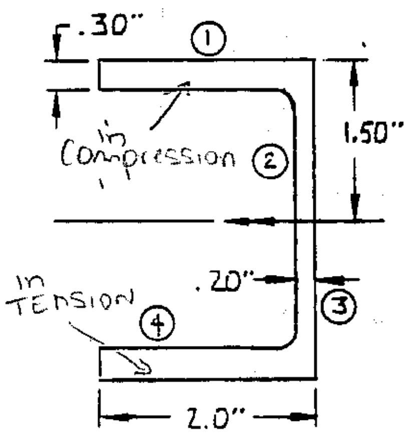
The thickness of cladding material varies with the alloy and material thickness. The following table shows the clad thickness per side as a percent of the sheet thickness.

Sheet Thickness \ Alloy	Clad Thickness per Side (%)		
	2024	7075	7178
Under .063	5	4	4
.063 through .187	2.5	2.5	2.5
Over .187	2.5	1.5	1.5

9.2.3 Compression Caps of Bending Beams

When the compression is due to bending, only the material in compression is included in the crippling analysis. In a case where part of a flange is in compression and part in tension, the width of the compression portion is used as b in the ratio b/t.

Example: Find the allowable crippling stress for the compression portion of the section shown assuming a 2024-T4 extrusion.



Element	b	t	b/t	bt	F_{cc}	btF_{cc}
①	1.90	.30	6.33	.57	44000	25080
②	1.35	.20	6.75	.27	57000	15390
				.84		40470

$$F_{cc} = \frac{\Sigma\,(btF_{cc})}{\Sigma\,(bt)} = \frac{40470}{.84} = 48200 \text{ PSI}$$

9.3 EFFECTIVE SKIN

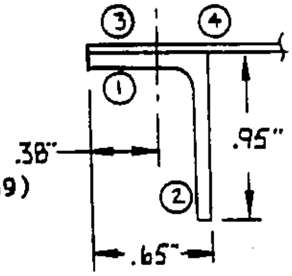
Axial members are usually attached to skins and/or shear webs which can carry a significant compression load because they are stabilized by the axial member. The procedure for analyzing effective skin in compression is explained in McDonnell Report 339 on pages 16.10 through 16.13. The procedure calculates an effective width of skin on either side of the skin fastener that can be assumed to act at the same stress level as the stiffener. Thus, the total width of effective skin is $2w_e$ unless reduced for a one edge free skin element or by having two rows of fasteners with less than $2w_e$ between them.

9.3.1 Non-Chem-Milled Skin, One Row of Fasteners

Example: .080 7075-T6511 extrusion and

.050 7178-T6 alclad skin

(Page numbers are from McDonnell Report 339)



Element	b	t	b/t	Edge Condition	F_{cc} (pg. 16.15)	bt	btF_{cc}
1	.61	.08	7.63	One Edge Free	58,000	.0488	2830
2	.91	.08	11.38	One Edge Free	42,000	.0728	3060
						.1216	5890

$$F_{cc} = \frac{\Sigma\,(btF_{cc})}{\Sigma\,(bt)} = \frac{5890}{.1216} = 48,440 \text{ PSI}$$

Since the average allowable crippling stress is based on elastic crippling allowable components, the plasticity factor, η, is one. (The plasticity factor, η is defined by the equation:)

$$\eta = \frac{E \text{ secant}}{E}$$

So, using the "effective width of stiffened sheet" chart on p. 16.13 of MAC 339 with:

$$\frac{(\eta E)\text{ skin}}{\sqrt{(\eta E)\text{ stiff}}} = \frac{(10.7 \times 10^3\text{KSI})}{\sqrt{10.7 \times 10^3\text{KSI}}} = 103 \approx 100$$

a value of $\frac{2w_e}{t} = 35$ is computed.

'34

Cladding material thickness per side is 2 1/2% of sheet thickness.

effective $t = .050'' (1 - 2 \times .025) = .0475$ in.

$2w_e = 35.0 \times .0475'' = 1.66''$

For a one-edge free skin element, use:

$\frac{w_e}{t} = .35 \left(\frac{2w_e}{t}\right)$ chart

So: $w_{e3} = .35\,(1.66'') = .58''$

but is actually limited to the edge distance of .38 in.

For the effective width of element 4 (no edge free)

$w_{e4} = 1/2\,(2w_e) = (0.5)(1.66 \text{ in}) = .83$ in.

Total width is $w_{e3} + w_{e4} = .38'' + .83'' = 1.21$ in.

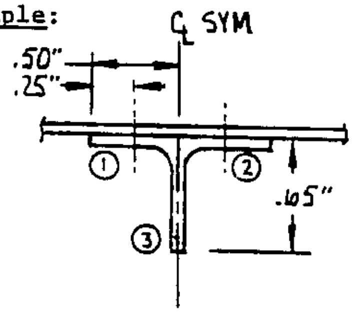
Effective skin area $= 1.21'' \times .0475'' = .0575 \text{ in}^2$.

$P_{c_{all}} = F_{cc}A = 48{,}440\text{psi}\,(.1216 \text{ in}^2 + .0575 \text{ in}^2) = \underline{8676^{\#}}$

9.3.2 Non-Chem-Milled Skin, Two rows of fasteners

The method developed in section 9.3.1 for determining the effective skin widths is also applicable to stiffeners attaching to a skin with two or more rows. The only change in the method is that any overlap of effective skin widths are neglected.

Example:



.063" 7178-T6 Skin

.063" 7075-T6511 Extrusion

Element	b	t	b/t	Edge Condition	F_{cc}(psi)	bt(in^2)	bt F_{cc}(lb)
1	.50	.063	7.9	One Edge Free	57,000	.032	1824
2	.50	.063	7.9	One Edge Free	57,000	.032	1824
3	.62	.063	9.8	One Edge Free	47,500	.039	1853
						.103	5501

$$F_{cc} = \frac{\Sigma F_{cc}bt}{\Sigma bt} = \frac{5501 \text{ lb}}{.103 \text{ in}^2} = 53{,}410 \text{ psi}$$

For $\eta = 1.0$

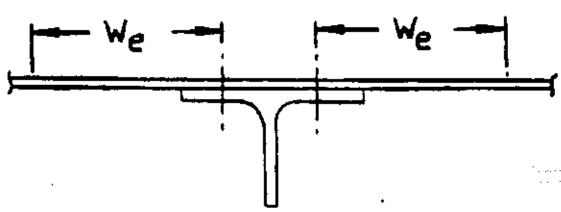
$$\frac{(\eta E) \text{ skin}}{\sqrt{(\eta E) \text{ stiff}}} = \frac{10.7 \times 10^3 \text{KSI}}{\sqrt{10.7 \times 10^3 \text{KSI}}} = 103 \approx 100$$

From p. 16.13 of MAC 339:

$$\frac{2w_e}{t} = 33$$

$$w_e = (\frac{33}{2})(.063 \text{ in.}) = 1.04 \text{ in.}$$

The effective skin used shall extend a distance "w_e" outboard of the two fasteners and also include the space between the fasteners. If this space exceeds $2w_e$, the effective material utilized shall be equal to a length "w_e" inboard of each fastener.



So the allowable crippling load for this section is:

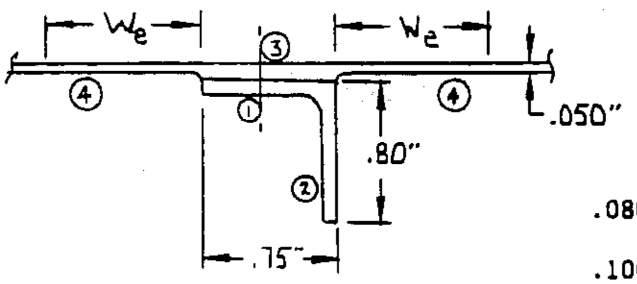
$$P_{c_{all}} = (53{,}410\text{psi})[.103\ \text{in}^2 + (.50")(.063") + (.063")(1.04")(2)]$$

$$P_{c_{all}} = 14{,}180\ \text{lb.}$$

9.3.3 Chem-Milled Skin, One Row of Fasteners

If the skin includes a chem-milled land that attaches to the axial member with one row of rivets, the effective skin widths are determined by the method of section 9.3.1, and are measured from the edge of the land.

Example:



.080" 7075-T6511 Extrusion

.100" 7178-T6 Chem-Milled skin as shown

Element	b	t	b/t	Edge Condition	F_{cc}(psi)	bt	bt F_{cc}
1	.71	.08	8.9	One Edge Free	51,500	.057	2936
2	.76	.08	9.5	One Edge Free	49,000	.061	2989
						.118	5925

$$F_{cc} = \frac{\Sigma\ btF_{cc}}{\Sigma\ bt} = \frac{5925\ lb}{.118\ in^2} = 50210\ PSI$$

$$\eta = 1.0$$

From p. 16.13 of MAC 339:

$$\frac{(10.7 \times 10^3\ KSI)}{\sqrt{10.7 \times 10^3\ KSI}} \approx 100$$

gives a value of $\frac{2w_e}{t} = 34$

So the effective width of elements 4 (no edge free)

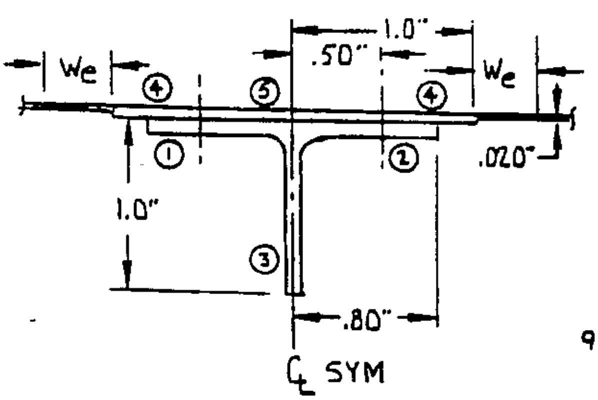
$$w_e = \frac{34t}{2} = (17)(.05\ in.) = .85\ in.$$

$$P_{c_{all}} = (50210\ psi)[(.118\ in^2) + (.10)(.75") + 2(.85")(.05")]$$

$$P_{c_{all}} = 13960\ lb$$

9.3.4 Chem-milled skin, two rows of fasteners

Example:



7178-T6 skin .050"

7075-T6511 extrusion .080"

The analysis procedure of this type of stiffener is slightly different than the method used up to this point. In this situation, use the skin material between the fasteners as a separate flange, and then treat the land material beyond the rivet lines as acting at the same stress level as the stiffener. That is:

Element	b	t	b/t	Edge Condition	F_{cc}(psi)	bt(in^2)	bt F_{cc}(lb)
1	.80"	.08"	10	One Edge Free	47,000	.064	3008
2	.80"	.08"	10	One Edge Free	47,000	.064	3008
3	.96"	.08"	12	One Edge Free	40,000	.077	3080
5	1.00"	.05"	20	No Edge Free	68,000	.050	3400

$$F_{cc} = \frac{Fcc_1 b_1 t_1 + Fcc_2 b_2 t_2 + Fcc_3 b_3 t_3}{b_1 t_1 + b_2 t_2 + b_3 t_3} = \frac{9096 \text{ lb}}{.205 \text{ in}^2} = 44{,}370 \text{ psi}$$

determine the effective width of skin from p. 16.13 of MAC 339

$$\frac{(\eta E)\text{ skin}}{\sqrt{(\eta E)\text{ stiff}}} = 100$$

$$\frac{2w_e}{t} = 36$$

$$W_e = \frac{1}{2}(36)(.020") = .36 \text{ in.}$$

This width, along with the distance between the fastener and the chem-mill, acts at the average stiffener stress. Its cross-sectional area is:

$$A_4 = (.50 \text{ in})(.05 \text{ in}) + (.36 \text{ in})(.020 \text{ in}) = .032 \text{ in}^2$$

Now, the total allowable crippling load can be determined:

$$P_{cc} = \left[\frac{Fcc_1 b_1 t_1 + Fcc_2 b_2 t_2 + Fcc_3 b_3 t_3}{b_1 t_1 + b_2 t_2 + b_3 t_3}\right](A_{stiff} + 2A_4) + [Fcc_5 b_5 t_5]$$

$$P_{cc} = (44{,}370 \text{ psi})[.205 \text{ in}^2 + 2\,(.032 \text{ in}^2)] + 3400 \text{ lb}$$

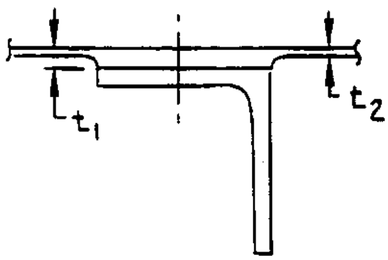
$$P_{cc} = 11940 \text{ lb} + 3400 \text{ lb} = 15340 \text{ lb}$$

For the preceding analysis of chem-milled skins, it was assumed that the thinned-down thickness was considerably less than the land thickness, so the effective width was measured off of the end of the chem-milled land. Some aircraft applications do not fall into this category. To determine whether to measure the effective width from the fastener or the edge of the chem-milled land, compare the magnitude of,

$$\left(\frac{W_e}{t}\right)(t_1) \text{ and } ED + \left(\frac{W_e}{t}\right)(t_2).$$ (ED = Distance from center of fastener to edge of land)

If $ED + \left(\frac{W_e}{t}\right)(t_2)$ exceeds $\left(\frac{W_e}{t}\right) t_1$, the effective width shall be measured from the fastener.

For example:



<u>Given:</u> $\frac{2W_e}{t} = 35$ from p. 16.13 of MAC 339

<u>Case 1:</u> ED =.50", t_1 =.063", t_2 =.050"

determine whether $\left(\frac{W_e}{t}\right)(t_1) \geq ED+\left(\frac{W_e}{t}\right) t_2$

$\left(\frac{W_e}{t}\right)(t_1) = (17.5)(.063") = 1.10$ in.

$\left(\frac{W_e}{t}\right)(t_2)+ED = (17.5)(.050")+.50" = 1.375$ in.

Since $ED + \left(\frac{W_e}{t}\right) t_2$ exceeds $\left(\frac{W_e}{t}\right)t_1$, the effective width should be measured from the fastener.

<u>Case 2:</u> ED = .50", t_1 = .063", t_2 = .032"

$\left(\frac{W_e}{t}\right)(t_1) = (17.5)(.063 \text{ in}) = 1.10$ in.

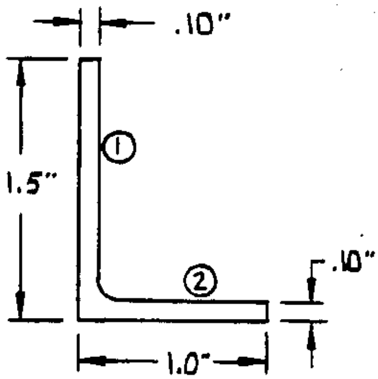
$\left(\frac{W_e}{t}\right)(t_2) + ED = (17.5)(.032 \text{ in}) + .50 \text{ in.} = 1.06$ in.

$ED + \left(\frac{W_e}{t}\right)t_2 < \left(\frac{W_e}{t}\right)t_1$, so the effective width should be measured from the edge of the chem-milled land.

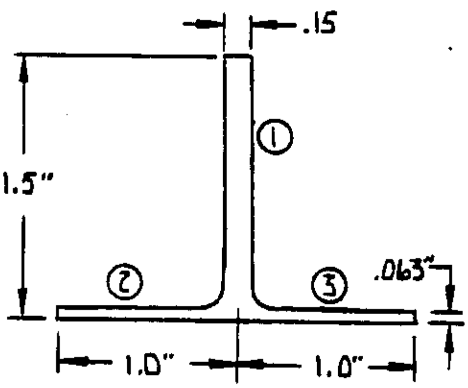
PROBLEMS

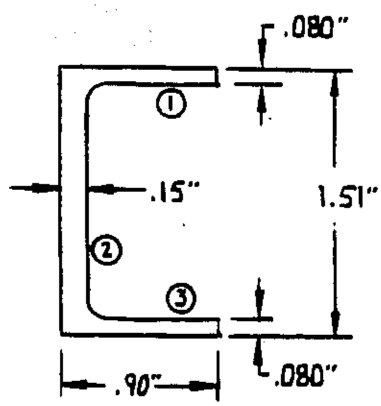
LESSON 9 CRIPPLING

9.1 Find the crippling stress and load for an alclad sheet metal angle, a bare sheet metal angle, and an extruded angle, all with the same dimensions. Material is 7075-T6.



9.2 Find the crippling stress and load of an extruded aluminum tee and a channel of the same area.



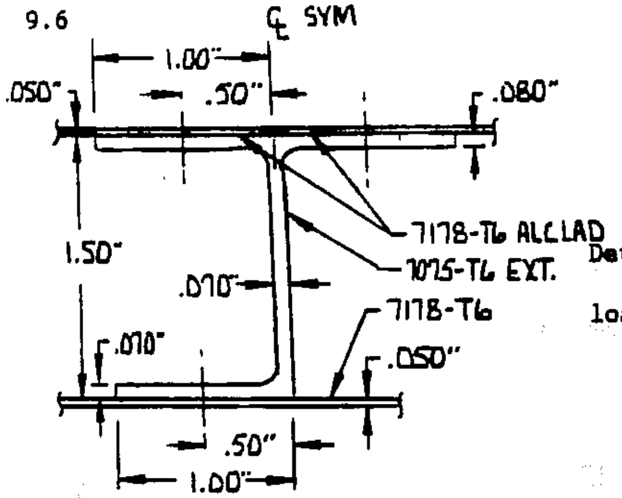


9.3 Find the crippling stress and load for the tee in problem 9.2 at a temperature of 300°F. (10 hr. exposure)

9.4 Find the crippling stress and load for the tee in problem 9.2 if the material is 6Al-4V titanium.

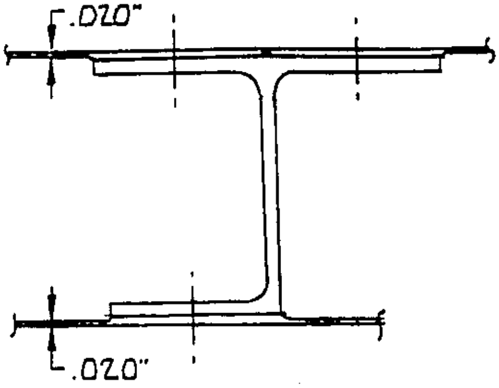
9.5 Find the crippling stress and load for the tee in problem 9.4 at a temperature of 300°F. (10 hr. exposure)

9.6



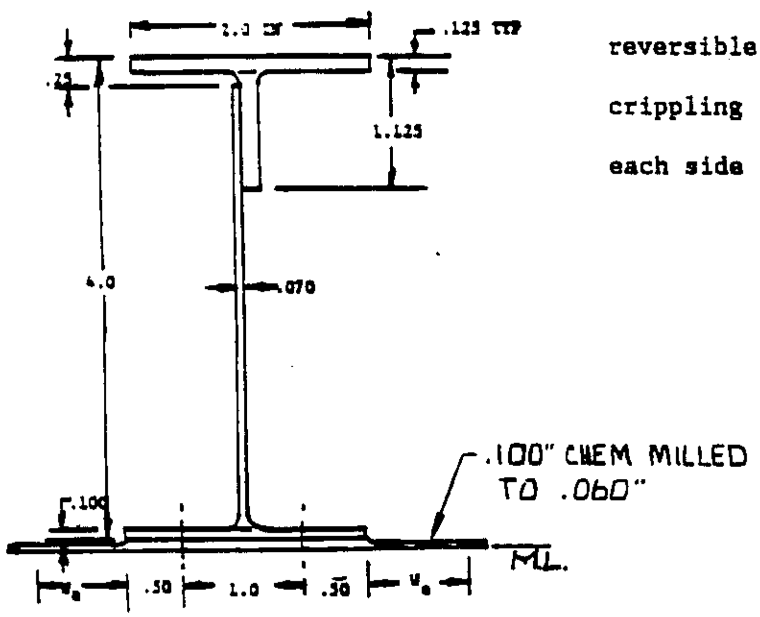
Determine the allowable crippling load for the section shown at left.

9.7



Determine the allowable crippling load for the section shown at left. All dimensions not shown are same as for problem 9.6.

9.8



If the section shown is loaded by reversible bending, find the crippling stress of the material on each side of the centroid.

LESSON 10

COLUMNS

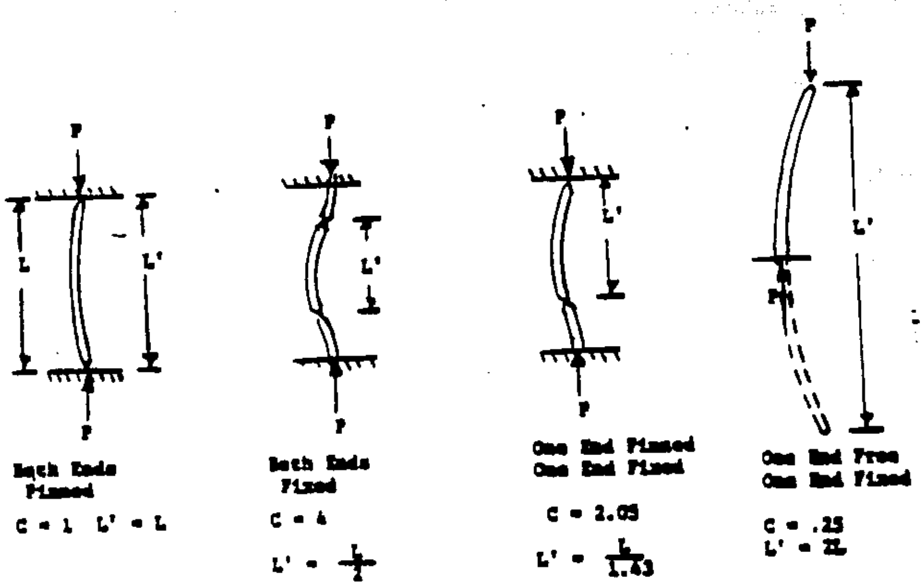
10.1 DEFINITIONS

A column is a straight slender bar which is acted upon by an axial compressive load. Buckling of the column arises out of a condition of neutral equilibrium, when the applied load on the member reaches a critical value, P_{cr}.

10.2 LONG COLUMNS

Long columns are analyzed by the Euler equation: $P_{cr} = \frac{\pi^2 EI}{L^2}$

where P_{cr} is the critical load and L is the length of a pin-ended column. When the column is not pin-ended, L is replaced by an effective pin-ended, length $L' = \frac{L}{C}$.



The equation for critical stress is obtained by dividing both sides of the Euler equation by the area and substituting ρ^2 for I/A:

$$F_c = \frac{\pi^2 E I}{(L')^2 A} = \frac{\pi^2 E \rho^2}{(L')^2} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2}$$

For stress levels above the proportional limit, the tangent modulus E_t is substituted for E.

10.3 SHORT COLUMNS

As the value L'/ρ decreases, the Euler equation indicates higher buckling stresses. At high stress levels, the Euler equation is not reliable because of an interaction between the column buckling mode of failure and the crippling mode. This interaction results in a lower allowable stress than predicted for either mode. In this short column range the Johnson parabola, an empirical column curve, is used. The Johnson column formula is:

$$F_c = F_{cc} - \frac{F_{cc}^2 (L'/\rho)^2}{4\pi^2 E}$$

where F_c is critical stress and F_{cc} is crippling stress

This formula is valid if:

$$\frac{L'}{\rho} < \pi \sqrt{\frac{2E}{F_{cc}}}$$

Pages 16.31 through 16.31.03 of McDonnell Report 339 presents plots of a series of Johnson parabolas super-imposed on plots of the Euler equation. Each page is for a different value of E. These plots can be used for any value of L'/ρ up to 100, whether short or long. The plots are used as follows:

1. Determine crippling stress F_{cc} of the column section. (See Lesson 9)

2. Locate F_{cc} on the vertical (F_c) axis of the appropriate plot.

3. Follow a parabolic curve, interpolating between plotted curves, to the value of L'/ρ on the horizontal axis.

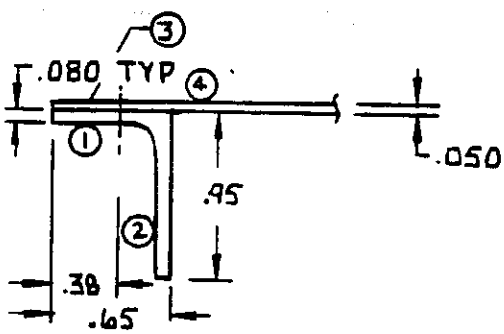
4. From that point, follow a horizontal line back to the vertical axis. This intercept is at the value of F_c.

Example: Using the section below, find the allowable column stress and load for L'/ρ of 45 and 70.

.080 7075-T6511 extrusion and

.050 7178-T6 alclad skin

Page numbers are from McDonnell Report 339.



Element	b	t	b/t	Edge Condition	F_{cc} (pg. 16.15)	bt	btF_{cc}
1	.61	.08	7.63	One Edge Free	58,000	.0488	2830
2	.91	.08	11.39	One Edge Free	42,000	.0728	3060
						.1216	5890

$$F_{cc} = \frac{\Sigma btF_{cc}}{\Sigma bt} = \frac{5890}{.1216} = 48{,}440 \text{ Psi}$$

$$\frac{2w_e}{t} = 35.0 \text{ (page 16.13)}$$

Cladding material thickness per side is 2 1/2% of sheet thickness.

effective t = .050″(1 - 2 x .025) = .0475 in. (page 16.11)

$2w_e$ = 35.0 x .045″ = 1.66″

w_{e_1} = .35 ($2w_e$) = .35 (1.66″) = .58 in. (but limited to actual edge distance of .38 in.) (page 16.11)

w_{e_2} = .5 ($2w_e$) = .5 x 1.66″ = .83 in. (page 16.11)

Total width is $w_{e_1} + w_{e_2}$ = .38″ + .83″ = 1.21 in.

Effective skin area = 1.21″ x .0475″ = .0575 in^2.

F_{cc} = 48,440 Psi A = .1216 + .0575 = .1791 in^2

The material is aluminum with E = 10,200,000 Psi

For L'/ρ = 45: F_c = 36,500 Psi; P_{cr} = 6540# (p. 16.31, MAC 339)

For L'/ρ = 70: F_c = 20,500 Psi; P_{cr} = 3670#

Note that for L'/ρ greater than about 63, the Johnson parabola and the Euler curve are colinear, showing that this is in the long column range.

10.4 COLUMNS WITH DISTRIBUTED AXIAL LOAD

Axial members, including columns, usually do not have a constant load over their length; but if loaded by shear flow, have a uniformly varying load. Analyzing for an average load is unconservative so a method of analysis is included in McDonnell Report 339 on page 16.38. Note that the smaller compression can actually be compression or tension. The method is valid for long and short columns.

10.5 STEPPED COLUMNS

So far, we have only considered columns of constant section where the value of EI is constant over the length. Such is often not the case. McDonnell Report 339 shows a method of analyzing stepped columns on pages 16.40 through 16.43. The examples shown are self-explanatory.

10.6 BEAM COLUMNS

Frequently, a column will have bending loads in addition to the column load. The bending deflections cause an eccentricity for the column load which, in turn, causes additional bending moment.

Analysis of beam columns requires the determination of the final bending moment and checking for the combination of bending and compression. A method of analysis is available in MAC 339, p.16.50 through p. 16.51. Various other methods are presented in aircraft engineering texts, such as Peery or Bruhn.

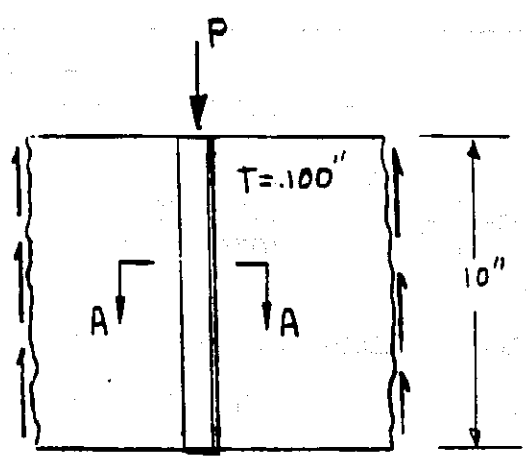
The principle of superposition of loads must be modified for beam columns. The total bending moment for a combination of transverse loads with a column load can be found by summing the bending moments for the separate transverse loads, each combined with the total axial load.

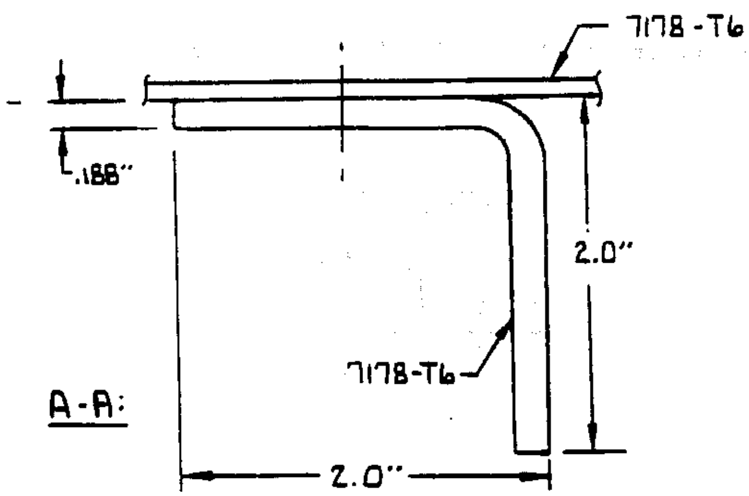
4. Determine the allowable ultimate compression load and stress for the strut in problem 1 if one end is fixed and one end pinned.

5. Determine the allowable ultimate compression load and stress for the strut in problem 1 if both ends are fixed.

6. Determine the allowable ultimate compression load and stress for the strut in problem 1 if one end is fixed and one end is free.

7. For the stiffener below, determine the allowable end load which can be applied to the end of the angle stiffener:

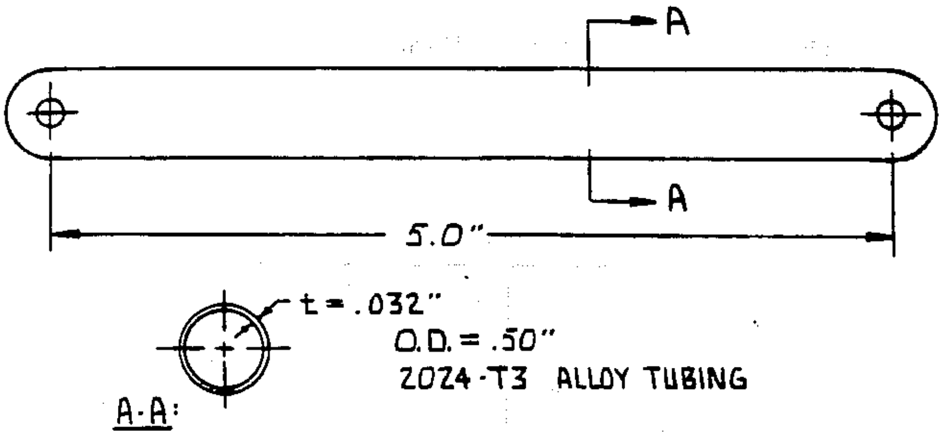




LESSON 10 - COLUMNS

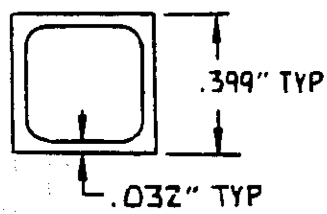
HOMEWORK PROBLEMS

1. Determine the allowable ultimate compression load and stress for the pin ended strut shown below.

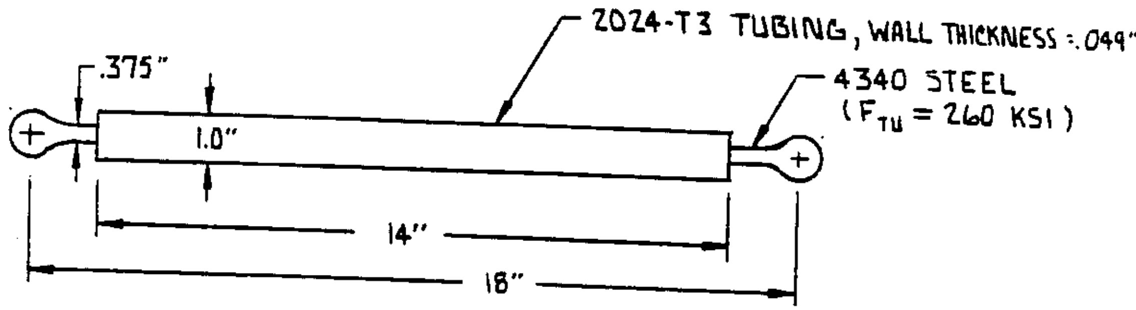


2. Determine the allowable ultimate compression load and stress for the pin-ended strut in problem 1, if the column length is 15 in.

3. Determine the allowable ultimate compression load and stress for the pin-ended strut in problem 1, if the column's cross section is as shown below:



8. For the strut shown below, determine the allowable compressive end load. Analyze it as a symmetrical stepped column.



9. Using beam column analysis, determine the margin of safety for the column above if a 100 lb load is applied perpendicular to the strut at mid span.

LESSON 11

SHEAR FLOW

11.1 Definition and Explanation

In lesson 5, Internal Loads, the concept of shear flow was introduced. Recalling the definition given earlier, shear flow is the shear load per unit length, generally expressed in pounds per inch. It is usually a more convenient parameter than load or stress to use in load and stress analysis. It is represented by the symbol "q" and a semi-headed arrow (⇀).

The shear flow in a web is the shear load divided by the web height. Thus:

$$q = \frac{V}{h}$$

The shear stress in a web is the shear load divided by the web area. Or:

$$f_s = \frac{V}{h} \cdot \frac{1}{t}$$

Substituting the expression for shear flow into the equation for shear stress gives:

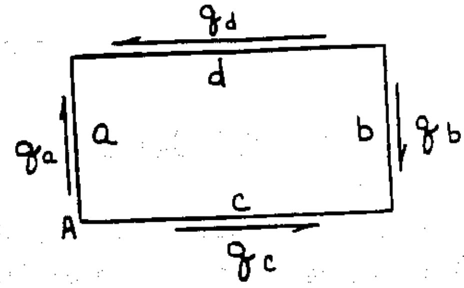
$$f_s = \frac{q}{t}$$

11.2 Balance of Panels Loaded in Shear

All shear panels are quadrilaterals, either rectangular, trapezoidal, or non-trapezoidal. When the shear flow on one edge of a panel is known, the shear flows at the other edges can be found using the laws of statics.

11.2.1 Rectangular Panels

Rectangular panels have the same shear flow on all four edges. The proof of this is as follows: For the summation of vertical and horizontal forces to equal zero, $q_a = q_b$ and $q_c = q_d$. For the panel to be in equilibrium, the summation of moments about any point must equal zero:



For example:

$$\Sigma M_A = (q_b \cdot b)c - (q_d \cdot d)a = 0$$

$q_b \cdot b \cdot c = q_d \cdot d \cdot a$

Since this is a rectangle: SUBST $q_b \cdot a \cdot d = q_d \cdot d \cdot a$

$a = b$, and $c = d$

$q_b = q_d$

Substitution gives:

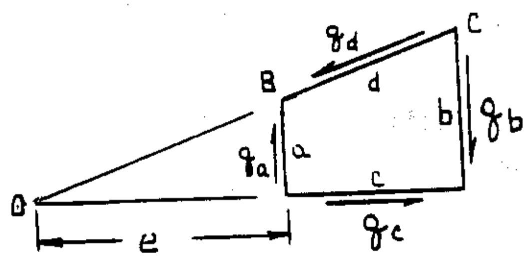
$$q_b = q_d$$

11.2.2 Trapezoidal Panels

Trapezoidal panels have the same shear flow on the two non-parallel sides. For the sum of the moments about point O to equal zero:

$$\Sigma M_O = 0: \quad q_a \times a \times e = q_b \times b(c + e)$$

$$\text{or:} \quad q_b = q_a \times \frac{ae}{b(c + e)}$$



Using the properties of similar triangles: $\dfrac{e}{c + e} = \dfrac{a}{b}$

Therefore, $q_b = q_a \left[\dfrac{a}{b}\right]^2$

Now for $\Sigma M_B = 0$:

$$q_c \times c \times a = q_b \times b \times c$$

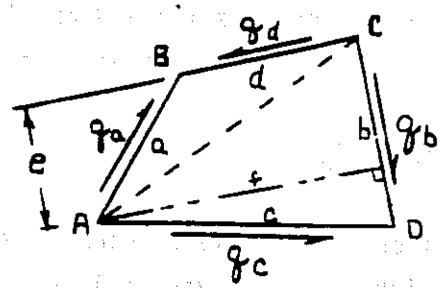
$$\text{or:} \quad q_c = q_b \frac{b}{a}$$

and by summing horizontal forces or taking moments about point c it can be shown that:

$$q_d = q_c$$

11.2.3 Quadrilateral, Non-Trapezoidal Panels

Non-trapezoidal shear panels have four different shear flows. If one shear flow is known, then the other shear flows can be found by taking moments about three of the corners, and checking the results by taking moments about the fourth corner.



For example, for $\Sigma M_A = 0$:

$0 = q_b \times b \times f - q_d \times d \times e$

Solving for q_b: $q_b = q_d \dfrac{de}{bf}$

Note that the first term in the moment equation above is equal to the product of the shear flow and twice the area of the triangle ACD:

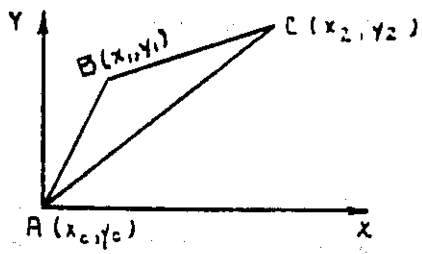
$$M_{qb} = q_b \times b \times f = q_b \times (b \times f)$$

$$\text{Or: } M_{qb} = 2A_{ACD} \times q_b.$$

Therefore the following statement can be made: A shear induced moment acting about <u>any</u> point in a field can be computed by multiplying the shear flow by twice the area of the triangle described by the two boundary points for the shear flow and the point about which the moment is being computed.

At least two methods exist to compute the area of a triangle. The first method as used in the previous example determines the area by finding the product of one-half the base and the altitude. The second method is useful if the coordinates of the corners are known. For example, consider the triangle ABC at right, with the coordinates as shown. The area of the triangle can be found by:

$$A_{ABC} = \left| \frac{(x_2-x_0)(y_1-y_0)-(x_1-x_0)(y_2-y_0)}{2} \right|$$

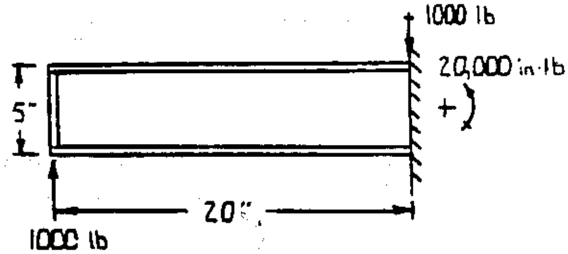


This can be proven, if desired, by using the principle of superposition.

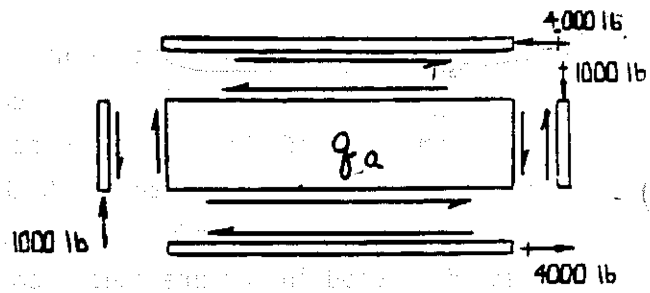
11.3 Shear Panel and Cap Balances

Since shear webs do not exist by themselves, but as a component of a larger detail, such as a channel or I section, the method of balancing these sections is of interest to us. The internal loads in sheet metal structure are found by making a "stick and panel" (or "cap and web") balance. With this arrangement, caps carry all axial loading and webs carry all shear loading.

11.3.1 Single Panel



Consider the cantilever beam shown at right with a 1000 lb vertical load acting upward at its end. For static equilibrium, a 1000 lb vertical upward force and a 20,000 in-lb ccw moment is required at the wall.



The beam can now be broken down into "sticks and panels" as shown at right. To balance the elements, first consider the externally loaded element. For it to balance, a 1000 lb force must be applied opposite to the load. The shear flow is then found:

$$q_a = \frac{F}{h} = \frac{1000 \text{ lb}}{5.0 \text{ in}} = 200 \text{ lb/in}$$

The balance for the panel is determined next. The shear flow on the left side is known from the preceding discussion. Since this panel is rectangular, the shear flow is the same on all four sides. Thus:

$$q_a = q_b = q_c = q_d = 200 \text{ lb/in.}$$

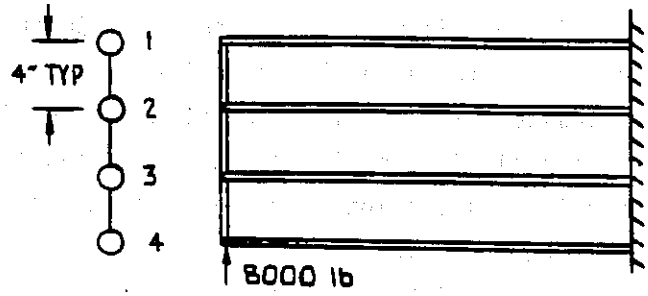
The shear flow in the upper and lower caps is equal to and opposite the shear flow along the top and bottom edges of the panel. Now, to balance the cap, a reaction of (200 lb/in)(20 in) is required. Note that the 4000 lb reactions form a couple producing a 20,000 in-lb moment, which was required to balance the beam initially. The only element remaining which needs to be balanced is the right vertical "stick". Its magnitude is found by

$$F_R = (200 \text{ lb/in})(5 \text{ in}) = 1000 \text{ lb}.$$

(Note that this is also the expected reaction).

11.3.2 Multiple Panel

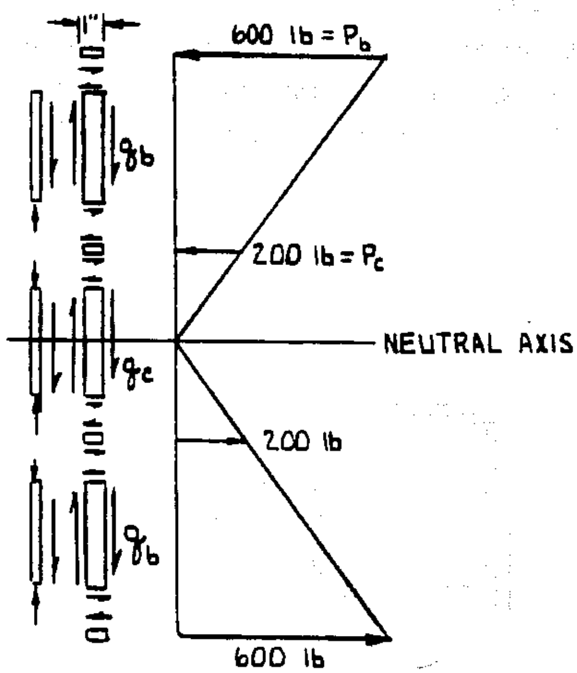
Analysis of multiple panels is similar to analysis of a single panel except that the cap loads need to be determined first. Consider the cantilever beam shown at right, with cap areas = .50 in^2 each. The moment of inertia about the center of the beam is:



$$I = 2A_1y_1^2 + 2A_2y_2^2$$

$$I = 2(.50 \text{ in}^2)(6 \text{ in})^2 + 2(.50 \text{ in}^2)(2 \text{ in})^2$$

$$I = 40 \text{ in}^4$$



For cap #1 at x = 1 in. :

$$f_b = \frac{Mc}{I} = \frac{(8000 \text{ lb} \times 1 \text{ in})(6 \text{ in})}{40 \text{ in}^4}$$

$$f_b = 1200 \text{ psi}$$

$$P_b = f_bA = (1200 \text{ psi})(.50 \text{ in}^2)$$

$$P_b = 600 \text{ lb}$$

For cap #2 at x = 1 in. :

$$f_c = \frac{M_c}{I} = \frac{(8000 \text{ lb} \times 1 \text{ in})(2 \text{ in})}{40 \text{ in}^4}$$

$$f_c = 400 \text{ psi}$$

$$P_c = f_cA = (400 \text{ psi})(.50 \text{ in}^2)$$

$$P_c = 200 \text{ lb}$$

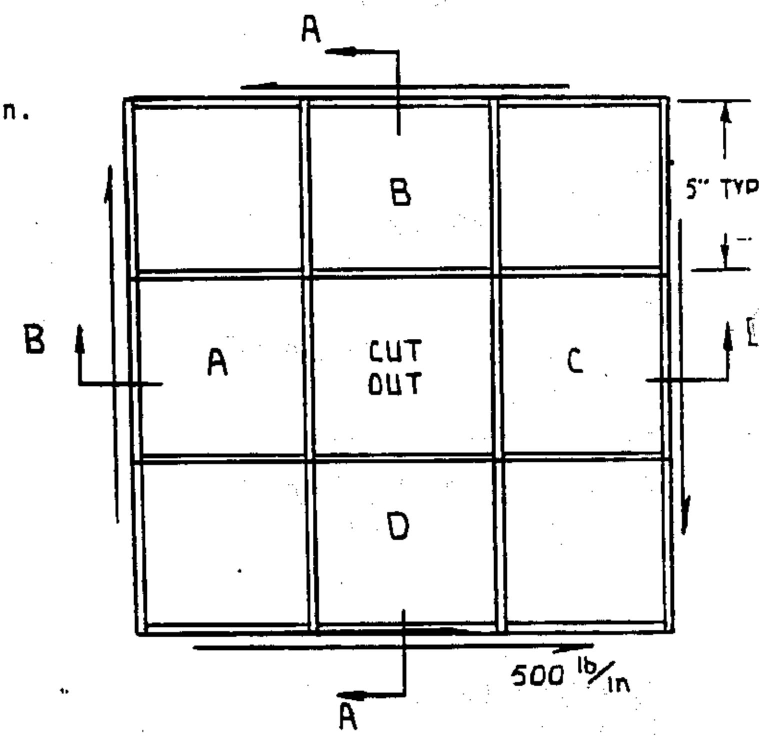
To determine the "stick and panel" shear flows, start with the uppermost cap. The shear flow is: q_b = (600 lb)/(1 in) = 600 lb/in. Since the uppermost panel is rectangular, the shear flows around the edges are equal. The shear flow for the cap second from the top, is found by:

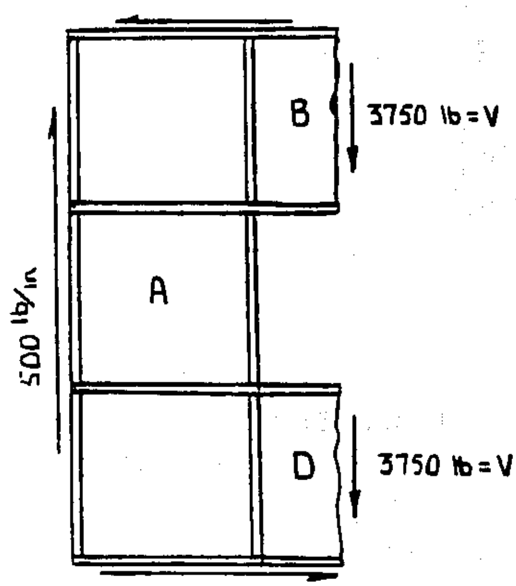
$$q_c = \frac{(P_c + (q_b)(1 \text{ in}))}{(1 \text{ in})} = \frac{(200 \text{ lb} + (600 \text{ lb/in})(1 \text{ in}))}{(1 \text{ in})} = 800 \text{ lb/in}.$$

Note that the shear flow in the middle panel is greater than in the outer panels. The shear flow for the other edges of the middle panel also equals q_c. The shear flows for the remaining "sticks and panels" are found in a similar manner.

11.3.3 Multiple Panel with Cutouts

When holes are placed in shear webs, the original strength of the web can be regained by using doublers or by "framing out" the holes. The use of doublers will be discussed in lesson 13. Cutouts that are too large for a doubler can be "framed out" with stiffeners which divide the panel into several smaller panels. The load distribution in these smaller panels is often statically indeterminant, but the redundancy can usually be resolved by geometrical considerations, such as symmetry or proportion. As an example of this technique, consider the panel at the right which has a known external shear flow, q = 500 lb/in. First, determine the internal shear flows of panels A,B,C, and D. Taking a section at A-A:





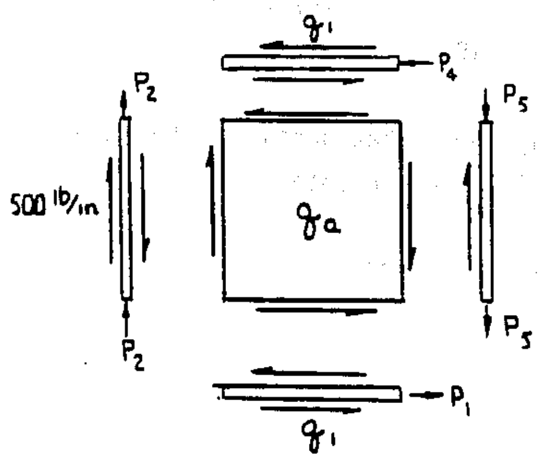
SECT. A - A

For panels B and D: $V = \frac{7500\ \text{lb}}{2} = 3750$ lb (due to symmetry)

$$q_{int} = \frac{3750\ \text{lb}}{5\ \text{in}} = 750\ \text{lb/in}$$

Because this panel is similar across section B-B, the same value of q_{int} occurs in panels A and C. From these internal shear flows and the external loads, the entire internal balance can be determined.

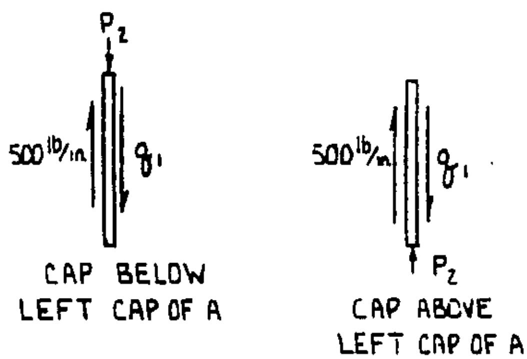
Begin with any panel adjacent to the cutout. Since panel A is rectangular, the shear flows on all four sides are equal. Now move to the bottom cap for this panel, and determine the internal force required to keep it in equilibrium:



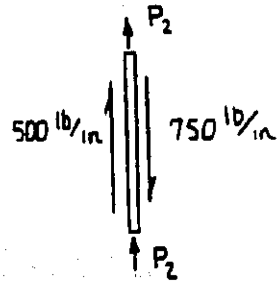
$$P1 = (750\ \text{lb/in})(5\ \text{in}) - (q_1\ \text{lb/in})(5\ \text{in})$$

The internal force required to balance the upper cap of panel A is $P_4 = P_1$, by symmetry. The panels directly above and below panel A will have shear flows = q_1. The cap above or below the left cap of A will be balanced by:

$$P_2 = (500\ \text{lb/in})(5\ \text{in}) - (q_1)(5\ \text{in})$$



By balancing the left cap for panel A, we can solve for the unknown, P_2:

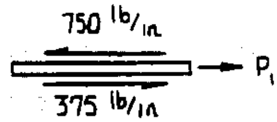


$$2(P_2) = (750 \text{ lb/in})(5 \text{ in}) - (500 \text{ lb/in})(5 \text{ in})$$
$$= 1250 \text{ lb}$$
$$P_2 = 625 \text{ lb}$$

Use this fact to solve for q_1:

$$625 \text{ lb} = (500 \text{ lb/in})(5 \text{ in}) - (q_1)(5 \text{ in})$$
$$q_1 = 375 \text{ lb/in}.$$

P_1 can now be determined:

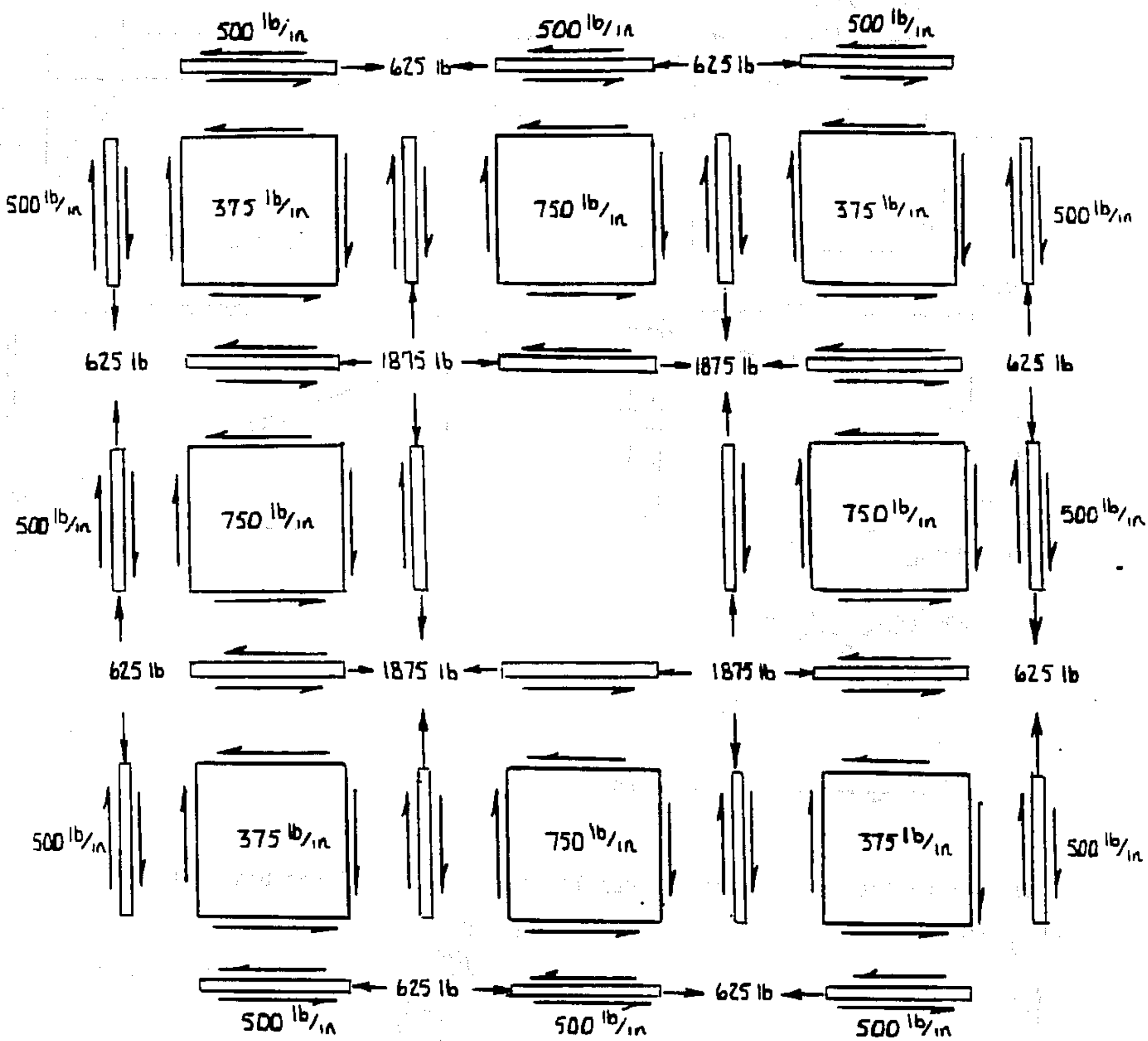


$$P_1 = (750 \text{ lb/in})(5 \text{ in}) - (375 \text{ lb/in})(5 \text{ in})$$
$$P_1 = 1875 \text{ lb}$$

Finally, note that panel D's left cap is similar to panel A's lower cap. This fact allows us to determine P_5:

$$P_5 = P_1 = 1875 \text{ lb}$$

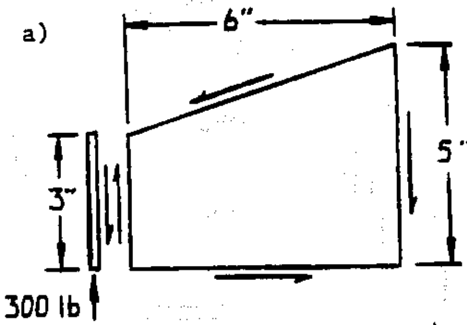
All other internal shear flows and loads can be determined from symmetry:



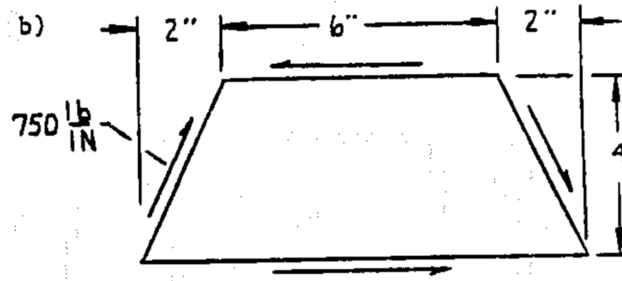
Note that the panels adjacent to the cutout have larger shear flows than that applied, and the panels located diagonally from the cutout have smaller shear flows.

11.1 Determine the shear flows on the other three edges of the shear panels shown.

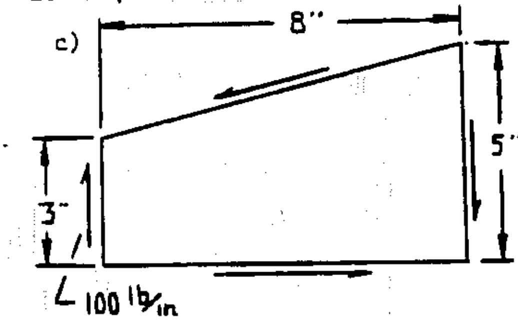
a)



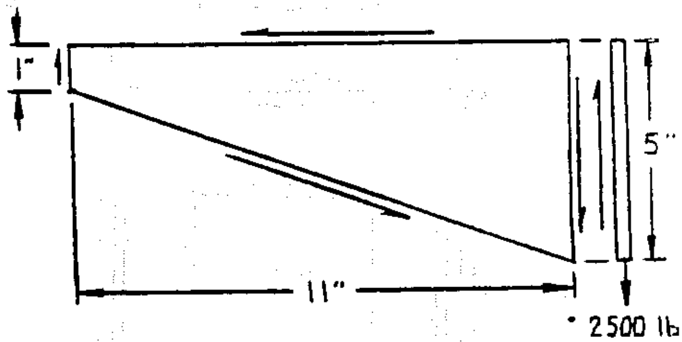
b)



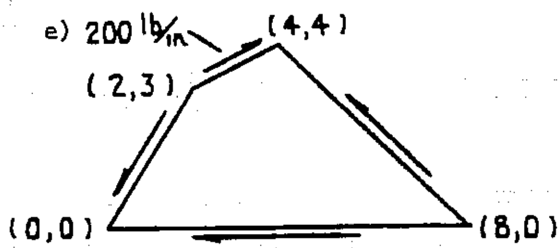
c)



d)

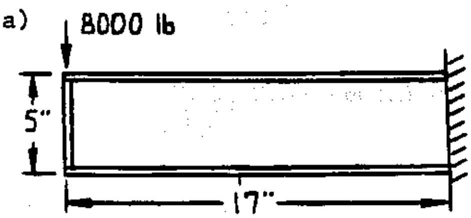


e)

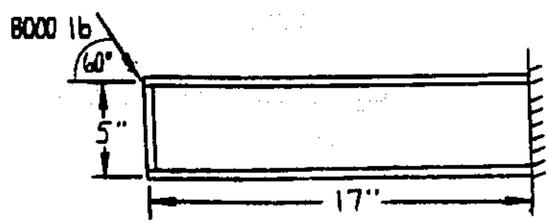


11.2 Find the shear flows and cap loads in the cantilever beams shown:

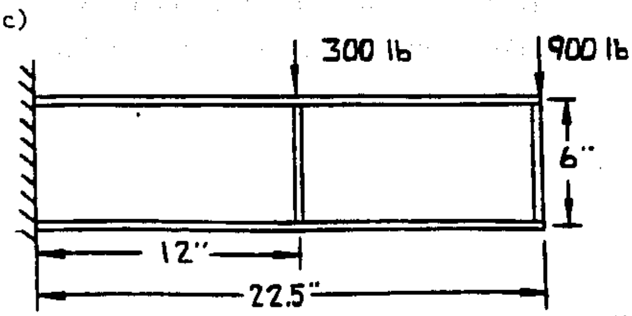
a)



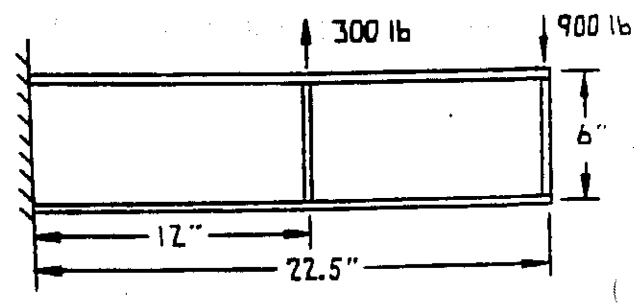
b)



c)

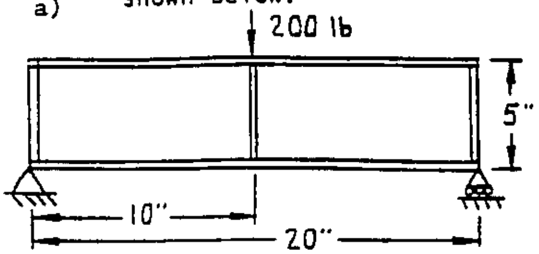


d)

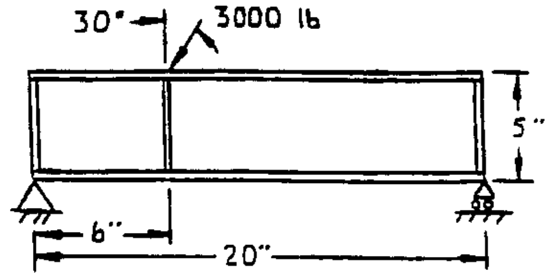


11.3 Determine the shear flows and cap loads in the simply supported beams shown below.

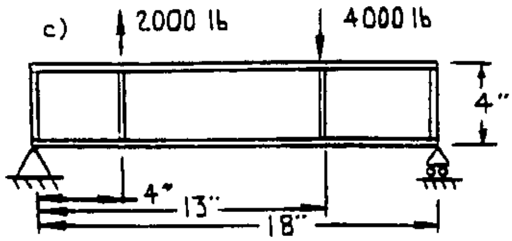
a)



b)

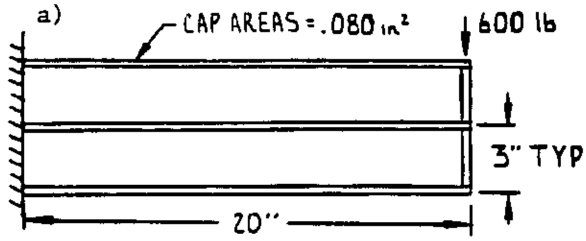


c)

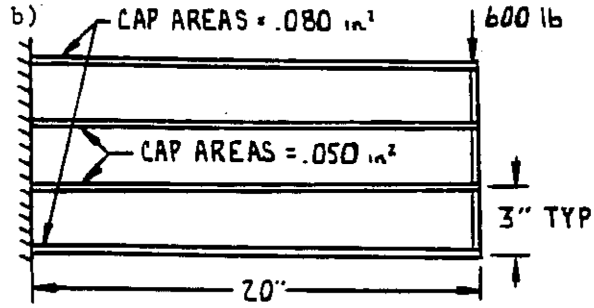


11.4 Determine the shear flows and cap loads for the webs shown below.

a)

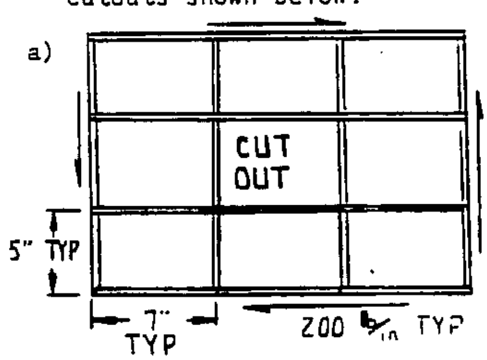


b)

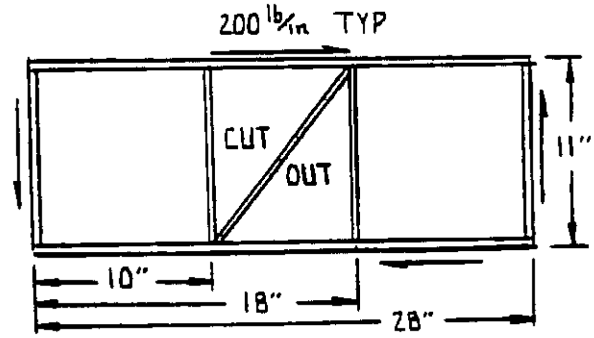


11.5 Determine the shear flows and cap loads for the multiple panels with cutouts shown below.

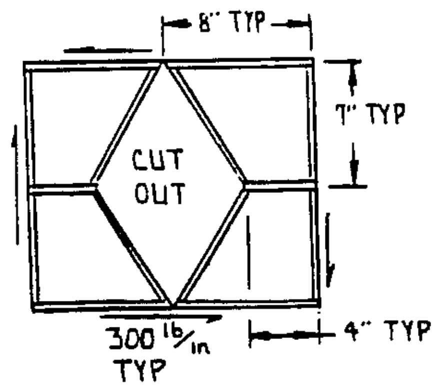
a)



b)



c)



LESSON 12

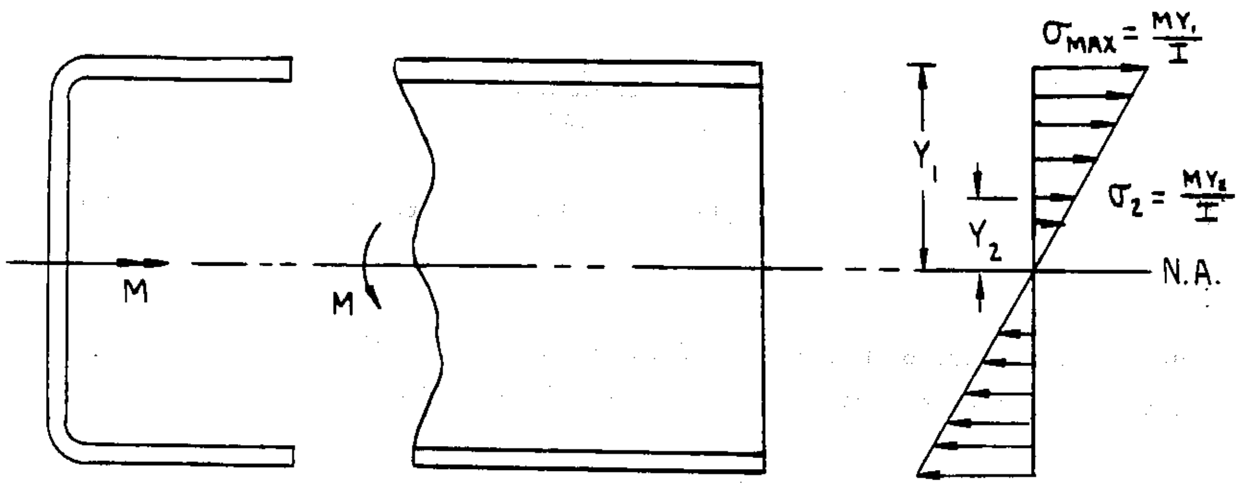
BEAMS AND OTHER BENDING MEMBERS

12.1 ELASTIC BENDING: When the maximum tension and compression stresses due to bending are less than the yield stress of the material the stress distribution follows the elastic bending theory and can be found from the equation

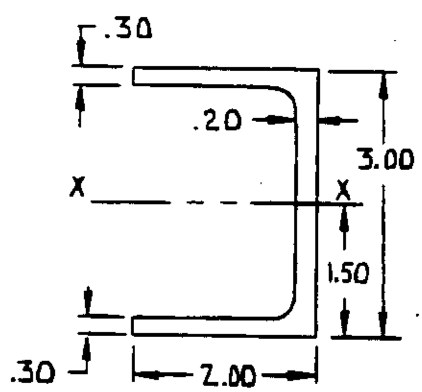
$$f_b = \frac{MC}{I}$$

where M = bending moment at the section.
c = distance from the neutral axis to the fiber in question.
I = moment of inertia of the section about the centroidal axis.

Thus the stress has a triangular distribution as shown:



EXAMPLE: Find the elastic bending stress distribution for the section defined below with a bending moment of 75,000 in-lbs about the x axis.



SOLUTION: The equation for elastic bending stress is:

$$f_b = \frac{Mc}{I} \quad \text{or} \quad \frac{My}{I}$$

where f_b is the stress, M is the applied moment, c or y is the distance from the neutral axis to the point of interest and I is the moment of inertia of the cross section. $I = \frac{2.0\ (3.0)^3}{12} - \frac{1.8\ (2.4)^3}{12} = 2.426$

At the extreme fiber $f_b = \frac{Mc}{I} = \frac{75,000 \times 1.50}{2.426} = \underline{46,400 \text{ psi}}$

At the midpoint between the extreme fiber and the neutral axis:

$$f_b = \frac{Mc}{I} = \frac{75,000 \times .75}{2.426} = \underline{23,200 \text{ psi}}$$

At the neutral axis: $f_b = \frac{Mc}{I} = \frac{75,000 \times 0}{2.426} = \underline{0}$

The three stress levels calculated illustrate the triangular distribution of elastic bending stresses.

Using the methods of Lesson 9 to find the crippling allowable of the compression flange a margin of safety can be written.

$F_{cc} = 48180$ psi

$$\underline{\underline{M.S.}} = \frac{F_{cc}}{f_c} - 1 = \frac{48,180}{46,400} - 1 = \underline{\underline{0.04}}$$

12.2 ELASTIC SHEAR STRESS DISTRIBUTION

12.2.1 Thick Web: When the "shear web" of a member is thick enough to carry bending stress the shear stress varies over the height of the section and can be found from the equation $f_s = \frac{VQ}{Ib}$

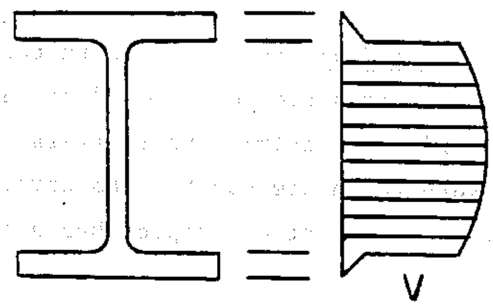
where V is the total shear at the section

Q is the static moment of the area between the point of interest and the extreme fiber.

I is the same as for bending stress

b is the width of the section at the point of interest

This produces a distribution similar to that shown in the sketch shown at the right.



EXAMPLE: Find the elastic shear stress distribution for the channel section of the preceding example for a shear load of 10,000 pounds.

SOLUTION: Calculate the values of Q for the points of interest

$$Q = \int dA\,(y) = \Sigma\,(Ay)$$

where A is the area of an element of the cross-sectional area and y is the distance from the centroid of that elemental area of the centroid of the section. At the upper edge:

$$A = 0 \quad \therefore Q = 0$$

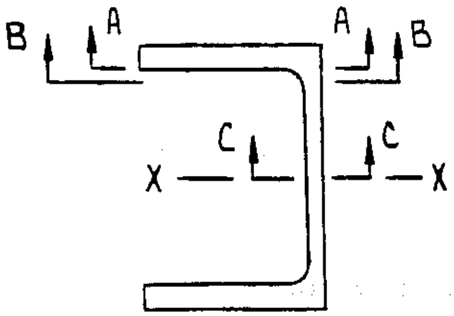
At section A − A and B − B: (See pg. 12-4)

$$Q = \Sigma(AY) = (2.0 \times .30)\,(1.35) = .81 \text{ in}^3$$

At section C - C:

$$Q = (AY) = Q_{A-A} + (1.20 \times .20)\ (.60) = .954\ in^3$$

The shear stresses at the several points can now be calculated. Because the stresses are required at several points it is convenient to do the calculations in tabular form.



location	Q	b	f_s $\left(\frac{VQ}{Ib}\right)$
up'r fiber	0	2.00	0
A-A	0.81	2.00	1670
B-B	0.81	0.20	16700
C-C	0.954	0.20	19660

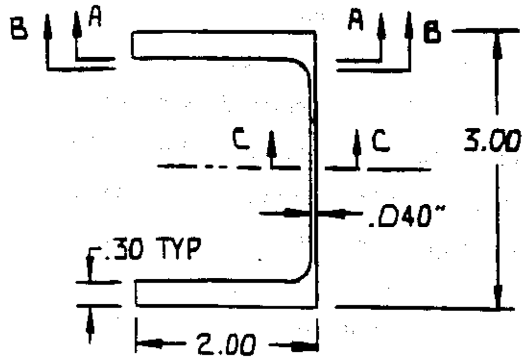
12.2.2 <u>Thin Web</u>: It was shown in the previous paragraph that for thick shear webs the shear stress does not deviate very far from a constant value. (In the example the minimum shear stress in the web was 85% of the maximum. For shear webs which are too thin to carry a significant portion of the bending moment the shear stress approaches a constant value.

<u>EXAMPLE</u>: This problem is the same as the previous example except that the web thickness is reduced from .20 to .040.

<u>SOLUTION</u>: The moment of inertia of the section is calculated and the values for Q are determined for the points where shear stresses are desired.

$$I_x = \frac{2.0\ (3.0)^3}{12} - \frac{1.96\ (2.4)^3}{12} = 2.242\ in^4$$

$Q_{A-A} = Q_{B-B} = 0.81\ in^3$ as in the previous example.



Q_{C-C}:

element	A	y	Ay
1	.60	1.35	.81
2	.048	.60	.0288
			.8388

$Q_{C-C} = .8388\ in^3$

The shear stresses are calculated by the same procedure as in the previous example:

location	Q	b	f_s $(\frac{VQ}{Ib})$
up'r fiber	0	2.0	0
A-A	.81	2.0	1806
B-B	.81	.04	90320
C-C	.8388	.04	93530

Comparison of this example and the previous example show that as the shear web is made thinner the web shear stresses approach a uniform value.

If the shear stress was assumed to be uniform between the centroids of the caps with no shear stress in the flanges, the shear stress would be:

$$f_s = \frac{V}{A} \quad (\text{or } \frac{S}{A})$$

$$= \frac{V}{ht}$$

where h = height between centroids of caps and t = web thickness. Then:

$$f_s = \frac{V}{ht} = \frac{10{,}000}{(3.00 - 2 \times .15).040} = 92{,}600 \text{ psi}$$

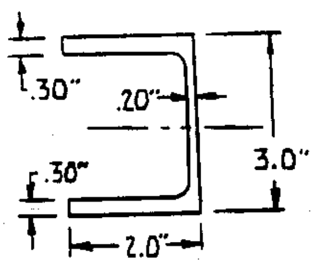
This value is only 1% less than the value calculated in the table, showing that this method is sufficiently accurate to be used for sections with thin webs.

12.3 PLASTIC BENDING: Once the bending stress at the extreme fiber of a bending section reaches the yield stress of the material, increases in bending moment will not increase the extreme fiber stress. The extreme fibers will yield, allowing additional deflection. Then the material between the yielded layer and the neutral axis will resist the larger bending moment by carrying increased stress levels. If the bending moment approaches the failing value, the stress distribution changes from the triangular shape of paragraph 12.1 to a curve approximating a trapezoidal shape as shown in McDonnell Report 338 on page 5.01.

In practice, it has been found convenient to assume a rectangular distribution and reduce the allowable stress to an artificial value, F_{rb}, which will produce the same allowable bending moment as F_{tu} does with the trapezoidal distribution. For plastic bending the neutral axis is at the center of the area rather than the centroid. This axis of equal areas is not at the centroid unless it is also an axis of symmetry.

The procedure for analyzing a section for bending in the plastic range is explained in section 5 of McDonnell Report 338. Effects of combined stresses are also considered. The tables and plots in section 14 of McDonnell Report 339 simplify calculations for bending about a single axis.

EXAMPLE: Find the allowable plastic bending moment and margin of safety for the example problem of paragraph 12.1



M = 75,000 in-lb

Material: 2024-T4 extrusion

SOLUTION: $M_{all} = (Q_1 + Q_2)F_{rb}$ (MAC 339, page 14.00) where Q_1 and Q_2 are the static moments about the neutral axis of the areas on each side of the neutral axis and F_{rb} is the value of a rectangular bending stress distribution that would have a bending moment equal to the failure moment.

Find the plastic bending neutral axis. This is the axis perpendicular to the plane of the bending moment that divides the cross section into two equal areas. In this example, due to symmetry, it coincides with the centroid.

Calculate the static moments, Q_1 and Q_2. Due to symmetry $Q_1 = Q_2$. Therefore, $Q_1 = Q_2 = 0.954\ in^3$.

Find the appropriate value of f_o/F_{tu} from the chart on page 14.02 of MAC 339. For 2024-T4 extrusions, longitudinal grain $f_o/F_{tu} = 0.84$.

Determine the section factor k either from the information on pages 14.03 and 14.04 of MAC 339 or from the general equation:

$$K = \frac{Q_1 + Q_2}{I/c} \quad \text{(MAC 339, page 14.00)}$$

where I is the moment of inertia of the entire section and c is the greater of c_1 and c_2, the distances from the neutral axis to the extreme fibers.

$I = 2.426\ in^4$ Ref. example in paragraph 12.1

$c = 1.5$ in

$$k = \frac{Q_1 + Q_2}{I/c} = \frac{2 \times .954}{(2.426/1.5)} = 1.18$$

From F_{rb} from page 14.01 of MAC 339.

$$\frac{F_{rb}}{F_{tu}} = .977$$

$F_{tu} = 57{,}000$ psi. However, since the allowable stress is determined by crippling rather than rupture use F_{cc} instead of F_{tu}.

$F_{cc} = 48{,}180$ psi

$F_{rb} = .977 \times 48{,}180 = 47{,}070$ psi

Calculate M_{all} and M.S.

$$M_{all} = (Q_1 + Q_2)F_{rb} = (2 \times .954)\ 47{,}070 = \underline{89{,}800 \text{ in lb}}.$$

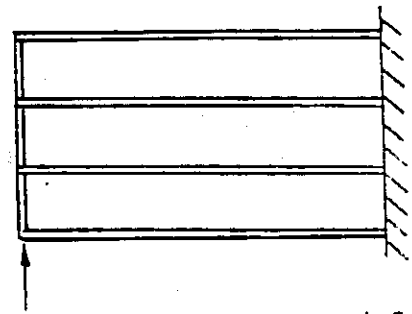
$\underline{M = 75{,}000 \text{ in lb}}$.

$$MS = \frac{M_{all}}{M} - 1 = \frac{89{,}800}{75{,}000} - 1 = \underline{\underline{+0.20}}$$

Note that the plastic M.S. is greater than the elastic bending M.S. determined for the same section in the example of paragraph 12.1.

12.4 PLASTIC SHEAR STRESS DISTRIBUTION: McDonnell Report 338 pages 5.09 and 5.10 details and explains the method of analyzing shear stresses in combination with plastic bending. For the example in the above paragraph the shear stress would be found by the formula $f_s = \frac{VQ}{Ib}$ and would not interact with the bending stresses.

12.5 BEAMS WITH THIN WEBS. A significant portion of aircraft structure is composed of thin webs supported by edge members (caps) and stabilized by stiffeners. Examples are keel webs, partial bulkheads and frames. Illustrated below is the method of determining loads and stresses in thin web beams.



Cap areas are 1.0 in^2 each.

EXAMPLE: Find the internal shear flows for the beam shown.

SOLUTION: Because there are more than three reactions the external reactions cannot be found by the laws of statics. However, the cap loads can be determined from the bending stress equation $f_b = \frac{Mc}{I}$ (or $\frac{My}{I}$) and the shear flows can be determined from the cap loads.

The moment of inertia (I) is found from the equation:

$$I_x = \Sigma Ay^2 + I_h - A(\bar{y})^2$$

Because the reference axis is the centroidal axis (the axis of symmetry), $\bar{y} = 0$ so the third term of the equation drops out. In a thin web beam, the web is assumed to carry shear but no bending moment so only the 1.0 in^2 caps are included in the calculation for moment of inertia. Also, because I_h, the moment of inertia of each cap about its own centroid is negligible compared to the I of the entire section, it is assumed that $I_h = 0$. Thus the second term of the equation drops out, leaving: $I = \Sigma Ay^2$.

The bending stresses in each cap is found at a cross section of the beam. Then the stress in each cap is multiplied by its area to get the cap load. Dividing this cap load by the distance from the end of the beam where all cap loads are zero gives the shear flow. For convenience, a section one inch from the free (left) end of the beam is used.

$$M = 1.00 \times 8000 = 8000''\#$$

$$I = \Sigma Ay^2 = 2A_1(y_1)^2 + 2A_2(y_2)^2$$

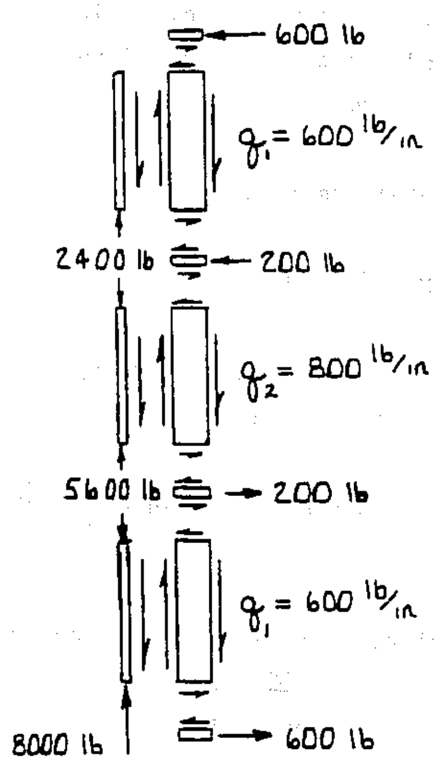
$$= 2(1.0)(6.0)^2 + 2(1.0)(2.0)^2 = 80 \text{ in}^4$$

For cap #1 at x = 1.0:

$$f_b = \frac{Mc}{I} = \frac{8000\ (6.0)}{80} = 600 \text{ psi}$$

$$P = f_bA = 600\ (1.0) = 600\#$$

$$q_1 = \frac{P}{l} = \frac{600}{1.0} = 600\ \#/''$$



For cap #2 at x = 1.0:

$$f_b = \frac{Mc}{I} = \frac{8000\ (2.0)}{80} = 200 \text{ psi}$$

$$P = f_bA = 200\ (1.0) = 200\#$$

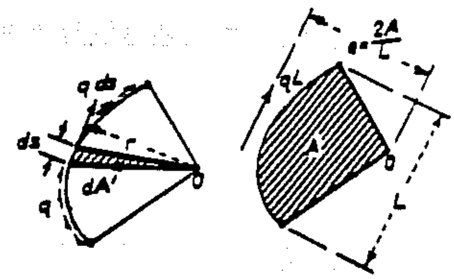
$$q_2 = \frac{P}{L} + q_1$$

$$= \frac{200}{1.0} + 600 = 800\ \#/''$$

The bottom portion of the beam is the same, due to symmetry, except that the cap loads are of opposite sign.

12.6 <u>Shear center:</u> The shear center of a beam cross section is the point at which applied shear loads will cause no torsion on the section. For sections with one or two axes of symmetry, the shear center lies on these axes. For "stick and panel" beams the shear center can be calculated by summing moments due to the shear flow.

The moment about any point 0, of the force qds is the product of the force and the moment arm r.



Thus the total moment is:

$$M = \int qr\, ds$$

The area of the shaded triangle is $r\,\frac{ds}{2}$ and rds = 2A. Thus:

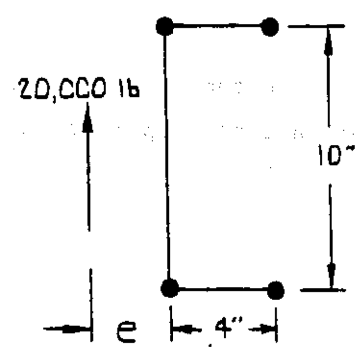
$$M = \int q\, 2dA = 2q \int dA \quad \text{or,}$$
$$M = 2Aq$$

Then the distance e from point 0 to the shear center is:

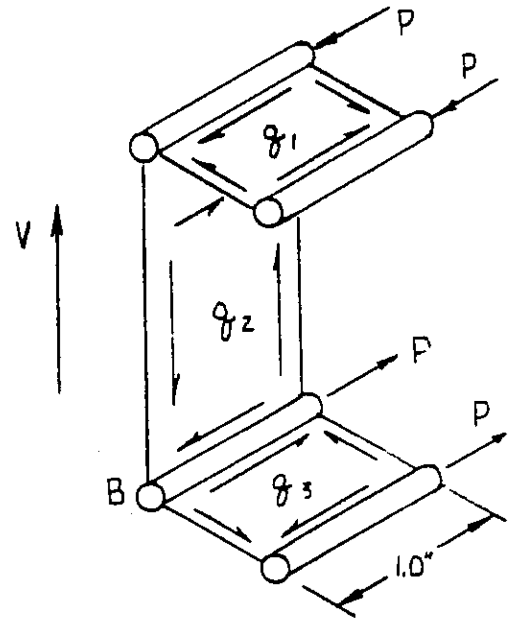
$$e = \frac{2Aq}{qL} = \frac{2A}{L}$$

12.6.1 <u>Cap and Thin Web Sections</u>

<u>EXAMPLE</u>: Find the shear flows in the webs of the beam shown. Flange areas are .50 in^2. Assume webs carry no bending stress. Locate the shear center.



SOLUTION: Two cross sections 1 in. apart are shown. The increase in bending moment in the 1 in. length is equal to the shear. The cap loads may be found as follows:

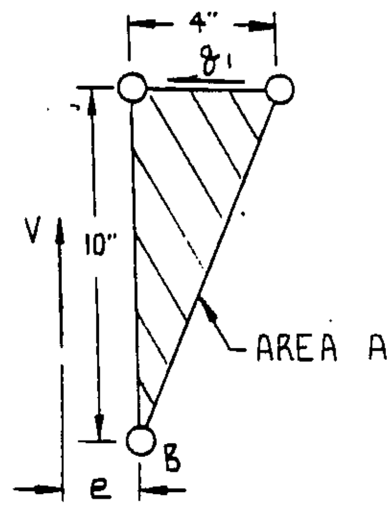


$$P = \frac{1}{2}\frac{M}{h} = \frac{1}{2} \times \frac{20{,}000 \times 1.0}{10.0} = \underline{1000\#}$$

By considering the loads on the right-hand caps, it can be seen that:

$$q_1 = q_3 = \frac{\Delta P}{L} = \frac{1000}{1.0} = \underline{1000\#/''}$$

Similarly, by considering the loads on one of the left-hand caps:



$$q_2 = \frac{\Delta P}{L} + q_1 = = \frac{1000}{1.0} + 1000 = \underline{2000\#/''}$$

$$\Sigma M_B = Ve - 2Aq_1 = 0$$

$$= 20{,}000\ e - (4.0'' \times 10.0'')\ 1000\ ^{lb}/_{in}$$

$$\underline{e = 2.0''}$$

12.6.2 <u>Compact Sections</u>

For compact or thick-walled sections shear centers can be found from formulas given in texts and handbooks such as "Formulas for Stress and Strain" by J. R. Roark.

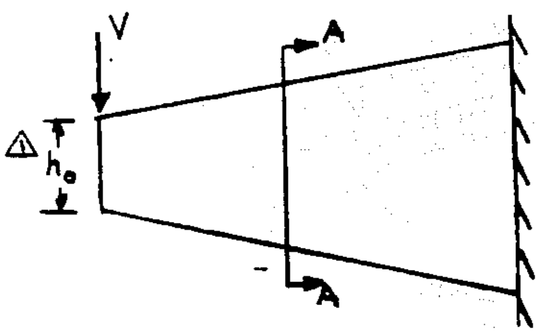
12.7 Tapered Beams: In the beams previously discussed it has been assumed that the cross section of the beam is constant along the length.

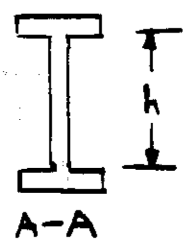
Since aircraft structures must be as light as possible, the beams are usually tapered so that the bending stresses remain constant. The depth and moment of inertia are greater at points where the bending moment is larger. While this variation in cross section may not cause appreciable errors in the application of the flexure formula for bending stresses, it can cause error in the shear stresses determined from the equation $f_s = \frac{VQ}{Ib}$ presented in paragraph 12.2.1.

Shear stresses determined for a tapered beam will tend to be higher in the caps and lower in the vertical web than for a beam of constant cross section. For the tapered beam shown below, the terms V_w or V_f can be substituted for V in the equation $f_s = \frac{VQ}{Ib}$ when determining the shear in the web or flanges respectively.

$$V_w = V \frac{h_o}{h}$$

$$V_f = V \frac{(h-h_o)}{h}$$

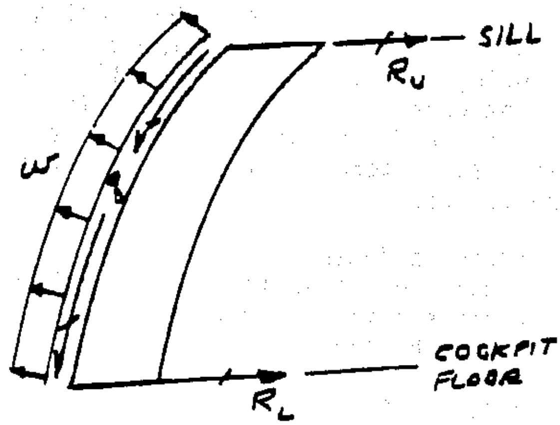




⚠ Web height

12.8.1 Frames and Rings: Some frames are complete rings as in the nose fuselage and engine air ducts and some are local formers that either are not continuous around the periphery or do not have bending continuity. Most fuselage frames are in the latter category. Frames that are not complete rings are usually statically determinant for external and internal loads.

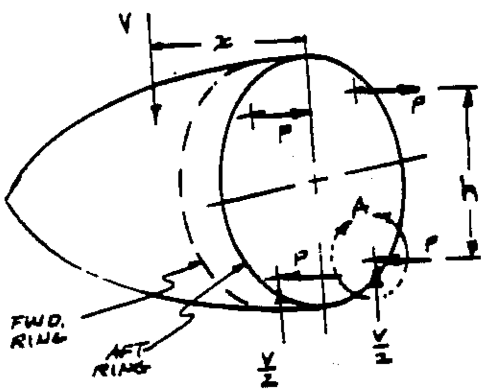
EXAMPLE: Loads on a typical cockpit frame. The balance shown is for internal cockpit pressurization. Other types of loading would have the same three reactions but with different magnitudes.

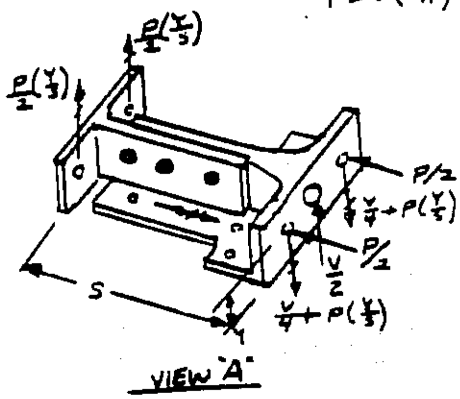


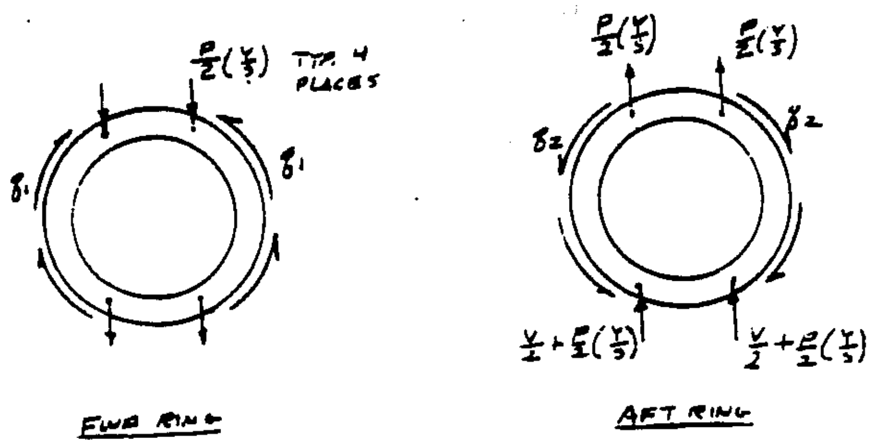
The internal loads in rings are seldom statically determinant even when the external loads are. Internal loads in circular rings can be found using conventional formulas available in texts and handbooks. Non-circular rings can be solved by use of any of the various redundant solution methods.

EXAMPLE: Loads on a circular radome ring.

$P = V\left(\frac{x}{h}\right)$



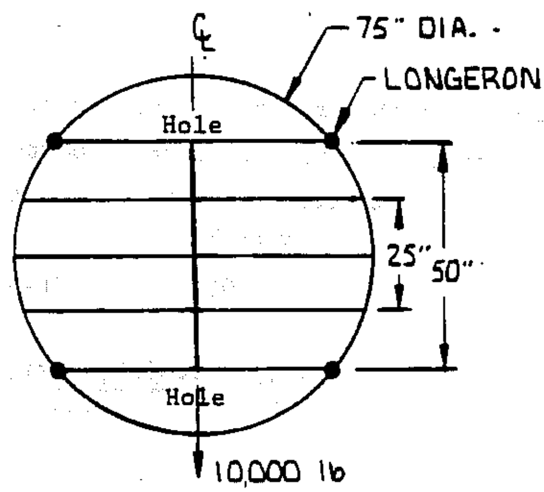




12.8.2 **Bulkheads**: Aircraft semi-monocoque structure is efficient in carrying high loads but problems arise when applying large loads normal to the thin shell. A piece of internal structure is required to distribute the load over a large portion of the circumference. In the fuselage the most efficient member is a bulkhead. In the wing the bulkhead is called a rib. As was mentioned in paragraph 5.2.4, bulkheads also redistribute skin and shear web shears and react internal pressures. In distributing applied loads or redistributing shear the bulkhead can be visualized as a beam. Internal loads in the bulkhead can then be found by the method used in the second example in Paragraph 12.5.

EXAMPLE:

Balance the bulkhead for the applied load by shear flows in the skin between the upper and lower longerons. Also find the axial load in the horizontal members and shear flows in the shear panels. The upper and lower stiffeners have an area of 1.0 in^2. The intermediate stiffeners have an area of .50 in^2.



SOLUTION: Find the reacting shear flows from the equation:

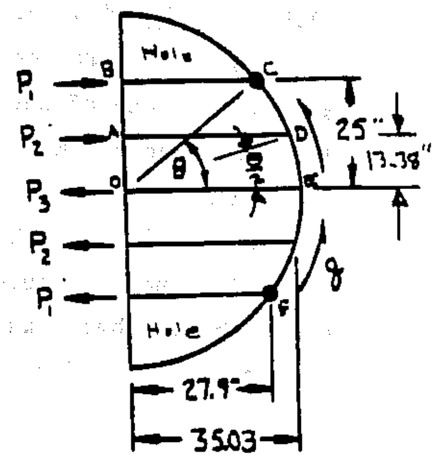
$$q = \frac{V}{h} = \frac{10000}{50.0} = 200 \text{ \#/in (total of both sides)}$$

Due to symmetry, the side skins share the shear flow equally:

$$q = 100 \text{ \#/in per side}$$

The loads in the horizontal members are found by treating the bulkhead as a beam. Find the bending moment at the center by taking the moment of the shear flow about any point on the vertical centerline of the bulkhead using the equation:

$$M = 2Aq$$



The area (A) is a function of angle θ .

$$\theta = \arcsin\left[\frac{25.0}{37.5}\right] = 41.80$$

Shear panel OADE and ABCD will be considered to be trapezoidal panels neglecting the ML curvature. For moments about point O the area is ;

$$A_{OCD} = (37.5)^2 \text{SIN}(\tfrac{\theta}{4})\cos(\tfrac{\theta}{4}) = 250.83 \text{ in.}^2$$

$$A_c = A_{OCF} = 4A_{OCD} = 4(250.83) = 1003.3 \text{ in.}^2$$

$$M = 2A_c q = 2(1003.3)(100) = 200660^{\#\text{in}}$$

Find the centerline cap loads from the equation:

$$P = f_b A = \frac{Mc}{I} A$$

Because the moments of inertia of the caps about their own centroid are negligible compared to the moments of inertia about the section centroid, and because the reference axis is at the section centroid:

$$I = \Sigma(Ad^2)$$

$$= 2[1.0(25.)^2] + 2[.50(13.38)^2] = 1429.02 \text{ in}^4$$

$$P_1 = \frac{MC_1A}{I} = \frac{200660(25)(1.0)}{1429.02} = 3510.4\#$$

$$P_2 = \frac{MC_2A}{I} = \frac{200660(13.38)(.5)}{1429.02} = 939.4$$

$P_3 = 0$, on ℄ of symmetry

Find the panel shear flows by putting the horizontal stiffeners on one side of the vertical centerline in static equilibrium (starting with the top or bottom stiffener) and by balancing the shear panels as trapezoidal panels, neglecting the ML curvature.

Upper Member:

$\Sigma F_H = 3510.4 - 27.9q = 0$

$q = \frac{3567.3}{27.9} = 125.8$ #/in

Upper Panel:

END $q_e = q_u \left[\frac{b}{a}\right] = 125.8 \left[\frac{27.9}{35.03}\right]$

$= 100.19$

LOWER $q_L = q_u \left[\frac{b}{a}\right]^2 = 125.8 \left[\frac{27.9}{35.03}\right]^2$

$= 79.8$

NEXT MEMBER:

$\Sigma F_H = 939.4 + 79.8 (35.03) - 35.03q = 0$

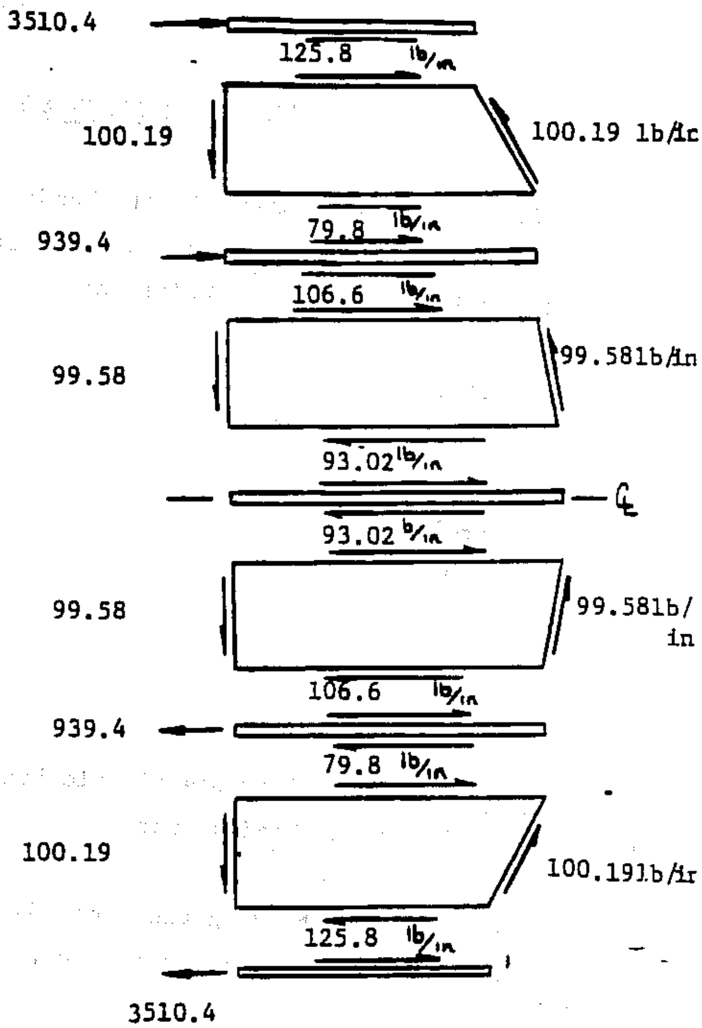
$q = \frac{3734.8}{35.03} = 106.6$ #/in

NEXT PANEL:

END $q_e = q_u \left[\frac{b}{a}\right] = 106.6 \left(\frac{35.03}{37.5}\right) = 99.58$

LOWER $q_L = q_u \left[\frac{b}{a}\right]^2 = 106.6 \left(\frac{35.03}{37.5}\right)^2 = 93.02$

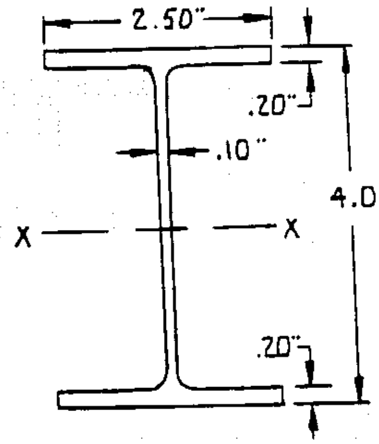
Next member is on the ℄ of symmetry.



PROBLEMS

LESSON 12 BEAMS AND OTHER BENDING MEMBERS

12.1 Find the maximum elastic bending and shear stress for the cross section shown when loaded about the X axis with a bending moment of 80000"# and a shear load of 11000#. Write the minimum margin of safety.



Material:

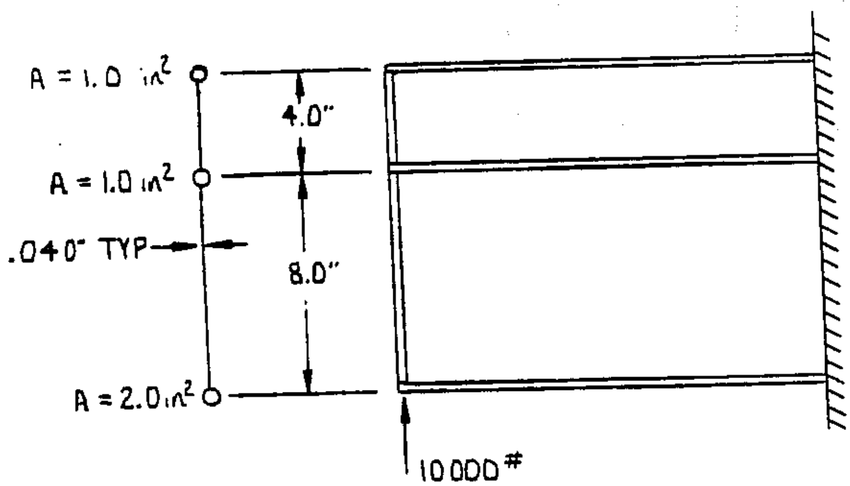
2024-T4 Extrusion

F_{TU} = 61 KSI

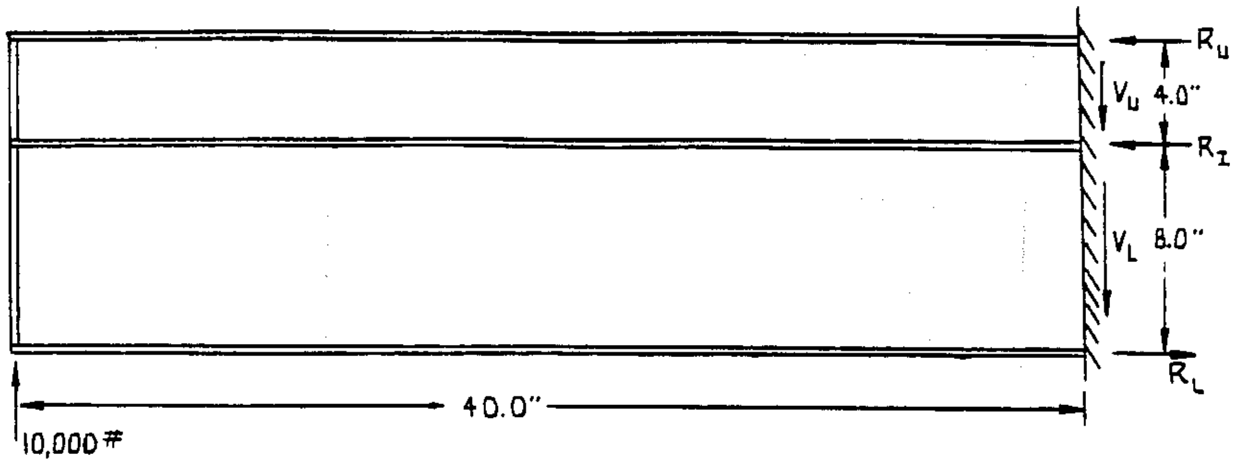
F_{SU} = 31 KSI

12.2 Find the allowable plastic bending moment for the section of problem 12.1. Write the bending margin of safety.

12.3 Find the bending and shear distribution of the thin web beam shown. Assume the web is ineffective in bending.



12.4 Find the axial and shear reactions at the beam support if the length of the beam in problem 12.3 is 40.0 in.



12.5 For the beam cross-sections shown find the shear flow in each web, location of the shear center and axial cap loads one inch from the loaded end of the cantilever beam. The cross-section area of each cap is 0.50 in^2.

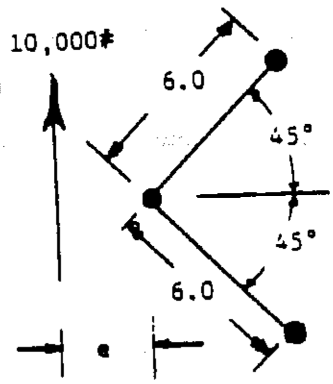


FIGURE 1

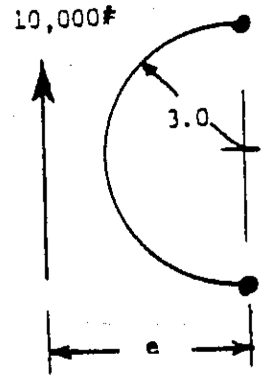


FIGURE 2

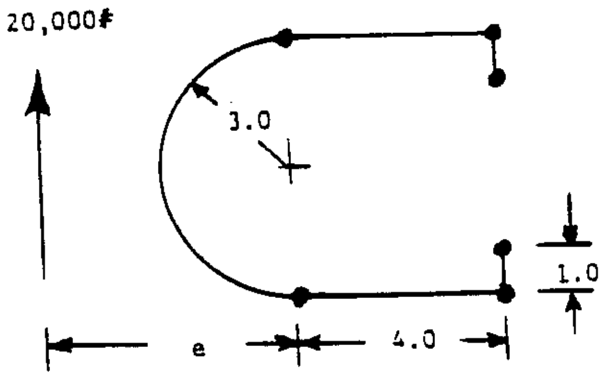
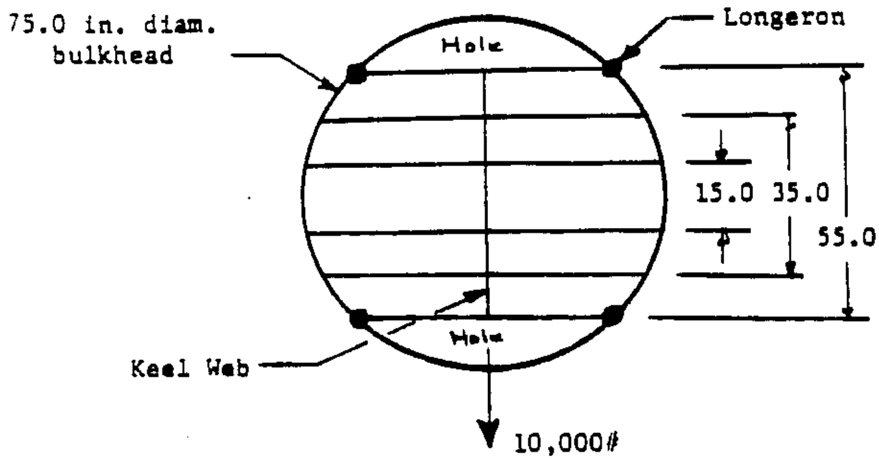


FIGURE 3

12.6 Balance the bulkhead in the sketch for the applied 10,000 lb. centerline load by shear flows in the skins between the upper and lower longerons. Also, find the axial loads in the horizontal members and shear flows in the shear panels. All horizontal stiffeners have the same area.



LESSON 13

ALLOWABLE SHEAR

13.1 General

In Lesson 11, the concept of shear flow was introduced, and in Lesson 12, it was applied to beams with thin webs. This lesson will discuss how shear flow is applied to shear webs. A shear web is a panel that carries shear in a beam or torque box. For a beam, the effective dimensions of the panel are the dimensions between the edging member centroids. For a torque box, the entire circumference is made up of shear panels. If a beam is also part of a torque box (like a wing spar), the shear flow is:

$$q = \frac{V}{h} + \frac{T}{2A} \quad (1)$$

where: q = shear flow (lb/in)
h = effective height of shear panel (in)
V = shear (lb)
T = Torque (in/lb)
A = circumferential area of torque box (in^2)

Shear webs are divided into two categories. Those that carry design ultimate load without buckling are termed "shear resistant", and those that buckle are called "diagonal tension" or "tension field" webs. Recall, that shear stress f_s, is often symbolized τ (tau). The initial shear buckling stress is then defined as F_{scrit} or τ_{cr}. The allowable shear stress τ_{all} is usually much greater than τ_{cr}.

13.2 Shear Resistant Webs

Theoretically, a shear web would be one with $\tau/\tau_{cr} < 1.0$. In practice, webs are considered shear resistant for τ/τ_{cr} up to 2.5, because below that value, the shear resistant analysis predicts failure with sufficient accuracy. τ_{cr} is determined by the equation:

$$\tau_{cr} = k_s E\,(t/d)^2 \qquad (2)$$

* Figures not found in text are at end of lesson.

where: E = modulus of elasticity

t = web thickness

d = smaller of the panel's dimensions

k_s = buckling constant (see fig. 1)

For a shear resistant web, the allowable stress is F_{all} (determined from figure 2.5). The necessary strength checks are:

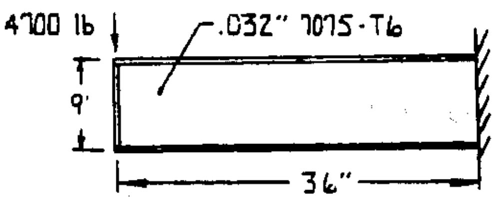
1. Maximum shear stress f_s. This is performed usually on the net section (through the web at a row of fasteners) or, for a web with varying thickness, through the thinnest part of the web. This value would then be compared to the value of F_{su} from MIL-HDBK-5.
2. Fastener Strength. The load per rivet is:

$$P = (q)(p)$$

where: p = rivet pitch

The allowables for fasteners are found in Section 1, MAC 339.

Example: Determine the stiffener spacing required to carry the end load shown in the figure below while maintaining a shear resistant web. Assume I_s = .009 in^4.



$$\tau = \frac{4700 \text{ lb}}{(9.0")(.032")} = 16.3 \text{ KSI}$$

WE WANT $\tau/\tau_{cR} < 2.5$ or $\tau_{cr} > 6.53$ KSI

TRY 11 STIFFENERS:

$$\frac{h_e}{d} = \frac{9"}{3"} = 3$$

FROM FIGURE 2

$$\frac{I_s}{h_e t^3} = 7.0 \quad ⚠1$$

FROM FIGURE 1: $K_s = 5.6$

$$\tau_{cR} = K_s E \left(\frac{t}{d}\right)^2 = (5.6)(10.3 \times 10^6 \text{psi})\left(\frac{.032"}{3.0"}\right)^2$$

$$\tau_{cR} = 6.56 \text{ KSI} \quad (OK)$$

Note from the previous example that in order to maintain a thin web, a relatively large number of stiffeners was required. In order to reduce manufacturing costs, other types of shear resistant webs have been developed. These are:

1. Webs with flanged lightening holes
2. Webs with beaded stiffeners
3. Webs with a combination of flanged lightening holes and beaded stiffeners.

A discussion of the analysis of each of these types follows.

⚠1 DIRECT COMPUTATION OF $\frac{I_s}{h_e t^3} = 30.5$. NOTE FROM FIGURE 2 HOWEVER, THAT NO INCREASE IN BUCKLING CONSTANT (FOR $\frac{h_e}{d} = 3$) OCCURS BEYOND $\frac{I_s}{h_e t^3} = 7$.

13.2.1 Panels with Flanged Lightening Holes

The figure below shows a typical beam with lightening holes having formed 45° flanges.

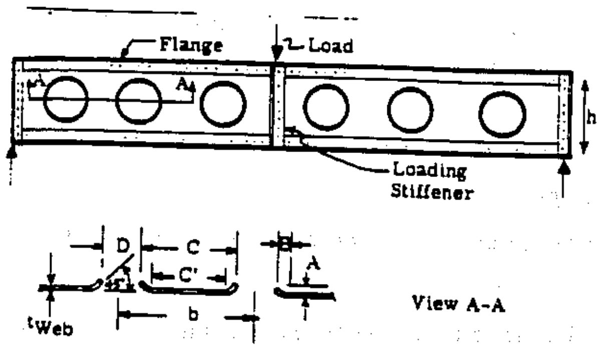


FIGURE 3: FLANGED HOLE GEOMETRY

An empirical formula has been developed which gives the allowable shear flow for webs having this type of hole. Referring to figure 3, the allowable shear flow is:

$$q_{all} = k\,t \left[f_{S_h}\left(1 - \left(\frac{D}{h}\right)^2\right) + f_{S_c}\sqrt{\frac{D}{h}} \right] \frac{C'}{b} \qquad (3)$$

where: $k = .85 - .0006\,\dfrac{h}{t}$

f_{Sh} = Collapsing shear stress of a long plate of width h and thickness t (from figure 4)

f_{Sc} = Collapsing shear stress of a long plate of width c and thickness t (from figure 4)

b = hole centerline spacing

C' = C-2B, where B is given in Table 1 for a typical hole

C = b-D

In general, it is found that larger holes (D≈.8h) with wider spacings, b, will give the lightest web in designing for a given shear flow. The stiffness, however, will be less. The allowables given by equation (3) are for pure shear only, that is, no normal loads on the web. Thus, a stiffener must be added wherever a normal load is introduced into the beam, or where the beam has significant curvature, as in round and elliptical bulkheads. In addition to the allowable shear flow check, a net section shear stress check through the holes should be performed. Finally, the rivets attaching the web to the flange above a hole need to be more closely spaced to take the higher "net" shear locally.

13.2.2. Webs with Beaded Stiffeners

Of the shear-resistant web systems available, the beaded stiffener web is the strongest of those not having separate stiffeners. Figure 5 below shows a web having "male" beads formed into it at the minimum spacing that forming will allow. The cross-section through the web shows the details of the bead, and is described in Table 2.

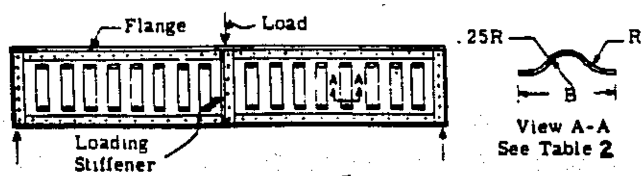


Fig. 5

t	B	R
.020	.95	.32
.025	1.05	.40
.032	1.16	.52
.040	1.27	.64
.051	1.42	.81
.064	1.55	1.02
.072	1.65	1.15
.081	1.73	1.30
.091	1.81	1.45
.102	1.92	1.63
.125	2.12	2.00

TABLE 2: BEADED HOLE GEOMETRY

The allowable shear flow, q (lb/in) at collapse, is given by the solid lines in figure 6. The dotted lines in this figure indicate initial buckling. Failure occurs when the beads collapse. The allowables are for pure shear only, that is, no normal loads. If a load is introduced normal to the beam flange, a loading stiffener must be used in place of the beaded stiffener. In addition, the allowables apply to webs with beads formed with a length long enough to extend as close to the beam cap as possible. Short beads, which end well away from the cap, will not develop the strength indicated by the figure.

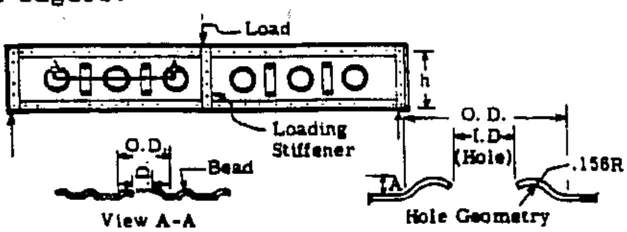
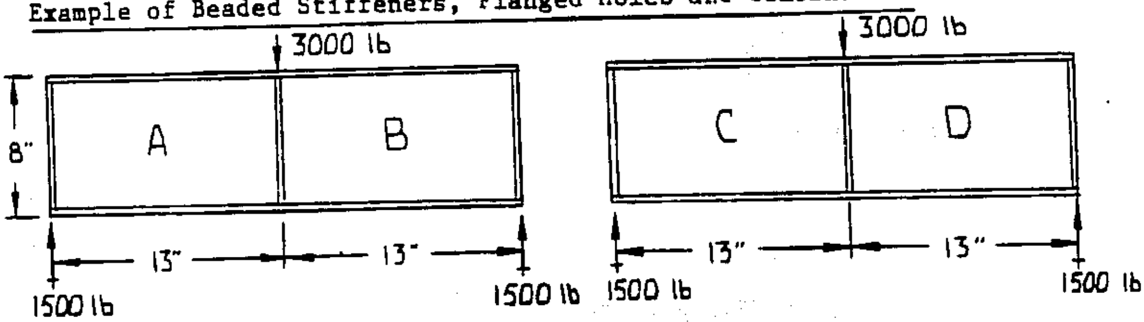


FIGURE 7: BEADED STIFFENERS AND FLANGED HOLES WEB

13.2.3 Webs with Beaded Stiffeners and Flanged Lightening Holes

A third type of web has round holes with beaded flanges and vertical "male" beads between the holes. This type of beam is shown in figure 7. The geometry of the flanged hole is shown in Table 3. For the particular case where $\frac{OD}{h} = .6$ and the hole spacing is equal to h, the allowable shear flows are shown in figure 8. The total collapsing strength of the web is given by the solid lines of this figure, in terms of allowable shear flow, q, as a function of web height, h. The dotted lines indicate the shear flow, q, at which initial buckling begins. Again, the allowables are based on pure shear loading (no normal loading). If the O.D. is greater than .6h, or if the spacing is reduced, the allowables will be reduced. Due to insufficient data, it is uncertain as to how much this reduction will be.

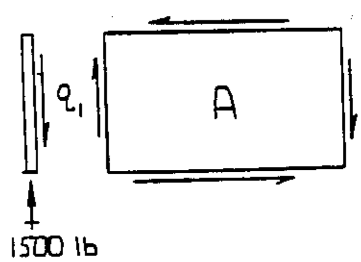
Example of Beaded Stiffeners, Flanged Holes and Combination



Design: Bay A - normal shear-resistant web
B - web with flanged lightening holes
C - web with beaded stiffeners
D - combination of B and C

Assume the web and stiffeners are 7075-T6

a) Shear Resistant Web



i) Try web = .050"

ii) Determine shear flow:

$$q_1 = \frac{1500 \text{ lb}}{8.00"} = 190 \text{ lb/in}$$

iii) Determine shear stress

$$\frac{q_1}{t} = \frac{190 \text{ lb/in}}{.050 \text{ in}} = 3800 \text{ psi}$$

iv) Determine Critical Shear Stress

$$\tau_{cr} = K_s E\left(\frac{t}{d}\right)^2$$

from MIL-HDBK-5:

for 7075-T6:

f_{su} = 43.0 KSI

Compute:

$$\frac{Is}{het^3} = \frac{.010\ in^4}{(13.00\ in)(.050\ in)^3} = 6.15$$

$$\frac{he}{d} = \frac{13.00\ in}{8.00\ in} = 1.63$$

From figure 1:

K_s = 6.8

So: $$\tau_{cr} = (6.8)(10.3 \times 10^6\ psi)\left(\frac{.050\ in}{8.00}\right)^2$$

τ_{cr} = 2740 psi

v) Check τ/τ_{cr}:

$$\frac{\tau}{\tau_{CR}} = \frac{3800\ psi}{2740\ psi} = 1.39 < 2.5$$

vi) Compare to τ_{all}:

from fig. 2.5

τ_{all} = 31.5 KSI

$$M.S. = \frac{31.5\ KSI}{3.80\ KSI} - 1 = 7.29\ \underline{HIGH}$$

b) Flanged Lightening Hole Web

i) Try 1 hole having D ≈ .8h or D ≈ 6.40 in

From Table 1, Choose D = 6.16 in

C = b-D = 13.00" - 6.16" = 6.84"

B = .25 in

C' = 6.84" - 2(.25") = 6.34 in

ii) Determine the allowable value of q for a trial t = .050 in

$$\frac{h}{t} = \frac{8.00\ in}{.050\ in} = 160$$

K = .85 - .0006(160) = .754

d) **Beaded and Flanged Hole Combination**

i) **Determine minimum thickness of web:**

Enter Figure 8 for q = 190 lb/in and h = 8.00 in

(This assumes a hole diameter = (.6)(8.00 in) = 4.80 in)

for t = .051 in, q_{int} = 230 lb/in

$$M.S. = \frac{230 \text{ lb/in}}{190 \text{ lb/in}} - 1 = .21$$

e) Weight Comparison

from MIL-HDBK-5D: Density = .101 lb/in^3

Web Type	Area	t (in)	Total Weight (lb)
A	104.0	.050	.53
B	74.2	.050	.37
C	104.0	.032	.34
D	85.9	.051	.44

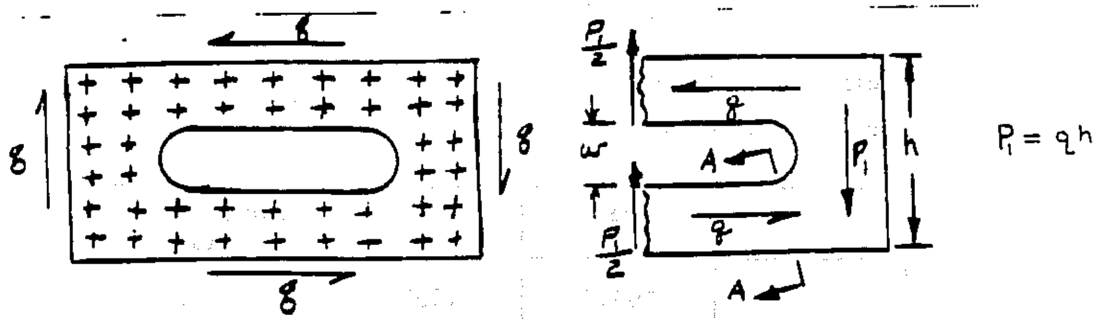
NOTE: If buckles would have been permitted, the web gage for part D could have been reduced to .032 in or .040 in (more conservative).

13.4 Doublers for Holes

Shear panels with reasonably small holes can be brought to full strength by adding sheet metal doublers. For round holes, the rule-of-thumb for the doubler size is based on meeting three conditions:

1. The doubler's cross-sectional area should be at least 1.5 times the cross-sectional area removed from the web.
2. The doubler gage should be at least one gage thicker than the web, and have two rows of rivets around the doubler.
3. The rivets in an area of the doubler enclosed by two parallel tangents to the hole must be capable of carrying a load of P = 2Dq.

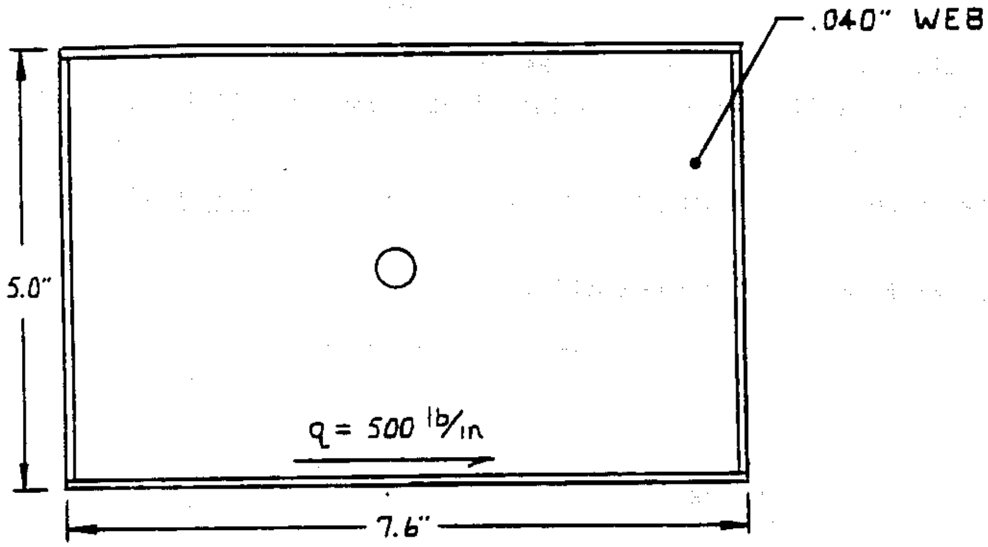
For large or non-circular holes, a doubler can be analyzed as a shear and moment carrying member. In the following sketch, the view shows one half of the doubler. It can be seen that the shear loads will create a bending moment in the doubler. The critical section for this doubler will be at or near Section A-A. Analysis of this section should include loading which is a combination of shear, axial load, and bending moment.



Recall, that cutouts that are too large for a doubler can be "framed out". This method was discussed in lesson 11.

Doubler for Small Hole Example

Given: A 0.50" diameter hole exists in the web below:



Design a doubler to restore the original strength to the web:

<u>Solution</u>:

1) Cross-Sectional Area of Doubler > (1.5) (web area removed)

> (1.5)(.040")(.50")

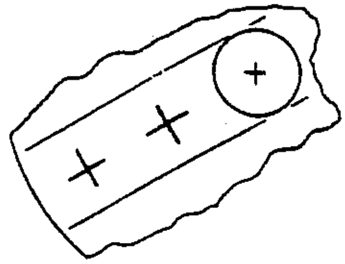
> $.03 \text{ in}^2$

2) Doubler gage

If web is .040", try doubler gage = .050"

$$\text{min. width} = \frac{.03 \text{ in}^2}{.050 \text{ in}} + .50 \text{ in} = 1.1 \text{ in}$$

3. Rivets between hole tangents:



$P = 2\,D_h q = (2)(.50\text{ in})(500\text{ lb/in})$

$= 500\text{ lb}$

Choose 2 MS20470AD4 Rivets:

$P_{all} = 300\text{ lb}$ (MAC 339 p.111)

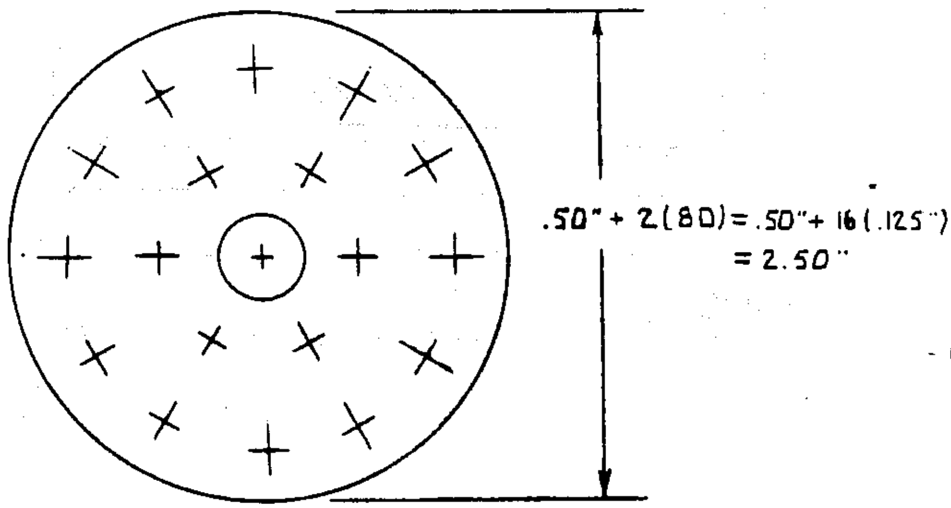
$M.S. = \frac{600\text{ lb}}{500\text{ lb}} - 1 = .20$

Now assuming 4D spacing around the perimeter:

for inside row @ 4D spacing = .50 in: # of rivets = $\frac{(2)(\pi)(.53\text{ in})}{(.50\text{ in})} = 6$ ⚠1

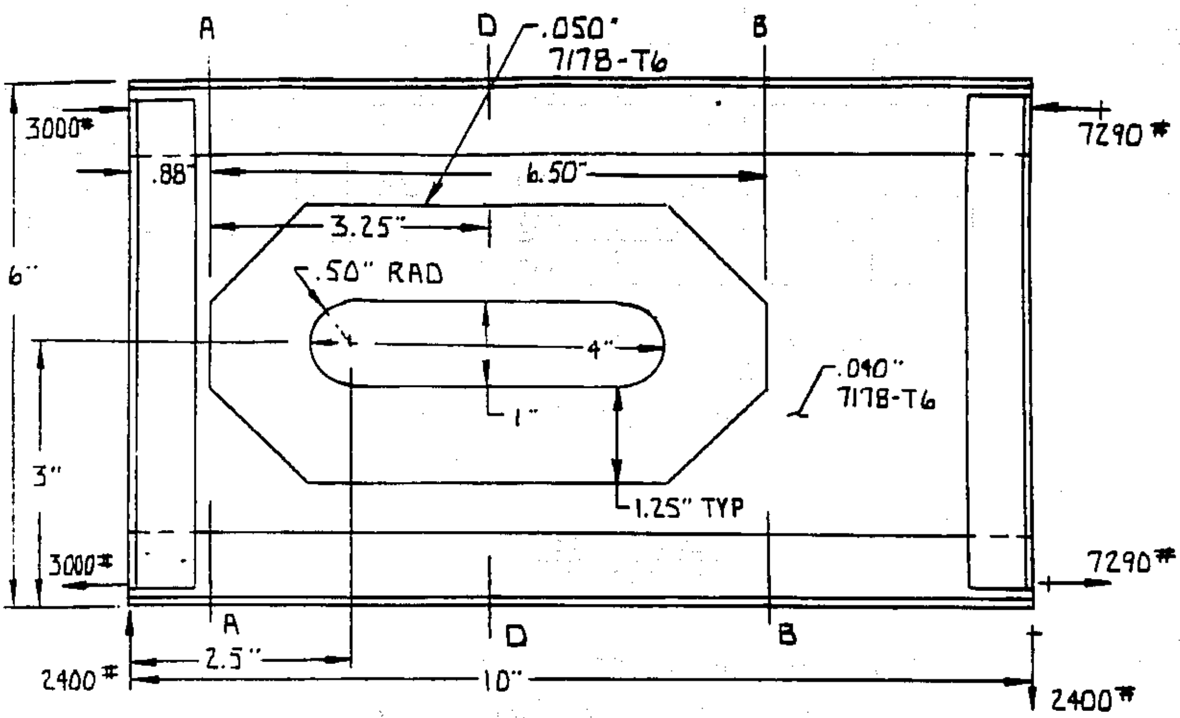
for outside row @ 4D spacing = .50 in: # of rivets = $\frac{2\pi(1.06\text{ in})}{(.50\text{ in})} = 13$

Thus the configuration of the doubler is:



⚠1 This is the maximum number of fasteners which can be used. However 6 fasteners in the inner row and 12 in the outer will provide a symmetric pattern with 2 fasteners enclosed by the area between two parallel tangents to the hole as is required for strength.

DOUBLER FOR ELONGATED HOLE EXAMPLE:



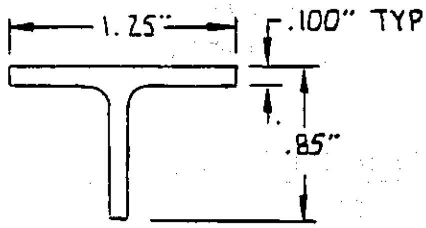
Determine shear flow:

a) Effective height

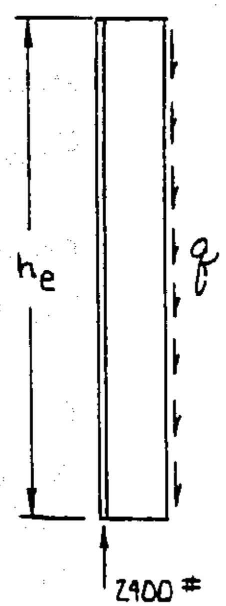
$\frac{b}{t} = \frac{1.25"}{.100"} = 12.5$

$\frac{d}{b} = \frac{.85"}{1.25"} = .68$

Fig. 14 (Lesson)



TYPICAL CAP



$\frac{x}{d} = .235 \quad \therefore x = .20"$

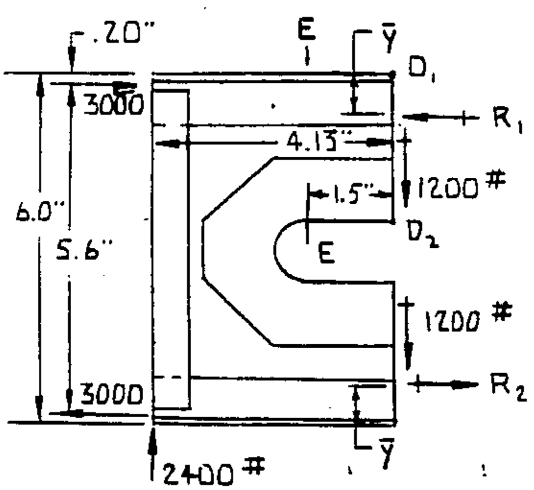
h_e = Vertical distance between upper & lower cap centroids.

Effective height = 6" - 2(.20") = 5.6"

$q = P \div h_e$

$q = \frac{2400^{\#}}{5.6"} = 429$ #/in

Free body of web from edge to D-D:



$\Sigma F_x = 0: \; 3000^{\#} - 3000^{\#} - R_1 + R_2 = 0 \quad (1)$

$+\circlearrowright \Sigma M_{D_1} = 0: -(3000^{\#})(5.8") + (3000^{\#})(.20") + R_2(6" - \bar{y}) - R_1\bar{y} - (2400^{\#})(4.13") = 0 \quad (2)$

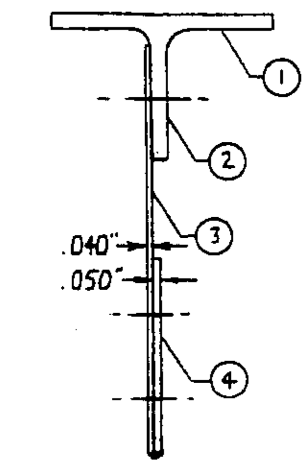
FROM EQN 1:

$R_1 = R_2$

SUBSTITUTING INTO EQN. 2:

$R_2(6" - 2\bar{y}) = 26712$ in·lb

SECTION PROPERTIES:



SECT. @ D_1-D_2 or E-E

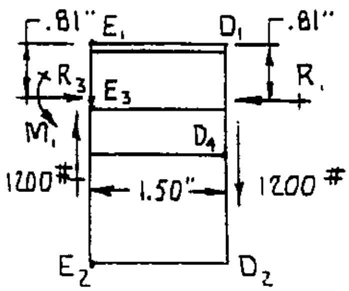
ELE	b	h	A	y	Ay	Ay^2	I
1	1.25	0.10	.125	.050	.006	.0003	.00
2	0.10	0.75	.075	.475	.036	.0171	.0035
3	0.04	2.25	.090	1.375	.124	.1705	.0380
4	0.05	1.25	.063	1.875	.118	.2213	.0081
			.353		.284	.4092	.0497

$$\bar{y} = \frac{Ay}{A} = \frac{.284}{.353} = .81''$$

$$I = .0497 + .4092 - (.81)^2\,(.353) = .23\ in^4$$

SO:

$$R_2 = R_1 = \frac{26712\ in\text{-}lb}{(6'' - 2(.81''))} = 6099\#$$



$\Sigma F_x = 0$: $R_3 - 6099\# = 0$

$+\circlearrowright \Sigma M = 0$ @ E_3: $M_1 - (1200\#)(1.5'') = 0$

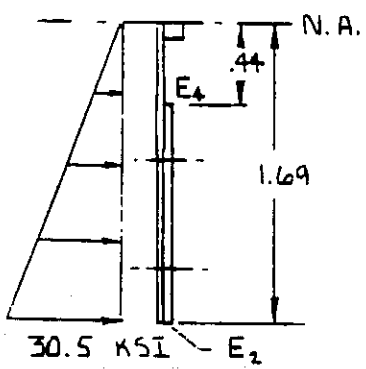
$R_3 = 6099\#$

$M_1 = 1800$ in·lb

Stress @ E_2:

$$\sigma_{comp} = \frac{(1800\ in\ lb)\ (1.69'')}{.23\ in^4} + \frac{6099\#}{.353} = 30500\ psi$$

Considering just the web & doubler combination as column:



$$\sigma_{avg} = \frac{\frac{(.44'')}{(1.69'')}(30.5\ KSI) + 30.5\ KSI}{2} = 19.22\ KSI$$

AVERAGE LOAD FROM E_2 TO E_4:

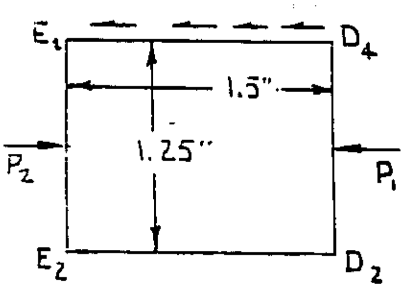
$P_2 = (19.22\ KSI)\ (1.25'')\ (.040'' + .050'') = 2.16$ KIP

AVERAGE LOAD FROM D_2 TO D_4:

$$P_T = \frac{6099}{.353\ in^2}\ (1.25'')\ (.04'' + .05'') = 1.94\ KIP$$

Using a Column with Distributed Axial Loading

From MAC 339 p. 16.38:



$$\frac{P_1}{P_2} = \frac{1.94}{2.16} = .90$$

$$\frac{P_2}{P_E} = 1.05 \qquad P_E = \frac{2.16\ KIP}{1.05} = 2.06\ KIP$$

Column Allowable:

Conservatively assume column pinned @ both ends:

$L' = L = 1.5"$

Determine least radius of gyration, ρ : ($\rho = \sqrt{\frac{I}{A}}$)

$I = \frac{(1.25")(.09")^3}{12} = .000076 \text{ in}^4$ $A = (1.25")(.040"+.050") = .113 \text{ in}^2$

$\rho = \sqrt{\frac{.000076}{.113}} = .026 \text{ in}$

Slenderness ratio:

$\frac{L'}{\rho} = \frac{1.5"}{.026"} = 57.7$

Euler Column Allowable:

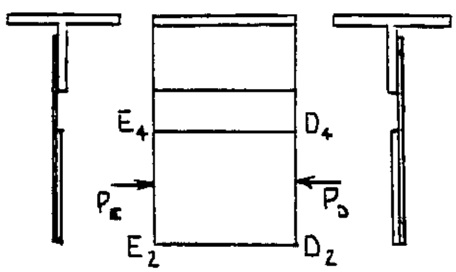
$F_C = \frac{\pi^2 E}{(L'/\rho)^2} = 30.5 \text{ KSI}$ (Where $E = 10.3 \times 10^6$ psi)

$P_{CR} = F_C A = (30.5 \text{ KSI})(.113 \text{ in}^2) = 3.45 \text{ KIP}$

Column Margin of Safety:

$M.S. = \frac{3.45 \text{ KIP}}{2.06 \text{ KIP}} - 1 = \underline{\underline{0.67}}$

Fastener Check:



Load in Doubler:

At E_2-E_4:

$P_E = (19.22 \text{ KSI})(1.25")(.05") = 1201 \text{ lb}$

At D_2-D_4:

$P_D = (17.28 \text{ KSI})(1.25")(.05") = 1080 \text{ lb}$

Load transfer = 121#

$q = \frac{121\#}{1.5"} = 81 \text{ \#/in}$

From MAC 339, p. 1.13:

Shear strength of BJ | 5 rivet in .040" 7178-T6 Alum sheet:

P = 575#

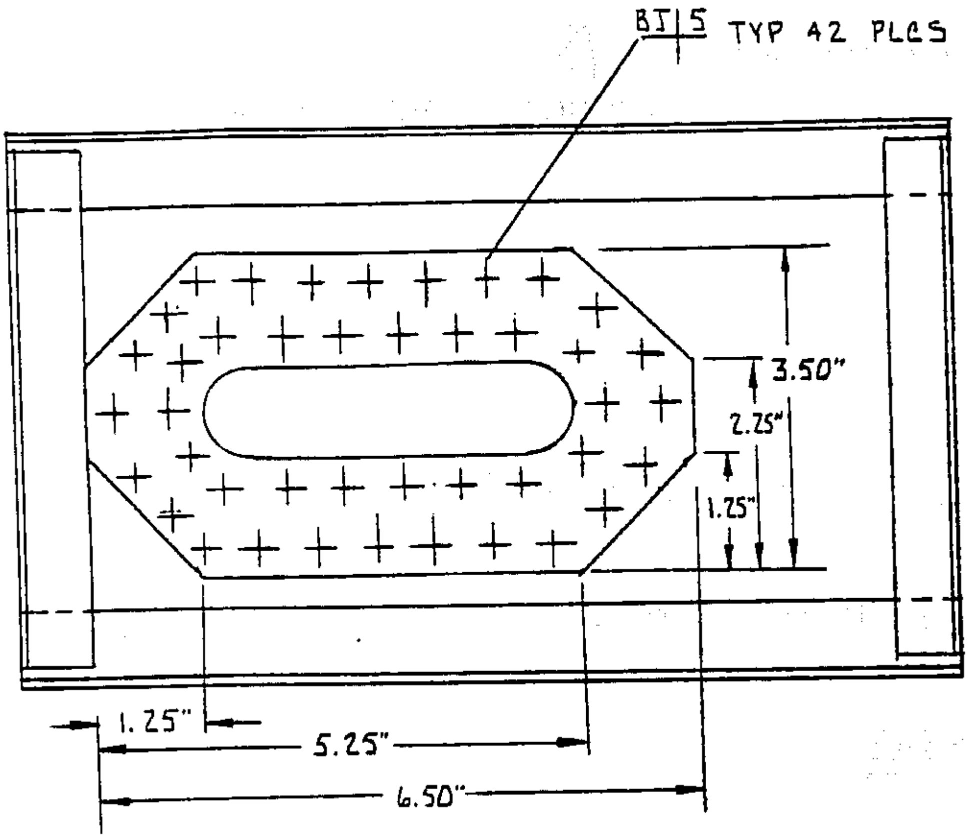
Assuming 4D fastener spacing: $q \text{ allow} = \frac{575\#}{.625"} = 920\#/"$

Margin of Safety:

$M.S. = \frac{920\#/in}{81 \#/in} - 1 = \underline{\underline{10.4}}$

13-13

Required Doubler:



13.3 Diagonal Tension Webs

Theoretically, a web can be in pure diagonal tension, with the web deeply buckled diagonally. In this configuration, all of the shear would be carried by tension parallel to the buckles. In practice, shear webs that are not shear resistant are in "incomplete diagonal tension". In this condition, some of the shear is carried by diagonal tension, and some as shear in the web. These are also called "tension field" webs. Panels with $\tau/\tau_{CR} > 2.5$ are in this category.

The analysis of tension field beams is based upon the superposition of two states of web stress, pure shear and pure diagonal tension. The figure below shows the stresses on web elements for each.

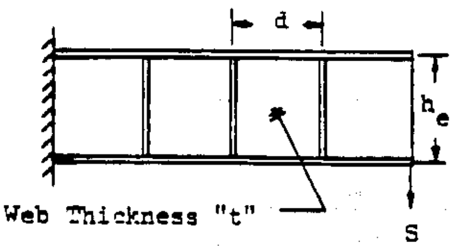
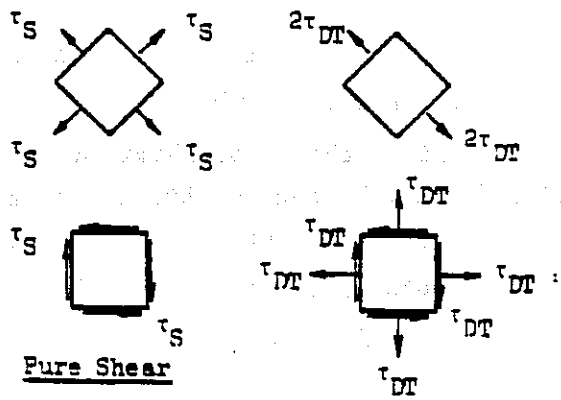


FIGURE 9.



Pure Shear

Diagonal Tension

The fraction of the total shear that is carried as diagonal tension is denoted by the symbol, k. If S represents the total shear, and S_{DT} and S_S represents the portion carried as diagonal tension and pure shear respectively, then:

$$S = S_{DT} + S_S$$

$$S_{DT} = kS \qquad (4)$$

$$S_S = (1 - k)\,S$$

The shear stress may be found from the shear flow equation:

$$\tau = q/t = S/h_e t$$

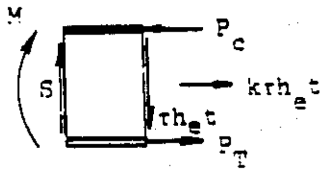
$$\tau = \tau_{DT} + \tau_S$$

Therefore:

$$\tau_{DT} = k\tau \qquad (5)$$
$$\tau_S = (1-k)\tau \qquad (6)$$

Now, consider the equilibrium of the infinitesimal length of beam shown in Figure 10:

FIG. 10:



The flange loads are:

$$P_T = M/h_e - \frac{1}{2}kS \qquad (7)$$
$$P_c = M/h_e + \frac{1}{2}kS$$

13.3.1 Analysis of Flat Webs

The analysis of flat diagonal tension shear panels includes the following checks:

1. Allowable Shear Stress in the Web

The stress at which the web buckles τ_{cr}, is determined by the method described above for shear resistant webs. The value of the diagonal tension factor, k, can be obtained from figure 2.5. The allowable web stress is also obtained from Figure 2.5. This figure was obtained for $\alpha = 45^o$ (the angle between the beam neutral axis and the direction of the diagonal tension), since for flat beams, the angle closely approaches this value near failure.

2. Stiffener Moment of Inertia

The stiffeners in a beam must be designed to carry the compression loading without buckling. To insure this, the moment of inertia of the stiffener is computed about its base, and then compared to the recommended stiffener moment of inertia for maximum buckling constant, I_s. The value for I_s is determined from figure 2. If the value of I for the stiffener exceeds the value of I_s, then the beam is "ok".

3. Flange Analysis

The flanges or caps of a beam are subjected to transverse loads due to diagonal tension in the web. Under such loads, they function as continuous beams supported at the stiffener(s). The secondary bending moments developed by this loading are normally less than would be expected from static analysis, because the flexibility of the caps results in non-uniform transverse loading. The following equation is recommended as representing the secondary moments at the stiffeners, and at the flange mid-span.

$$M_{ST} = \frac{q\,kd^2}{16} \qquad (8)$$

Where M_{ST}: = Secondary Moment due to diagonal Tension

The value computed from this equation must be compared to the allowable bending moment that would be found by treating the cap as a beam, using plastic bending analysis (For a review of this method, see Lesson 12). In the case of an end stiffener of a simple beam, the above equation may be used, provided that the tie between the end stiffener and the flange members is also designed for the moment given by the equation. In practice however, the end bay is usually made shear resistant in order to eliminate secondary effects.

The primary loads in the flange are found by the equations developed in Section 13.3. These loads should be used for checking the column and crippling strength of the flange. The critical crippling stress, F_{cc} is found by using the curve of figure 11. This curve is applicable to all ductile aircraft alloys, at both room and elevated temperatures.

4. Stiffener Analysis

a) The applied stiffener stress due to tension field effects may be computed from the equation:

$$\sigma_s = -\frac{k\,\tau\,\mathrm{TAN}\,\alpha}{\frac{A_{se}}{dt} + .5\,(1-k)} \qquad (9)$$

Where A_{se} = effective stiffener area

$$A_{se} = \frac{A_s}{1 + (e/\rho)^2} \text{ for single stiffeners}$$

$A_{se} = A_s$ (of both stiffeners), for double stiffeners.

and

e = distance from medium plane of web to centroid of stiffener, (in).

ρ = stiffener radius of gyration about the centroidal axis parallel to the web, (in).

In the equation above for the applied stress due to tension field effects, the load in the stiffener has been taken to be: $K\,\tau\,dt\,\tan\alpha$. This is only true in the case of a simple beam where two panels, of the same dimensions and stress level, contribute to the stiffener load! For all other cases, (different stress levels, different dimensions, or additional panels) the applied load should be taken as the summation of the loads contributed by the individual panels. The equation is:

$$P_{DT} = \sum \frac{k_1 \tau_1 d_1 t_1 \text{TAN } \alpha_1}{2} + \frac{k_2 \tau_2 d_2 t_2 \text{TAN } \alpha_2}{2} + \ldots \frac{k_n \tau_n d_n t_n \text{TAN } \alpha_n}{2} \quad (10)$$

The applied stiffener stress is then:

$$\sigma_s = -\frac{\sum \frac{k_1 \tau_1 d_1 t_1 \text{TAN } \alpha_1}{2} + \frac{k_2 \tau_2 d_2 t_2 \text{TAN } \alpha_2}{2} + \ldots + \frac{k_n \tau_n d_n t_n \text{TAN } \alpha_n}{2}}{A_{se} + .5 \sum \frac{(1-k_1) d_1 t_1}{2} + \frac{(1-k_2) d_2 t_2}{2} + \ldots + \frac{(1-k_n) d_n t_n}{2}} \quad (11)$$

where the subscripts, 1,2...,n, denote the panel members.

The individual terms ($k \cdot \tau \cdot d \cdot t \cdot \tan \alpha$) in the equation for the load due to diagonal tension are found by the relationship:

$$k \tau dt \text{ TAN } \alpha = [A_{se} + .5 (1-k) dt] \tau \, \sigma_{s/\tau} \quad (12)$$

where σ_s/τ is found by use of figure 12.

The maximum stress in the stiffener σ_{s-max}, occurs at the midpoint of the stiffener, and is obtained by the use of Figure 13.

b) Allowable Stresses for Single Stiffeners

To avoid forced crippling failure, the stiffener stress, σ_{s-max}, should not exceed the allowable stress, σ_o, defined by the empirical equation:

$$\sigma_o = C\, k^{2/3} (t_s/t)^{1/3} \quad (13)$$

where: C = material constant

t_s = stiffener thickness, in.

Values of C are included in Table 4. If the allowable stress, σ_o, exceeds the proportional limit of the material, multiply by the plasticity factor, η:

$$\eta = E_{sec}/E \quad (14)$$

where: E_{sec} = Secant Modulus, psi

E = elastic modulus, psi

The stress in the stiffener, σ_s, should not exceed the column yield stress. The average stress over the cross section of the stiffener, $\sigma_{s_{av}}$, given by:

$$\sigma_{s_{av}} = \frac{\sigma_s A_{se}}{A_s} \qquad (15)$$

should not exceed the allowable stress for a column with a slenderness ratio:

$$L'/\rho = h_s/2\rho \qquad (16)$$

c) Allowable Stresses for Double Stiffeners

Double stiffeners must be checked for forced crippling in the same manner as single stiffeners, that is, by the use of equation 13, and substituting the appropriate forced crippling constant, C.

The stiffener stress, σ_s, should not exceed the allowable column stress for a column with a slenderness ratio, L'/ρ, where:

$$L' = \frac{h_s}{\sqrt{1 + k^2\left(3 - \frac{2d}{h_s}\right)}} \quad (d < 1.5\,h_s) \qquad (17)$$

$$\text{or: } L' = h_s \quad (d > 1.5\,h_s)$$

5. Attachment Analysis

The final check is for the attachment of: a) the web to the caps; b) the stiffener ends to the caps; and c) the stiffeners to the web.

a) Web to Flange Connection

The web to flange attachments carry a running load per inch, R, where R is:

$R = S/h$ (for shear resistant beams)

$R = \sqrt{2}\;S/h$ (for a beam in pure diagonal tension)

Linear interpolation between these extremes gives the running load per inch for incomplete diagonal tension:

$$R = \frac{S(1 + .414k)}{h_r} \qquad (18)$$

where: h_r = distance between the rivet lines from the top to bottom flange (for a single row of rivets)

h_r = distance between the centroids of the rivet patterns (for multiple row rivet patterns)

b) Stiffener to Flange Connection

The end rivets of the stiffener must carry the stiffener load into the cap. These loads are:

$$P_{DT} = \sigma_s A_s \text{ (for double stiffeners)} \qquad (19)$$
$$P_{DT} = \sigma_s A_{se} \text{ (for single stiffeners)}$$

c) Stiffener to Web Connection

The strength necessary to avoid tension failure of the rivets in a stiffener is given by:

Tensile strength of rivets per inch $> .10\,\sigma_{ult}\,t$ (for double stiffners) (20)

$> .15\,\sigma_{ult}\,t$ (for single stiffners)

The tensile strength of a rivet is defined as:

1) The tensile load that will cause tensile failure of the rivet

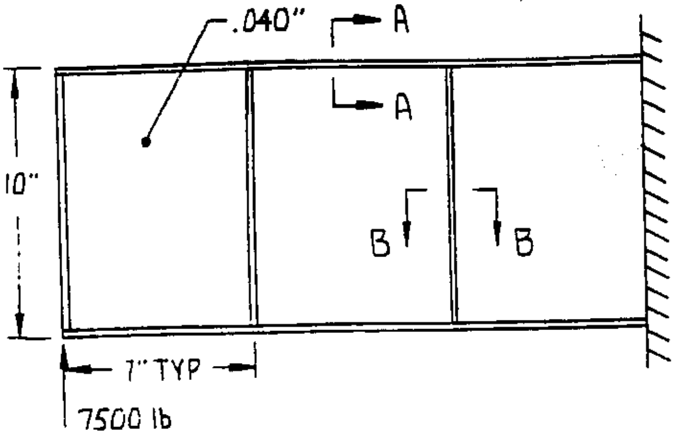
or:

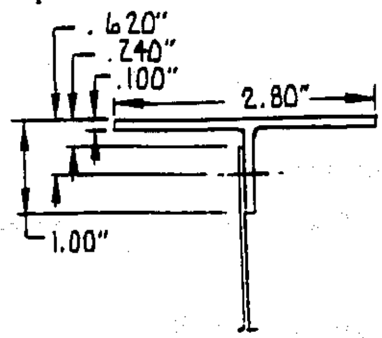
2) The tensile load that will cause the rivet to pull out of the attaching structure.

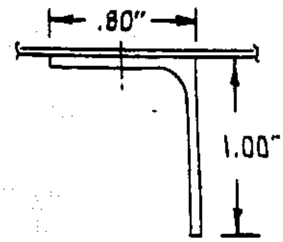
Normal rivet spacing of four to five rivet diameters will, in general, prevent permanent buckles from forming between the rivets, and also satisfy the tensile strength requirement.

13.3.2 Diagonal Tension Web Analysis Example

Given:







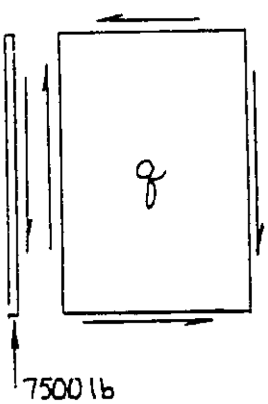
Sect A-A
t = .100 in
(typ for cap)

Sect B-B
t = .063 in
(typ for stiffener)

Web is 7075-T6 ALCLAD sheet

Tees and angles are 7178-T6 extrusion

Solution:



1. Web Analysis

a. Determine shear flow, q

$$q = \frac{V}{h_e}$$

Determine the effective height of the web

$$\frac{b}{t} = \frac{2.80 \text{ in}}{.100 \text{ in}} = 28.0$$

$$\frac{d}{b} = \frac{1.00}{2.80} = .360$$

from fig. 14:

$$\frac{x}{d} = .179$$

$$x = (.179)(1.00") = .179 \text{ in}$$

$$he = 10.00 - 2\,(.179) = 9.64 \text{ in}$$

Having h_e allows us to compute q:

$$q = \frac{7500 \text{ lb}}{9.64 \text{ in}} = 778 \text{ lb/in}$$

b. Determine the shear stress, τ :

$$\tau = \frac{q}{t} = \frac{778 \text{ lb/in}}{.040 \text{ in}} = 19450 \text{ psi}$$

c. Determine critical shear stress, τ_{cr}

$$\frac{qd^2\ 10^{-8}}{t^3} = \frac{(778 \text{ lb/in})(7.00 \text{ in})^2\ (10^{-8})}{(.040 \text{ in})^3} = 5.96$$

$$\frac{he}{d} = \frac{9.64 \text{ in}}{7.00 \text{ in}} = 1.38$$

From figure 2.5:

$$\frac{\tau}{\tau_{cr}} = 8.5$$

$$\tau_{cr} = \frac{(19.45 \text{ KSI})}{8.5} = 2.29 \text{ KSI}$$

d) Determine τ_{all} and K

From figure 2.5:

$$\tau_{all} = 28.2 \text{ KSI}$$

K = .44

e) Determine the Margin of Safety

$$\text{M.S.} = \frac{\tau_{all}}{\tau} = \frac{28.2 \text{ KSI}}{19.45 \text{ KSI}} - 1 = .45$$

f) Net section Shear:

Assuming 4D rivet spacing:

$$\frac{\text{Area}_{net}}{\text{Area}_{tot}} = \frac{\text{Area} - \text{Area}_{rivets}}{\text{Area}_{tot}} = .750$$

$$\text{Net } f_s = \tau \left[\frac{\text{Area}_{tot}}{\text{Area}_{net}}\right] = \frac{19.45 \text{ KSI}}{.750} = 25.9 \text{ KSI}$$

Compare to allowable ultimate shear stress:

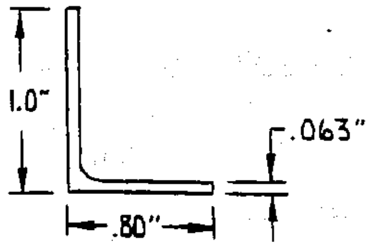
From: MIL-HDBK-5D: $F_{su} = 44$ KSI ∴ Not critical

2. Stiffener Moment of Inertia:

a) Determine the stiffener moment of inertia about its base:

$$I = \frac{(.063\ in)(1.00\ in)^3}{3} + \frac{(.737")(.063")^3}{3}$$

$$I = .021\ in^4$$



b) Required moment of inertia, I:

Recall: $\frac{he}{d} = \frac{9.64}{7} = 1.38$

From figure 2:

$$\frac{I_s}{h_e t^3} = 1.60$$

$$I_s = 1.60(9.64\ in)(.040\ in)^3$$

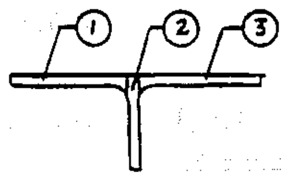
$$I_s = .001\ in^4$$

c) Compare I to I_s

is: $I > I_s$?

$.021\ in^4 > .001\ in^4$ ✓ OK

3. Flange Analysis



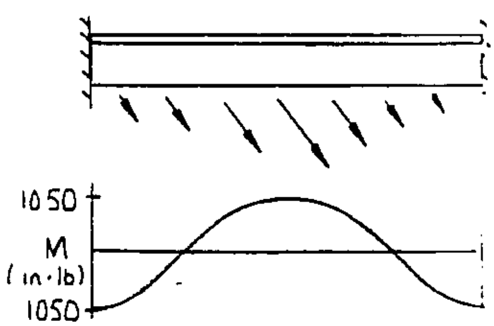
a) Compute the Cap Bending Moments:

$$M_{ST} = \frac{qkd^2}{16}$$

$$M_{ST} = \frac{(778\ lb/in)(.44)(7.00\ in)^2}{16}$$

$$M_{ST} = 1050\ in \cdot lb$$

b) Analyze the Cap as a beam:



Ele	b	t	A	y	Ay	Ay^2	I
1	1.36	.10	.136	.05	.007	.0003	.0001
2	1.00	.10	.100	.50	.050	.0250	.0083
3	1.36	.10	.136	.05	.007	.0003	.0001
			.372		.064	.0256	.0085

$$\bar{y} = \frac{.064\ in^3}{.372\ in^2} = .172\ in$$

$$I_{NA} = .0085\ in^4 + .0256\ in^4 - (.172)^2(.372\ in^2)$$

$$I_{NA} = .023\ in^4$$

Outer Fiber Stress:

$$\sigma_{max_1} = \frac{Mc_1}{I} = \frac{(1050\ in \cdot lb)(.828\ in)}{.023\ in^4}$$

$$\sigma_{max_1} = 37800\ psi$$

$$\sigma_{max_2} = \frac{Mc_2}{I} = \frac{(1050\ in \cdot lb)(.172\ in)}{.023\ in^4}$$

$$\sigma_{max_2} = 7852\ psi$$

F_{tu} = 84 KSI (From MIL-HDBK-5D)

Determine the Margin of Safety for the cap subjected to secondary bending moments:

$$M.S. = \frac{84 \text{ KSI}}{37.8 \text{ KSI}} - 1 = 1.22$$

Forced Crippling Check:

$$\sqrt{\frac{F_{cy}}{E}}\ \frac{b}{t} = \sqrt{\frac{(76 \text{ KSI})}{(10.7 \times 10^3 \text{ KSI})}}\left(\frac{(.828 \text{ in})}{(.100 \text{ in})}\right) = .70$$

From Figure 11: $\frac{F_{cc}}{F_{cy}} = .75$

$$F_{cc} = (.75)(76 \text{ KSI}) = 57 \text{ KSI}$$

Since the compressive stress on the section is non-uniform, the average stress = 2/3 (37.8 KSI) = 25.2 KSI. This value is considerably less than the maximum allowable crippling stress.

c) Apply Primary Loads to Cap:

$$P_T = \frac{M}{h_e} - 0.5\,(K)(V)$$

$$P_C = \frac{M}{h_e} + 0.5\,(K)(V)$$

The flange loads for analysis of column crippling are highest at the end of the first bay.

$$P_T = \frac{(7500 \text{ lb})(14.00 \text{ in})}{9.64 \text{ in}} - 0.5\,(.44)(7500 \text{ lb}) = 9240 \text{ lb}$$

$$P_C = \frac{(7500 \text{ lb})(14.00 \text{ in})}{9.64 \text{ in}} + 0.5\,(.44)(7500 \text{ lb}) = 12{,}540 \text{ lb}$$

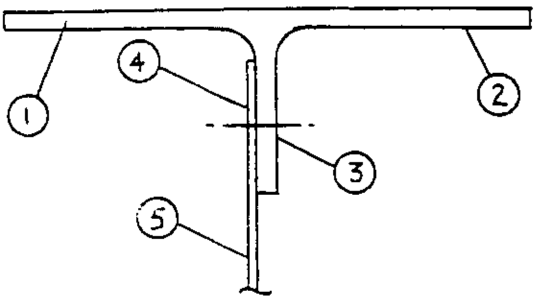
Computing the column loads at the reactions:

$$P_T = \frac{(7500 \text{ lb})(21.00 \text{ in})}{9.64 \text{ in}} = 16.34 \text{ KIP}$$

$$P_C = \frac{(7500 \text{ lb})(21.00 \text{ in})}{9.64 \text{ in}} = 16.34 \text{ KIP}$$

Determine the crippling stress of the flange:

Ele	b	t	b/t	Edge Condition	bt	Fcc	Fccbt
1	1.40	.100	14	One edge free	.140	37300	5222
2	1.40	.100	14	One edge free	.140	37300	5222
3	.95	.100	9.5	One edge free	.095	50500	4798
					.375		15242



$$F_{cc} = \frac{\Sigma F_{ccbt}}{\Sigma bt} = \frac{15242}{.375} = 40650 \text{ KSI}$$

Since $F_{cc} < F_{cy}$, $\eta = 1$

Using figure on page 16.13 of MAC 339:

$$\eta \frac{w_e}{t} = 38$$

Since element 4 is a one edge free skin element:

$$W_e = .35\left(\frac{\eta\, w_e}{t}\right) t = .35\,(38)(.040 \text{ in}) = .53''$$

(The actual effective width is .375 inches due to edge distance)

Effective width for element 5:

$$W_e = .50\left(\frac{\eta\, w_e}{t}\right) t = .50\,(38)(.040 \text{ in}) = .76''$$

The allowable crippling load is:

$$P_{cc} = (40.65 \text{ KSI})\left[.375 \text{ in}^2 + (.38'' + .76'')(.040'')\right] = 17.10 \text{ KIPS}$$

The crippling Margin of Safety:

$$M.S. = \frac{17.10 \text{ KIP}}{16.34 \text{ KIP}} - 1 = .047$$

The cap in the first bay should be analyzed as a column with distributed axial loading:

Using figure 15 with $P_2 = 16.34$ KIP and $P_1 = 12.54$ KIP

$$\frac{P_1}{P_2} = \frac{12.54 \text{ KIP}}{16.34 \text{ KIP}} = .77$$

$$\frac{P_2}{P_E} = 1.15$$

So the equivalent euler column load is:

$$P_E = \frac{P_2}{1.15} = \frac{16.34 \text{ KIP}}{1.15} = 14.21 \text{ KIP}$$

or

$$F_E = \frac{P_E}{A} = \frac{14.21 \text{ KIP}}{.375 \text{ in}^2} = 37.9 \text{ KSI}$$

The maximum compression end load can be determined:

$$\rho = \sqrt{\frac{.023 \text{ in}^4}{.372 \text{ in}^2}} = .249 \text{ in}$$

$$L' = (.7)(7.0 \text{ in}) = 4.9 \text{ in}$$ ⚠1

$$\frac{L'}{\rho} = \frac{4.9 \text{ in}}{.249 \text{ in}} = 19.7$$

Recall $F_{cc} = 40.65$ KSI

From p. 16.31 of MAC 339:

$$F_{cr} = 39 \text{ KSI}$$

Compute Margin of Safety for column:

$$M.S. = \frac{39 \text{ KSI}}{37.9 \text{ KSI}} - 1 = .029$$

4. Stiffener Analysis

a) Determine crippling allowables and effective skin widths:

Element	b(in)	t(in)	b/t	Edge Condition	Fcc(KSI)	bt	Fccbt
1	.968	.063	15.4	1 E. F.	34.5	.061	2100
2	.768	.063	12.2	1 E. F.	41.5	.048	2010
						.109	4110

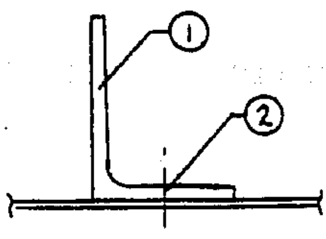
$$F_{cc-av} = \frac{\Sigma F_{cc}bt}{\Sigma bt} = \frac{4110 \text{ lb}}{.109 \text{ in}^2} = 37{,}710 \text{ psi}$$

Since F_{cc} is less than F_{cy}

$$\eta = 1.0$$

From p. 16.13 of MAC 339

$$\frac{2w_e}{t} = 39$$



⚠1 The edge fixity factor .7 is due in the web providing elastic constraint

Since on both sides of the stiffener, the web is continuous, the effective width is:

$$W_e = \left(\frac{2\,we}{t}\right)(.5)(t) = (39)(.5)(.040\text{ in}) = .78\text{ in}$$

and the allowable crippling load is:

$$P_c = (37710\text{ psi})[.109\text{ in}^2 + (.78")(.040")(2)] = 6463\text{ lb}$$

b) Determine the section properties for the effective stiffener:

Ele	b	t	A	y	Ay	Ay^2	I
1	1.00	.063	.063	.540	.034	.018	.005
2	.737	.063	.046	.071	.003	.000	.000
3	.780	.040	.031	.020	.001	.000	.000
4	.780	.040	.031	.020	.001	.000	.000
			.171		.039	.018	.005

$$\bar{y} = \frac{.039\text{ in}^3}{.171\text{ in}^2} = .228"$$

$$I = I + (Ay^2) - A(\bar{y})^2 = .005\text{ in}^4 + .018\text{ in}^4 - (.171\text{ in}^2)(.228\text{ in})^2$$

$$I = .014\text{ in}^4$$

$$\rho = \sqrt{\frac{I}{A}} = \frac{.014\text{ in}^4}{.171\text{ in}^2} = .286\text{ in}$$

$$e = .228\text{ in} - .020\text{ in} = .208\text{ in}$$

$$\frac{e}{\rho} = \frac{.208\text{ in}}{.286\text{ in}} = .727$$

Using figure 12, $\frac{A_{se}}{A_s} = .65$

Compute A_s/dt:

$$\frac{A_s}{dt} = \frac{(.171\text{ in}^2)}{(7.00\text{ in})(.040\text{ in})} = .61$$

Using figure 12 again gives:

$$\frac{A_{se}}{dt} = .395$$

Recall $\tau/\tau_{cr} = 8.5$, using figures 12 gives:

$$\sigma_s/\tau = .65$$

The applied stiffener stress due to tension field effects is:

$$\sigma_s = (.65)(19.45\text{ KSI}) = 12.6\text{ KSI}$$

$$F_{cy} = 75.0\text{ KSI (MIL-HDBK-5D)}$$

Compute Margin of Safety:

$$M.S. = \frac{F_{cy}}{\sigma_s} - 1 = \frac{75.0\ KSI}{12.6\ KSI} - 1 = 4.95$$

The maximum stiffener stress occurs at the midpoint at the stiffener and is obtained from figure 13.

$h_e = h_s$ so $h_s = 9.64$ in

$$\frac{d}{h_s} = \frac{7.00\ in}{9.64\ in} = .726$$

recall $\tau/\tau_{cr} = 8.5$

$$\frac{(\sigma_s)\ max}{\sigma_s} = 1.8$$

$(\sigma_s)\ max = (12.6\ KSI)(1.8) = 22.7\ KSI$

b) To prevent forced crippling at the stiffener, the maximum stress should not exceed the allowable crippling stress. Use the following equation to compute the allowable crippling stress:.

$$\sigma_o = C K^{2/3} \left(\frac{t_s}{t}\right)^{1/3}$$

From table 4:

C = 36400 psi for single stiffener of 7178-T6

So: $\sigma_o = (36400\ psi)\sqrt[3]{(.44)^2 \quad \frac{.063\ in}{.040\ in}}$

$\sigma_o = 24.5$ KSI

Compute the Margin of Safety:

$$M.S. = \frac{24.5\ KSI}{22.7\ KSI} - 1 = .079$$

Compute the average stiffener stress:

The average stress over the cross-section of the stiffener shall not exceed the allowable stress as a column.

$$\sigma_{s\ av} = \frac{\sigma_s\ A_{se}}{A_s} = (12.6\ KSI)(.65) = 8.19\ KSI$$

Recall, Fcc = 37.71 KSI

$$\frac{L'}{\rho} = \frac{h_a}{2\rho} = \frac{9.64 \text{ in}}{2\,(.286 \text{ in})} = 16.85$$

Using p.16.31 of MAC 339:

F_c = 36 KSI

Compute Margin of Safety: $M.S. = \frac{36 \text{ KSI}}{8.19 \text{ KSI}} - 1 = 3.40$

5. Attachment Analysis

a) Web to Flange Connection

Running Load:

h_r = 10.0" - 2(.62") = 8.76 in

$$R = \frac{S(1 + .414\,K)}{h_r} = \frac{(7500 \text{ lb})\,(1 + (.414)(.44))}{8.76 \text{ in}}$$

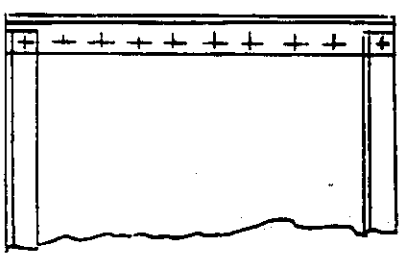
R = 1012 lb/in

Rivet Spacing

Assume 4D spacing, and 3/16" rivets (MS20470DD6)

(7 in)/.75 in = 9.33

This gives 8 rivets after the cap to stiffener rivets are thrown out.



The Load per Rivet:

$P_R = P_R\,(4d)$

P_R = (1012 lb/in)(.750 in) = 759 lb

From 339 p. 1.13: for e/d = 2

P_{all} = 1085 lbs

$M.S. = \frac{1085 \text{ lb}}{759 \text{ lb}} - 1 = .43$

b) Stiffener to Cap Connection:

Recall: $\frac{A_{se}}{dt} = .395$

So: Ase = (.395)(7")(.040") = .111 in^2

$P_{DT} = \sigma_s A_{se}$ = (12.6 KSI)(.111 in^2) = 1399 lb for MS20470DD8 rivets in .063" material (339 p.1.13)

for 1.5 diameter edge distance:

P_{all} = 1838 lb

$$M.S. = \frac{1838\ lb}{1399\ lb} - 1 = .314$$

c) Web to Stiffener Connection

P_t req = .15 σ_{ULT} t

P_t req = (.15)(72.0 KSI)(.040 in)

P_t req = 432 lb/in

For MS20470DD6 rivets with 4D spacing: (339, 1.11)

P_{all} = 450 lb

$$\frac{P_{tall}}{spacing} = \frac{450\ lb}{.750\ in} = 600\ lb/in$$

Compute Margin of Safety:

$$M.\ S. = \frac{600\ lb/in}{432\ lb/in} - 1 = .39$$

13.3.2 Curved Webs

The analysis of diagonal tension in curved web beams utilizes the methods developed for flat web systems, with minor modifications. For example, if a structure is built as a polygonal cylinder, and subjected to shear flows, the majority of the tension diagonals lie in the surface planes of the polygonized cylinder. In this case, analysis using flat web theory is applicable. In actual cylindrical structures having curved webs, the webs tend to "flatten" after buckling, and the structure approaches a polygonal condition. In this case, however, the majority of the tension diagonals lie on a hyperboloid of revolution, and the flat web theory is not directly applicable. Furthermore, the induced tension field causes radial loads on the rings which tend to put the rings in hoop compression. Lateral loads are also induced on the stringers between rings in the radial direction.

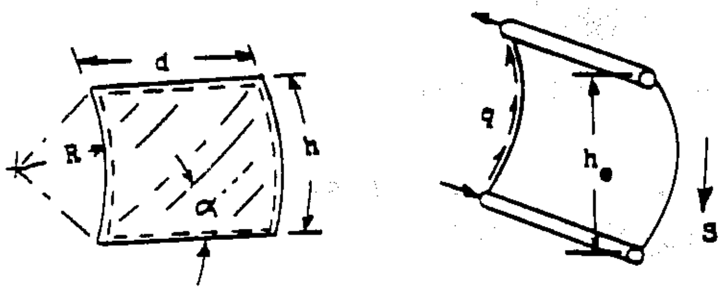


FIGURE 18: GEOMETRY OF CURVED WEBS

1. Curved Web Analysis

The stress at which the web buckles, F_{cr}, is determined by:

$$F_{cr} = K_s E \left(\frac{t}{b}\right)^2 \qquad (20)$$

where K_s is determined from fig. 17.

For a beam load, as in fig. 18, the applied shear stress is:

$$\tau = \frac{q}{t} = \frac{S}{h_e t} \qquad (22)$$

For other types of loading, such as pure torsion of a cylinder, the shear stress is found by conventional methods of analysis.

The relationship between the tension field factor, k, and the loading ratio, τ/τ_{cr} is given by:

$$K = \tanh \left[(0.5 + 300 \frac{td}{Rh}) \log \tau/\tau_{cr}\right] \qquad (23)$$

This relationship is plotted in figure 19. The allowable web stress, τ_{all}, for the diagonal tension angle, $\alpha = 45^o$, is shown in figure 20.

2. Diagonal Tension Angle

Recall that the angle of diagonal tension, α, is the angle between the neutral axis of a beam and the direction of diagonal tension. Although this angle does not affect the web stresses appreciably near the point of failure, it has considerable influence on stringer and ring stress in curved beams. Consequently, a method of computation is necessary, and follows. If the ring spacing is greater than the stringer spacing, (h < d), then:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_{ST}}{\epsilon - \epsilon_{RG} + \left(\frac{1}{24}\right)\left(\frac{h}{R}\right)^2} \quad (24)$$

On the other hand, if ring spacing is less than the stringer spacing, (d < h), then:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_{ST}}{\epsilon - \epsilon_{RG} + \left(\frac{1}{8}\right)\left(\frac{d}{R}\right)^2 \tan^2\alpha} \quad (25)$$

where: ϵ = web strain (from figure 21)

ϵ_{ST} = stringer strain = $\frac{\sigma_{ST}}{E}$ (comprehensive strain negative)

ϵ_{RG} = ring strain = $\frac{\sigma_{RG}}{E}$ (comprehensive strain negative)

The angle α, must be determined by the use of equation 26, 38, figure 21, and the equations above, by successive approximations, using $\alpha = 30^\circ$ as the first trial value.

3. Stringer Analysis

The angle of diagonal tension is smaller in curved webs than in flat webs. This causes the stringers to receive a relatively higher axial load than the rings. Although the stringers of a cylinder correspond to the flanges of a flat web beam geometrically, under load, the stringers act essentially like the stiffeners of a flat web beam. Therefore, the analysis of stringers is similar to the analysis of flat web stiffeners.

The portion of stringer stress due to diagonal tension is:

$$\sigma_{ST} = -\frac{k \tau \cot \alpha}{\frac{A_{ST}}{ht} + .5(1-k)} \quad (26)$$

For cases other than those involving two panels of the same dimensions and stress level, use the following equation:

$$\sigma_{ST} = -\frac{\sum \frac{k_1 \tau_1 h_1 t_1 \cot\alpha_1}{2} + \frac{k_2 \tau_2 h_2 t_2 \cot\alpha_2}{2} + \ldots \frac{k_n \tau_n h_n t_n \cot\alpha_n}{2}}{A_{ST} + .5\sum \frac{(1-k_1)h_1 t_1}{2} + \frac{(1-k_2)h_2 t_2}{2} + \ldots \frac{(1-k_n)h_n t_n}{2}} \quad (27)$$

The individual terms in the numerator of the equation above may be found by:

$$k\tau ht \cot\alpha = \left[A_{ST} + .5\,(1-k)ht\right]\sigma_{ST} \qquad (28)$$

where σ_{ST} is found for each term by solution of equations 26, 38, and 24 or 25 by successive approximations.

The stress from equations 26 and 27 is an average stress. The maximum stress, $\sigma_{ST\text{-}MAX}$, is obtained by the use of figure 13.

Because of the polygonal shape acquired by the cross section of a cylinder as diagonal tension develops, and due to the change in direction of the tension diagonal at each stringer, a radial pressure is exerted on the stringer which produces secondary moments. This radial pressure is not uniformly distributed due to: sagging of the stringer; possible sagging of the ring; and nonuniformity of skin stress. Therefore, the radial loading on the stringer will be approximated as:

$$M_{ST} = \frac{k\tau thd^2 \tan\alpha}{24R} \qquad (29)$$

An approximate beam column loading may be defined using the above expression.

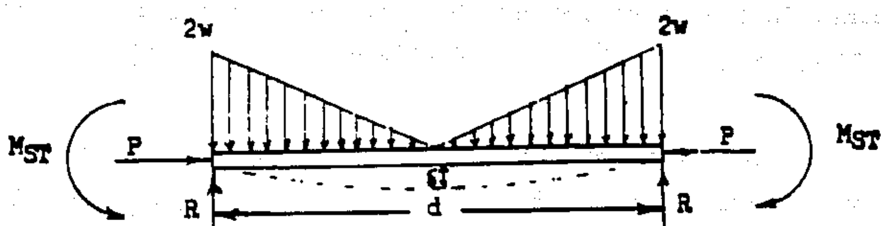


Figure 22

$$w = \frac{k\tau th \tan\alpha}{R} \qquad (30)$$

$$M_{ST} = -\frac{wd^2}{24} = -\frac{k\tau thd^2 \tan\alpha}{24R} \qquad (31)$$

$$R = \frac{wd}{2} \qquad (32)$$

$$P = \sigma_{ST}(A_{ST})_e + P_{primary} \qquad (33)$$

$$\delta = \frac{wd^4}{240EI} \qquad (34)$$

To avoid forced crippling failure, $\sigma_{ST\text{-}MAX}$ should not exceed the allowable forced crippling stress, σ_o, from equation 13. If σ_o exceeds the proportional limit, multiply by the plasticity correction factor.

$$\eta = E_{SEC}/E \qquad (14)$$

The combined loading on the stringer requires that it be checked as a beam column. Using the following relationships, the stringer may be checked by the method shown on p.16.50 of MAC 339.

$$M_o = -(M_{ST}) \qquad (35)$$

$$Y_o = \delta \qquad (36)$$

$$P = \sigma_{ST} \; (A_{ST})_e + P_{PRIMARY} \qquad (37)$$

4. Analysis of Rings

Rings which are riveted to the skin will act as stiffeners in flat web beams. The ring stress due to diagonal tension is:

$$\sigma_{RG} = - \frac{k \tau \, TAN \, \alpha}{\frac{A_{RG}}{dt} + .5\,(1-k)} \qquad (38)$$

For cases where all the panels of a curved web are not the same stress level, or are of different dimensions, use:

$$\sigma_{RG} = - \frac{\sum \frac{k_1 \tau_1 d_1 t_1 TAN \, \alpha_1}{2} + \frac{k_2 \tau_2 d_2 t_2 TAN \, \alpha_2}{2} + \ldots \frac{k_n \tau_n d_n t_n TAN \, \alpha_n}{2}}{A_{RG} + .5 \sum \frac{(1-k_1) d_1 t_1}{2} + \frac{(1-k_2) d_2 t_2}{2} + \ldots \frac{(1-k_n) d_n t_n}{2}} \qquad (39)$$

The individual terms, in the numerator of the equation above are found from:

$$K \tau \, dt \;\; \tan \alpha = \left[A_{RG} + \frac{1}{2}(1-K)dt\right] \sigma_{RG} \qquad (40)$$

where σ_{RG} is found for each term by the solution of eqns. 38, 26, and 24 or 25 by successive approximations. The maximum stress, σ_{RG-MAX}, may be found from figure 13.

To avoid forced crippling failure, σ_{RG-MAX} should not exceed the allowable forced crippling stress, σ_o, from equation 13. If σ_o exceeds the proportional limit, multiply by the plasticity factor:

$$\eta = \frac{E_{SEC}}{E} \qquad (14)$$

It is also necessary for rings to carry loads other than those due to diagonal tension. Therefore the following interaction equation is used:

$$R_1 + R_2 = 1.0 \qquad (41)$$

where

$$R_1 = \frac{f_b}{F_b} = \frac{\text{bending stress}}{\text{allowable bending stress}}$$

$$R_2 = \frac{\sigma_{RG\,MAX}}{\sigma_o}$$

5. Attachment Analysis

a) Web to Stringer

For an edge of a panel riveted to a stringer or longeron, the required rivet shear strength per inch is:

$$R'' = q \left[1 + k \left(\frac{1}{\cos \alpha} - 1 \right) \right] \qquad (42)$$

When a web is continuous across a stringer, the rivets need only be designed to carry the difference in shear flow, $q_1 - q_2$, between adjacent panels. The criteria for tensile strength of a stiffener (stringer) to a web should also be fulfilled.

b) Web to Ring

For an edge of a skin rivets to a ring, the required rivet shear strength per inch is:

$$R'' = q \left[1 + k \left(\frac{1}{\sin \alpha} - 1 \right) \right] \qquad (43)$$

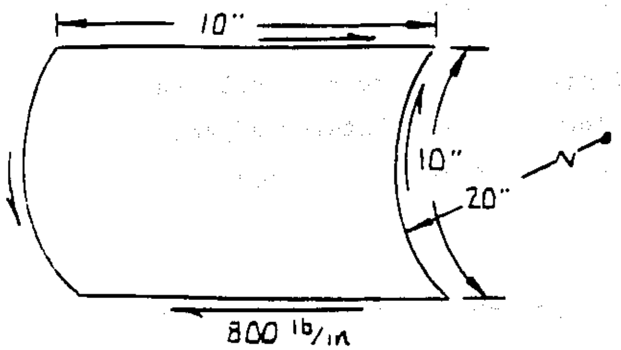
When the web is continuous across a ring, the rivets need to be designed to carry only the difference in shear flows, $q_1 - q_2$, between adjacent panels. Also, the criteria for tensile strength of a stiffener (ring) to a web should be fulfilled.

Curved Web Analysis

Consider the given curved panel:

Determine the allowable shear

Web Material: .040" 7075-T6



1. Web Analysis:

a) Find τ_{CR} :

$\frac{a}{b} = 1.0$

$$\frac{b^2}{rt} = \frac{(10.00\ )^2}{(20.00\ in)(.040\ in)} = 125$$

from figure 17:

$$K_s = 34.0 = \frac{F_{CR}}{E}\left(\frac{b}{t}\right)^2$$

$$F_{cr} = \frac{(10.3 \times 10^6\ psi)(34.0)}{\left(\frac{10.00\ in}{.040\ in}\right)^2}$$

$F_{cr} = 5600$ psi

b) Find the applied shear stress:

$$\tau = \frac{800 \text{ lb/in}}{.040 \text{ in}} = 20{,}000 \text{ psi}$$

c) Loading Ratio:

$$\frac{\tau}{\tau_{cr}} = \frac{20{,}000}{5600} = 3.57$$

d) Diagonal Tension Factor:

$$\frac{300td}{Rh} = \frac{300(.040 \text{ in})(10.00 \text{ in})}{(20.00 \text{ in})\ (10.00 \text{ in})} = .60$$

from figure 19;

$K = .54$

or computing directly

$$K = \tanh\left[\left(\frac{1}{2} + 300\frac{td}{Rh}\right)\log \tau/\tau_{cr}\right] = \tanh\left[(1.10)(\log 3.57)\right] = .54$$

e) Allowable Shear

from figure 20:

$\tau_{all} = 27.9$ KSI

TABLE 1: WEBS WITH FLANGED 45° LIGHTENING HOLES

D	B	A	D	B	A	D	B	A
1.31	.13	.13	3.61	.19	.19	6.16	.25	.25
1.56	.13	.13	3.86	.19	.19	6.66	.25	.25
1.81	.13	.13	4.11	.19	.19	7.16	.25	.25
2.06	.16	.16	4.36	.19	.19	7.71	.32	.25
2.33	.16	.16	4.61	.19	.19	8.21	.32	.25
2.58	.16	.16	4.86	.19	.19	8.71	.32	.25
2.83	.16	.16	5.11	.19	.19	9.27	.37	.25
3.08	.16	.19	5.36	.19	.19	9.77	.37	.25
3.36	.19	.19	5.61	.19	.19	10.27	.37	.25

TABLE 3: GEOMETRY OF BEADED FLANGE LIGHTENING HOLE
FOR WEBS WITH FLANGED LIGHTENING HOLES AND BEADED STIFFENERS

O.D.	I.D.	A
1.69	81	.19
1.81	94	.19
1.94	1.06	.19
2.06	1.19	.19
2.19	1.31	.19
2.31	1.44	.19
2.63	1.69	.25
2.75	1.81	.25
2.88	1.94	.25
3.00	2.06	.25
3.13	2.19	.25
3.38	2.44	.25
3.63	2.69	.25

O.D.	I.D.	A
3.88	2.94	.25
4.43	3.31	.38
4.94	3.81	.38
5.43	4.31	.38
5.94	4.81	.38
6.44	5.31	.38
6.94	5.81	.38
7.44	6.31	.38
7.94	6.81	.38
8.44	7.31	.38
8.94	7.81	.38
9.45	8.31	.38

TABLE 4: C Values for Various Materials

Mat'l.	Single Stiffener	Double Stiffener
7075-T6	32500	26000
7178-T6	36400	29600
2020 T6	37500	30000
Magnesium HK31A-H24	12500	10000
Titanium AMS 4908	58500	47000
2024-T3	26000	21000

FIGURE 1: SHEAR BUCKLING CONSTANTS FOR SHEAR RESISTANT BEAMS

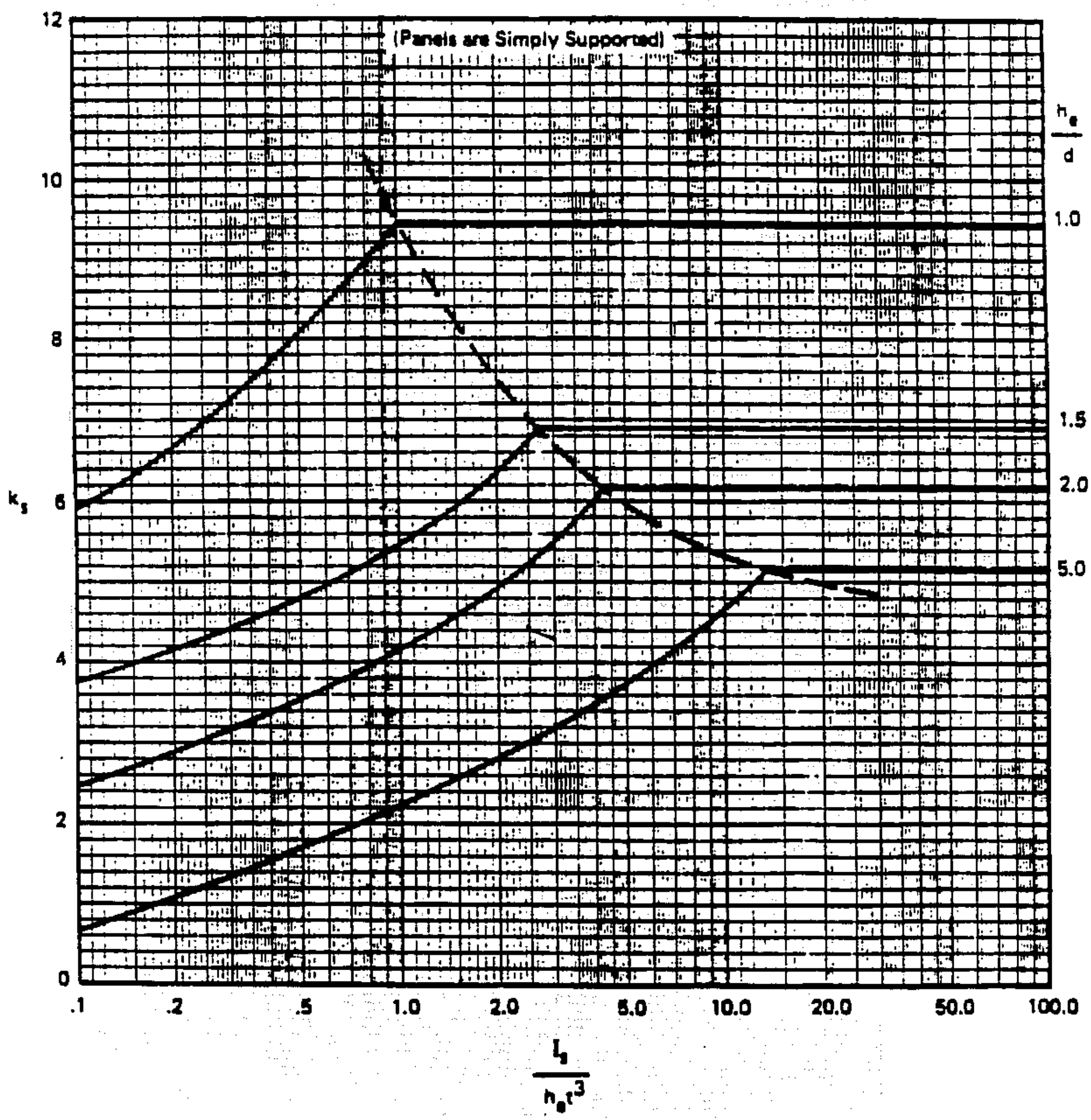


FIGURE 2 REQUIRED STIFFENER PROPERTIES FOR MAXIMUM BUCKLING CONSTANT

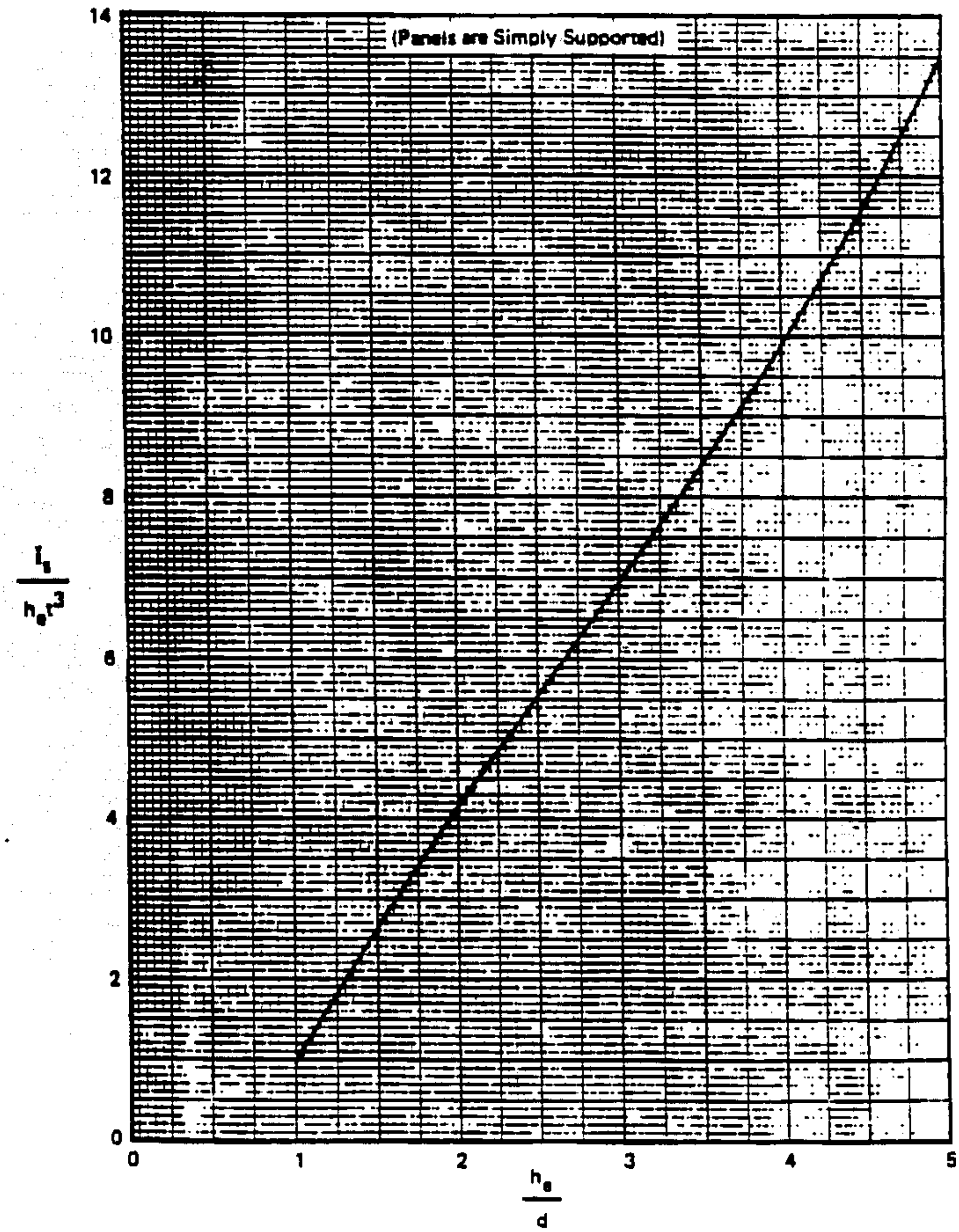
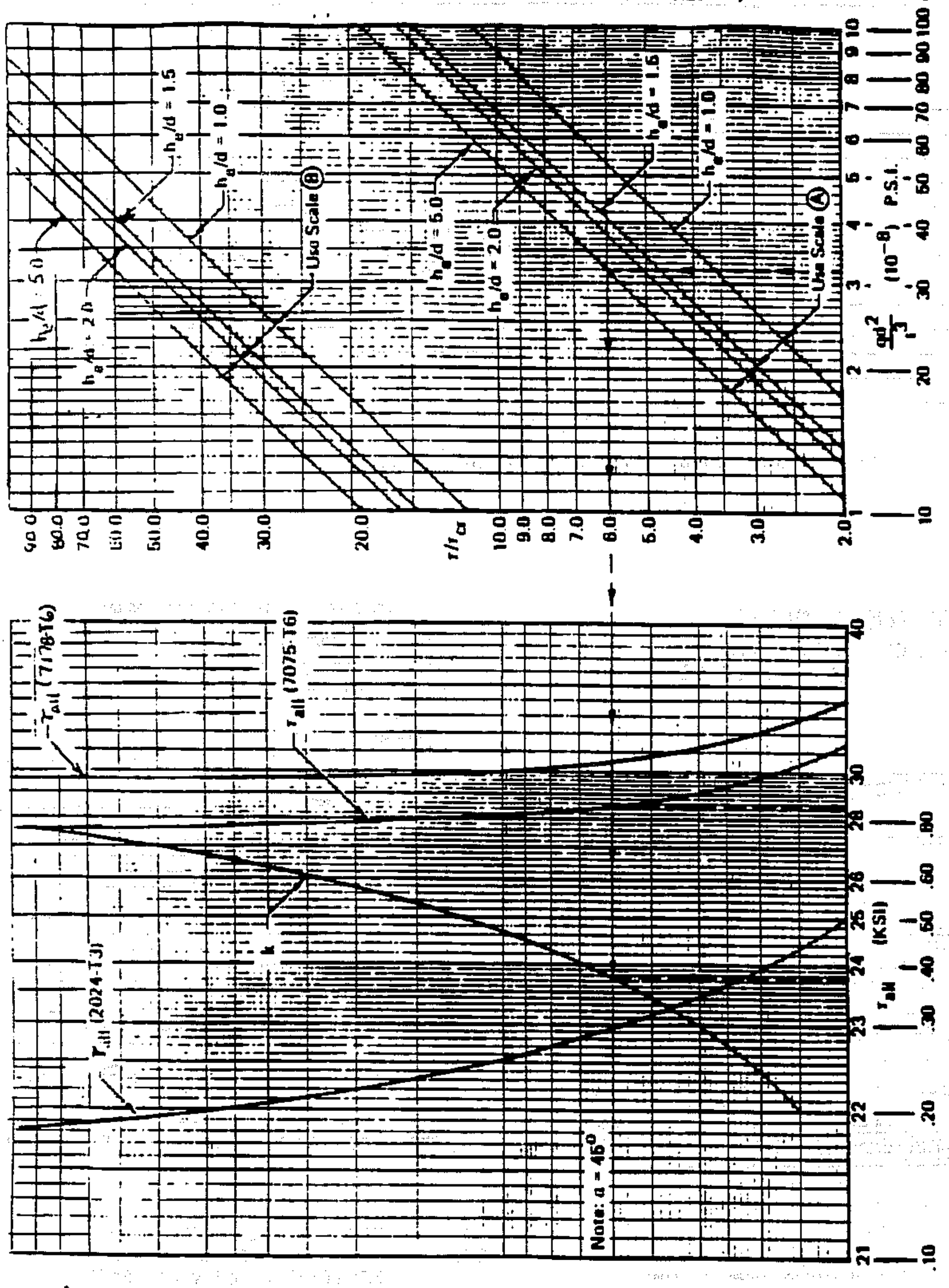


FIGURE 2.5

FLAT WEB ANALYSIS (ALUMINUM)



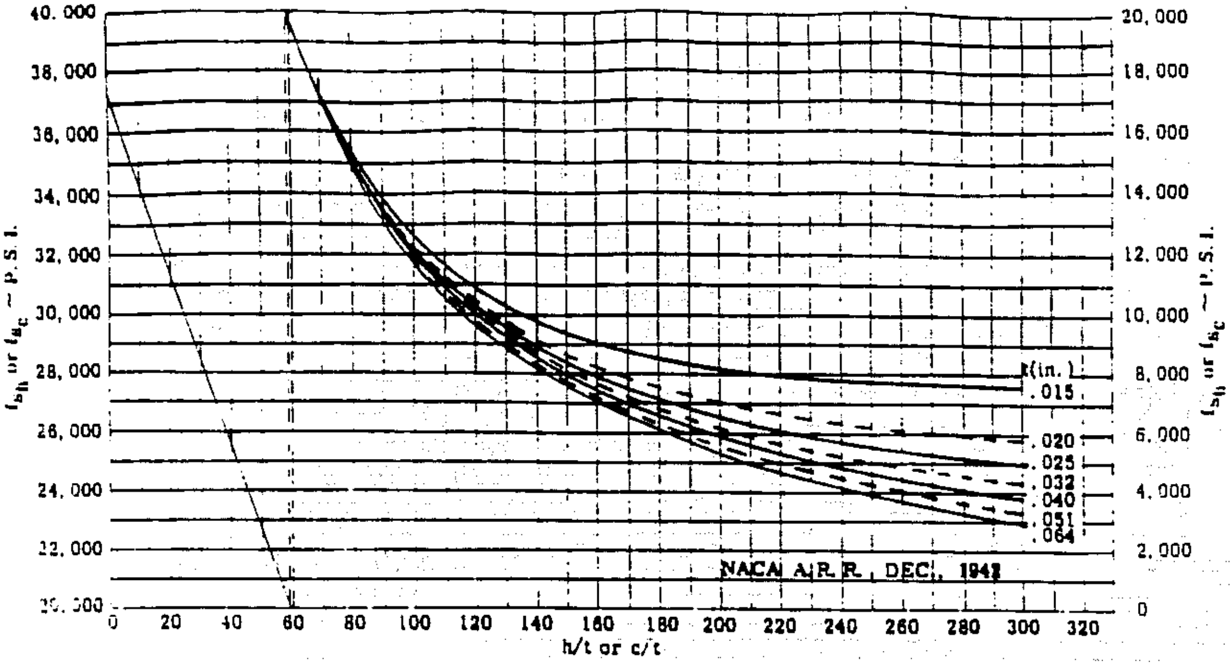
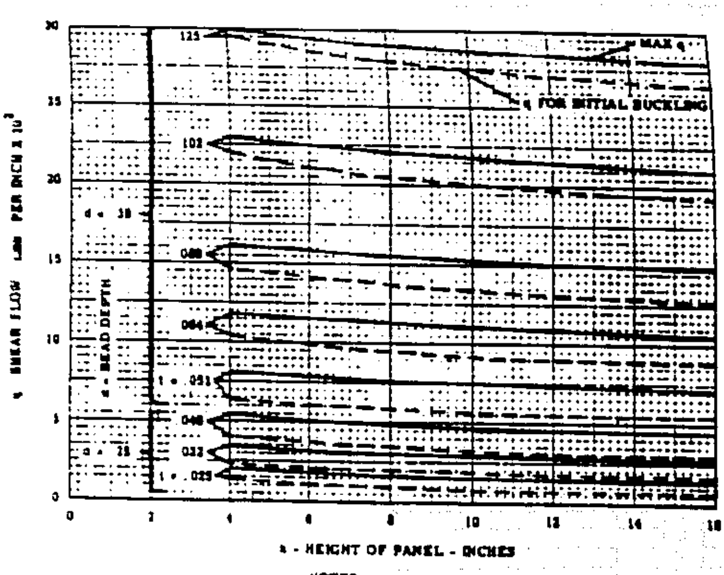


FIGURE 4: COLLAPSING SHEAR STRESSES, f_{s_h} OR f_{s_c}, FOR SOLID WEBS OF 24S-T ALUM. ALLOY

FIG 6: BEADED SHEAR PANELS

DESIGN CURVES FOR CLAD 2024-T4 AND 7075-T6 SHEAR PANELS STIFFENED BY MALE BEADS AT MINIMUM SPACING



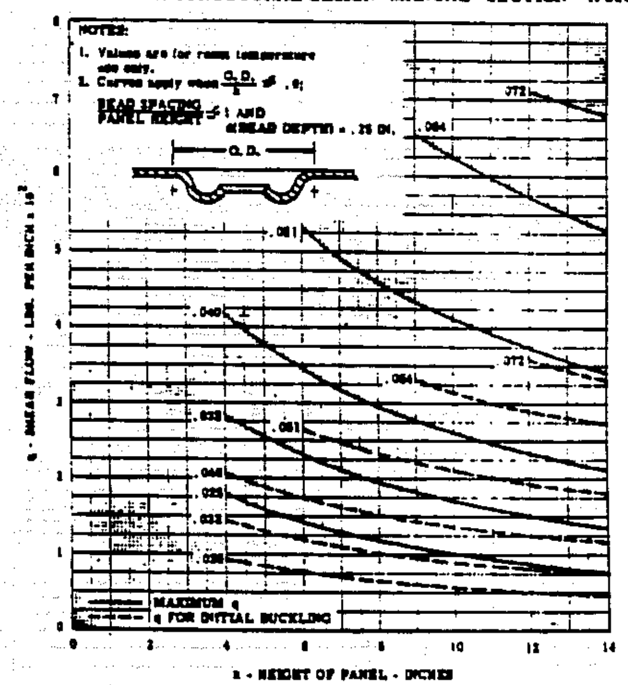
NOTES

1. For visible permanent set available test data indicate that q >= .90 q MAX.
2. Values are for room temperature use only.

REFERENCE: STRUCTURAL DESIGN MANUAL SECTION 4.230

CHANCE VOUGHT AIRCRAFT, INCORPORATED

REFERENCE: STRUCTURAL DESIGN MANUAL SECTION 4.230



DESIGN CURVES FOR CLAD 2024-T4 AND 7075-T6 SHEAR PANELS WITH CVC-3030 LIGHTENING HOLES AND STIFFENED BY MALE BEADS

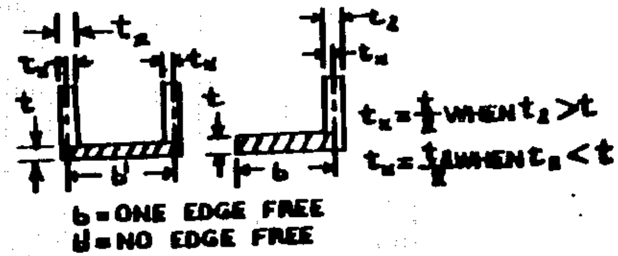
FIG. 8: BEADED SHEAR PANELS WITH LIGHTENING HOLES

FIGURE 11:

NONDIMENSIONAL CRIPPLING CURVES

REF. MAC 2721, PAGE 63

THIS CURVE APPLICABLE TO ALL DUCTILE AIRCRAFT ALLOYS AT BOTH ROOM AND ELEVATED TEMPERATURES.



NOTE: USE BARE THICKNESS ON CLAD MATERIALS

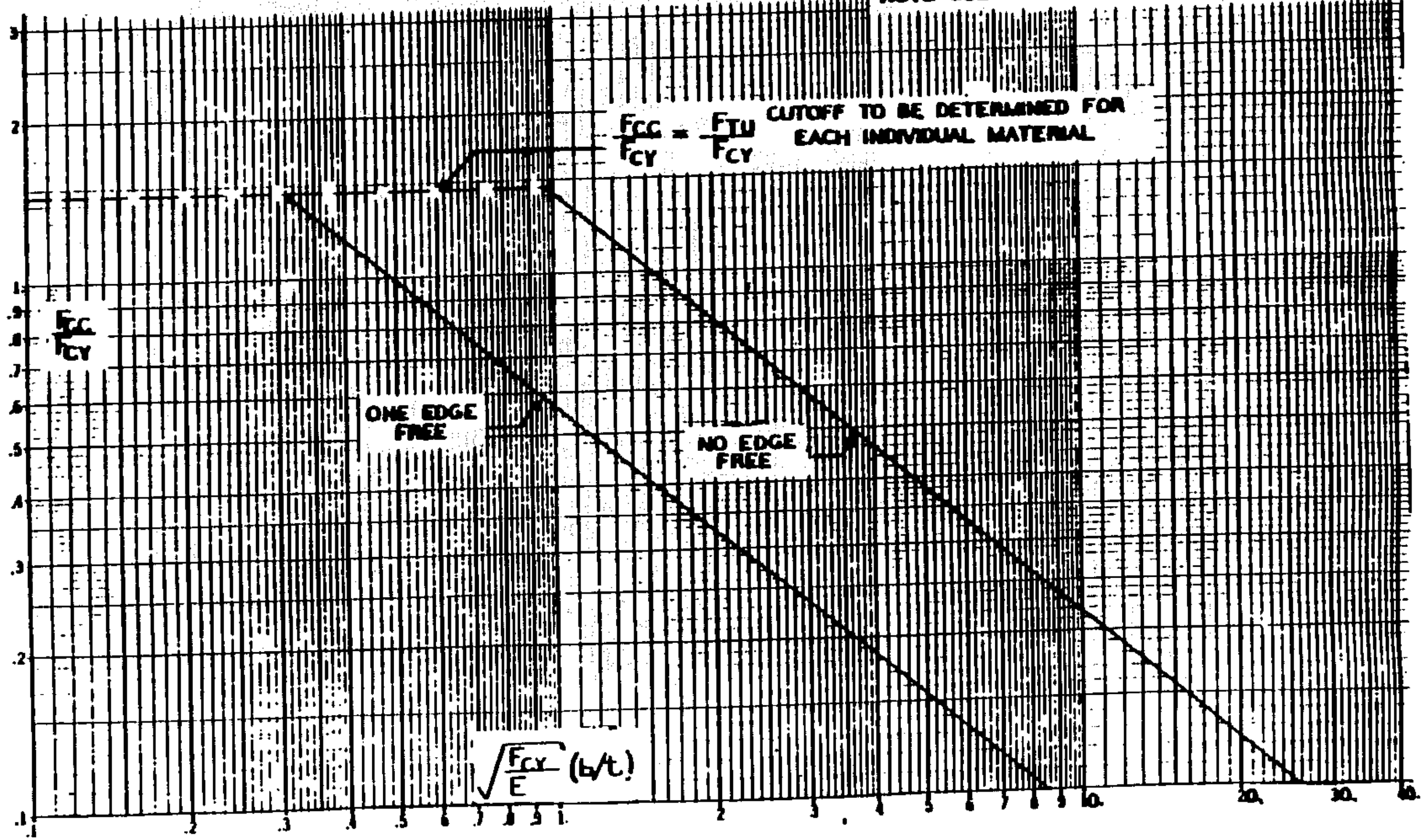


FIGURE 12
STIFFENER ANALYSIS

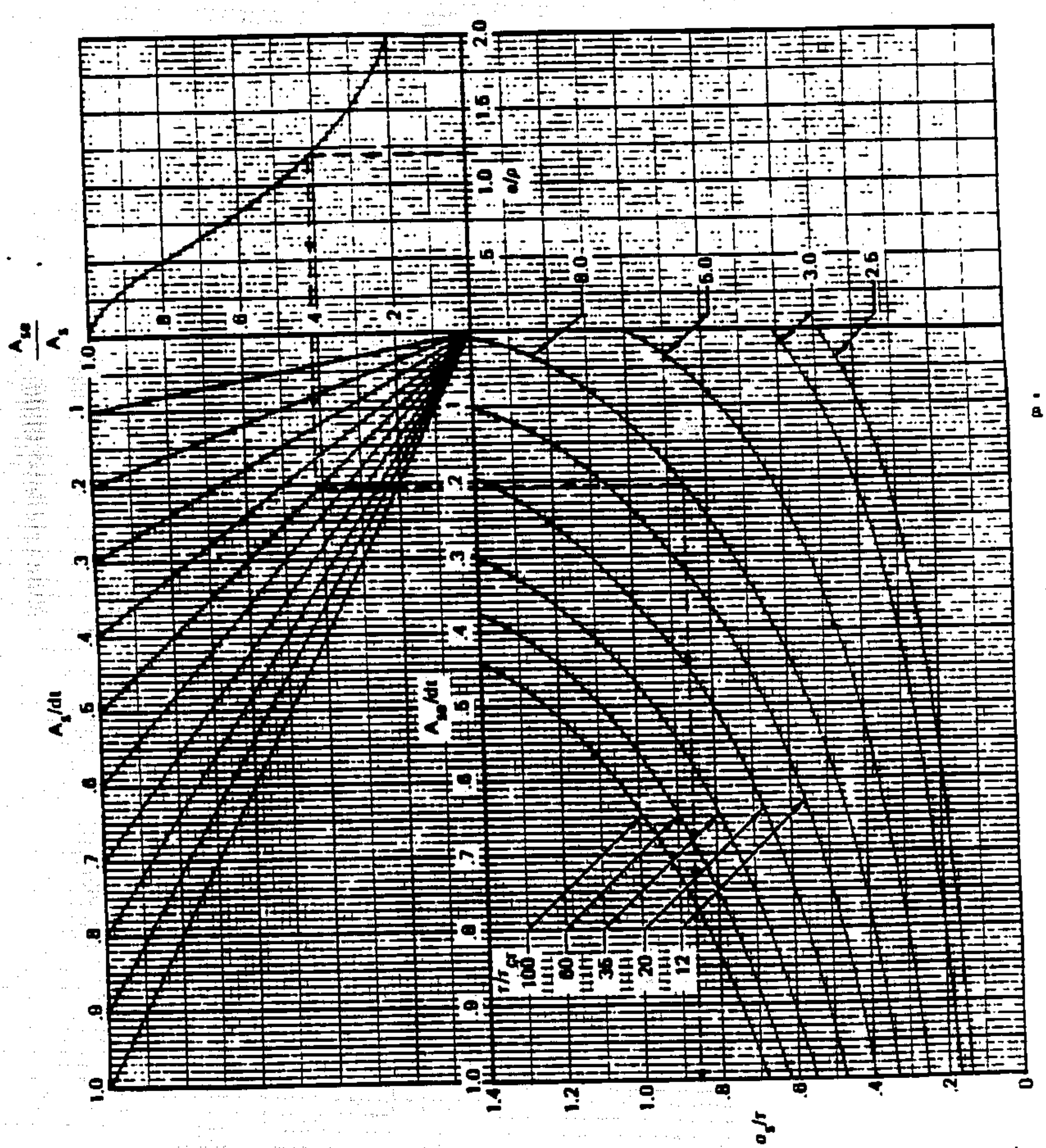
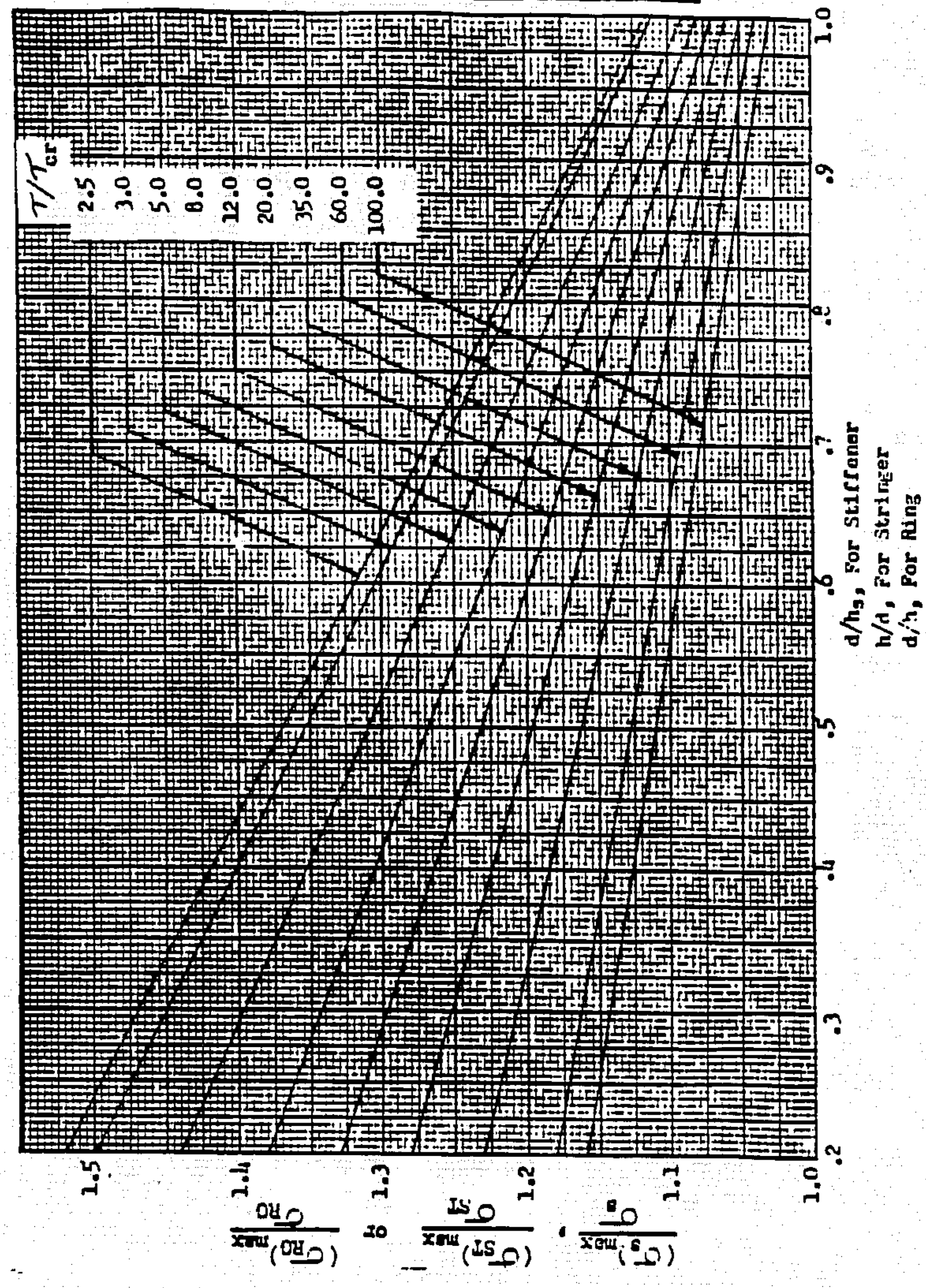


Figure 13
RATIO OF MAXIMUM STRESS TO AVERAGE STRESS IN STIFFENER, STRINGER OR RING



NEUTRAL AXIS LOCATION OF ANGLES OR TEES

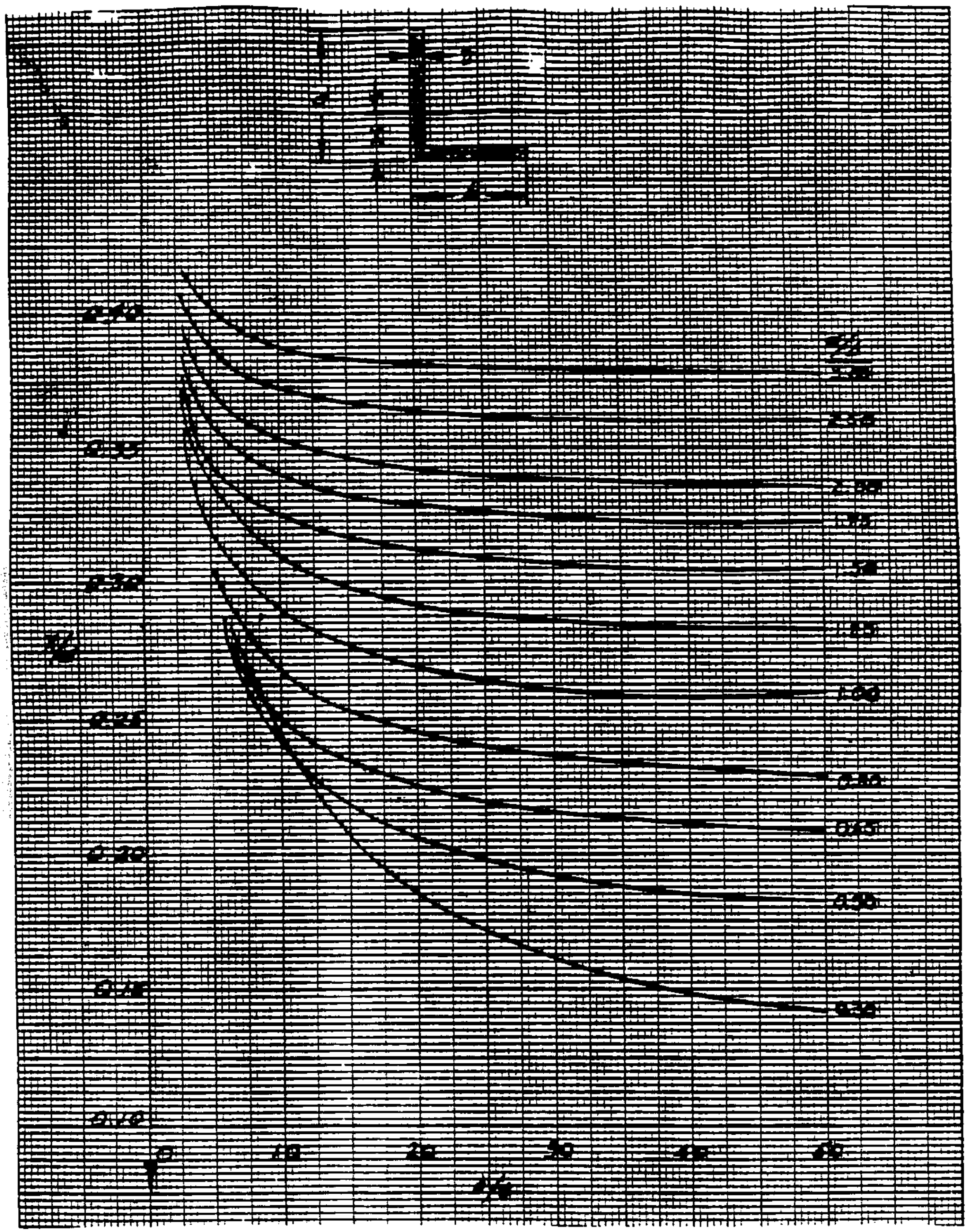
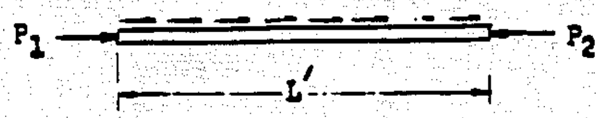


FIGURE 14

FIGURE 15:

COLUMN WITH DISTRIBUTED AXIAL LOADING HAVING A FIXED LINE OF ACTION



$P_2 > P_1$

$$P_E = \frac{\pi^2 EI}{L'^2} \text{ (lbs)}$$

For Stresses Beyond The Elastic Limit, Use E_T

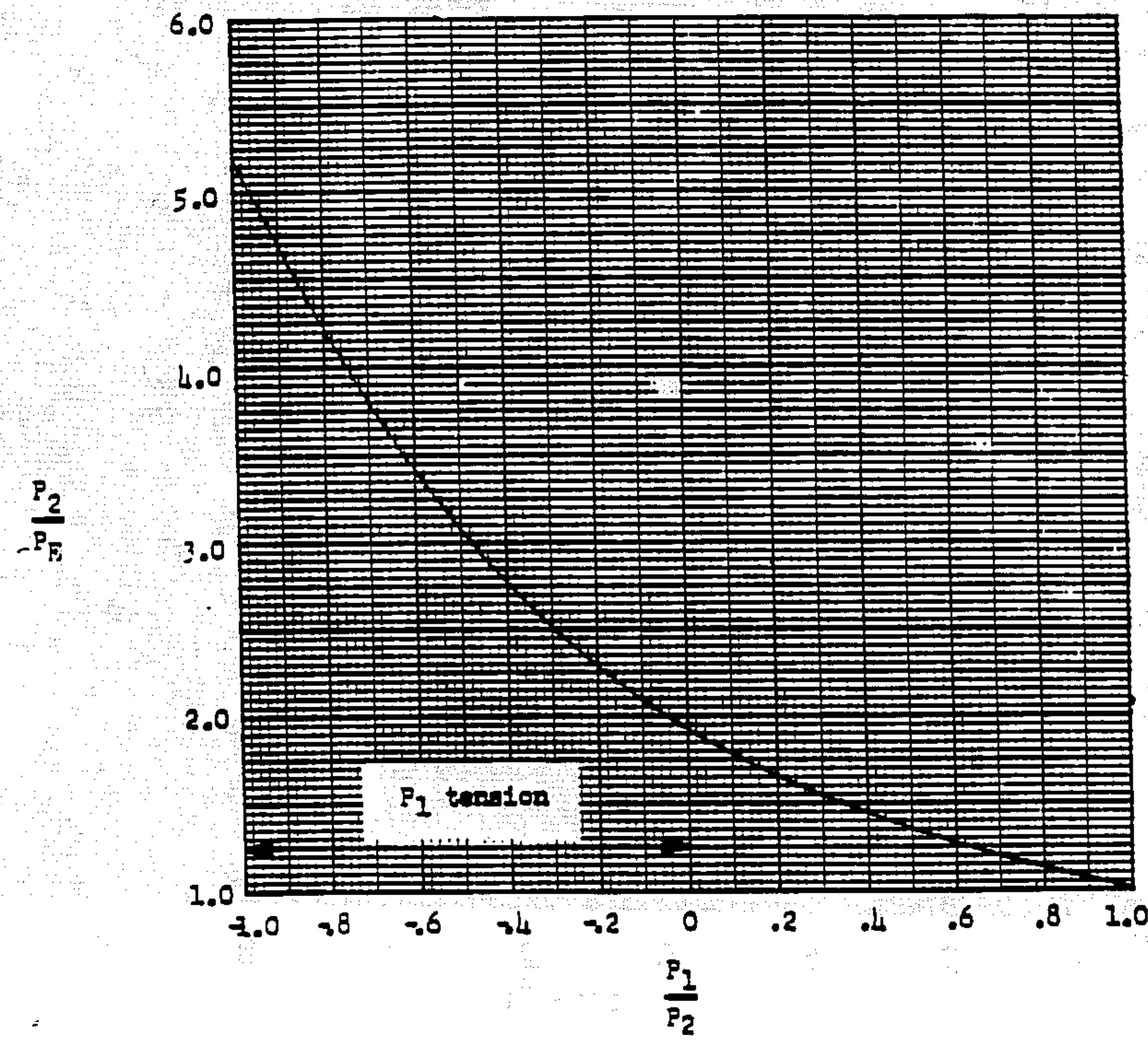


FIGURE 16.

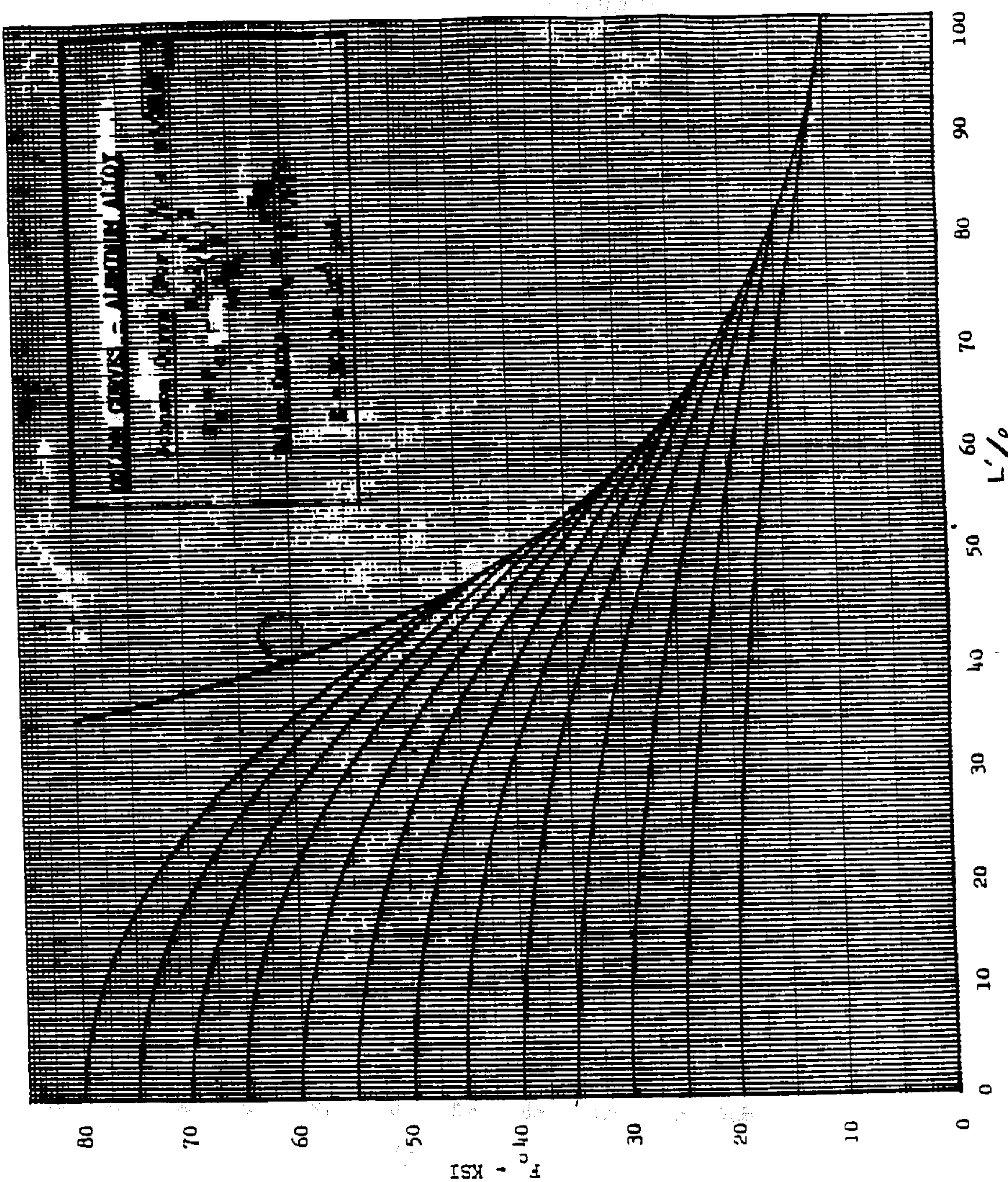


FIGURE 17:

CRITICAL SHEAR STRESS COEFFICIENTS

FOR SIMPLY SUPPORTED CURVED PANELS

$$K_s = F_{cr}/E \left(b/t\right)^2$$

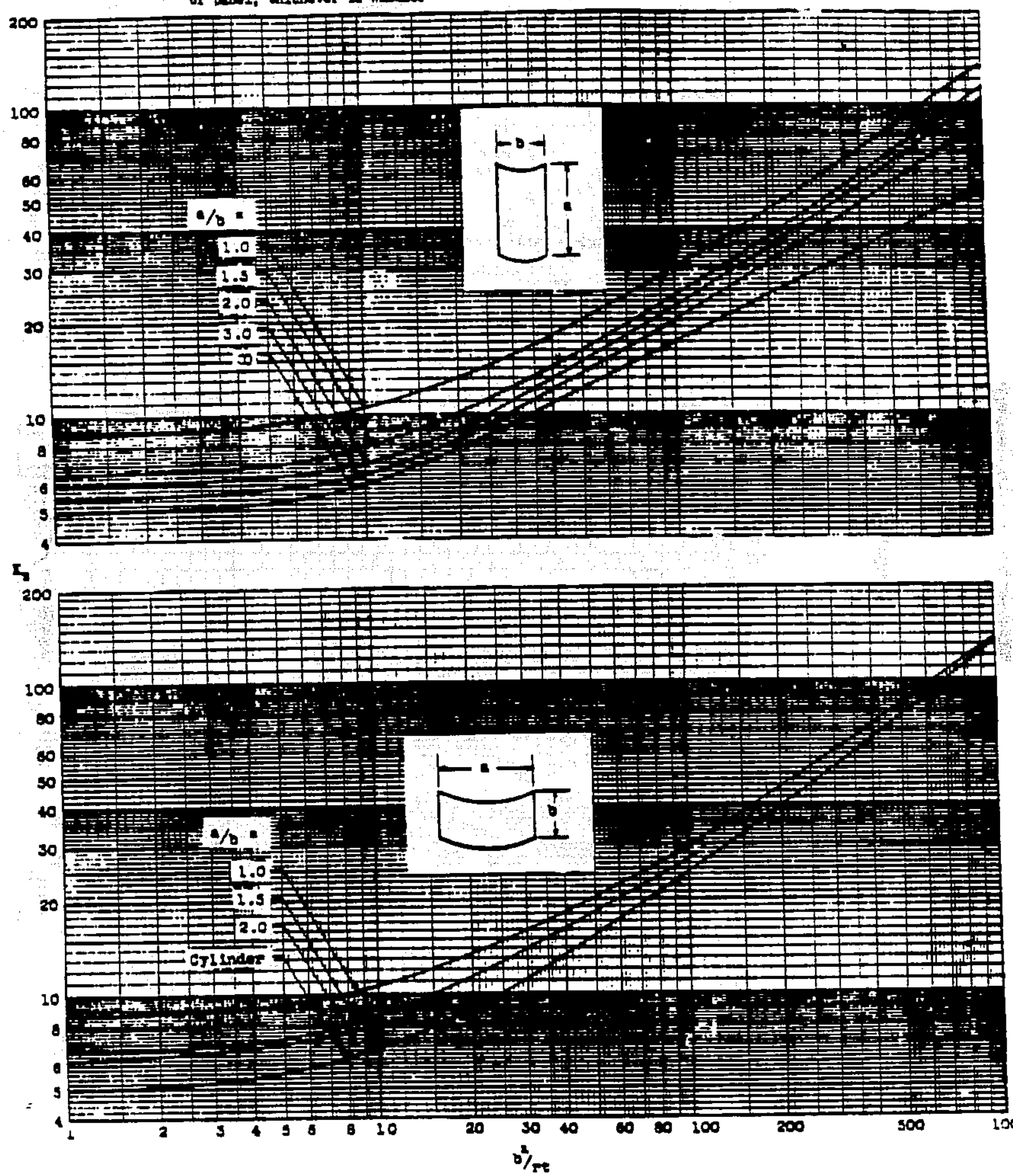
a = axial or circumferential dimension of panel, whichever is larger

b = axial or circumferential dimension of panel, whichever is smaller

r = radius of curvature of panel

t = thickness of panel

F_{cr} = critical shear stress



DIAGONAL TENSION FACTOR k FOR CURVED WEBS

Figure 19

$$K = \tanh\left[\left(0.5 + \frac{300\,t\,d}{R\,h}\right)\log\frac{\tau}{\tau_{cr}}\right]$$

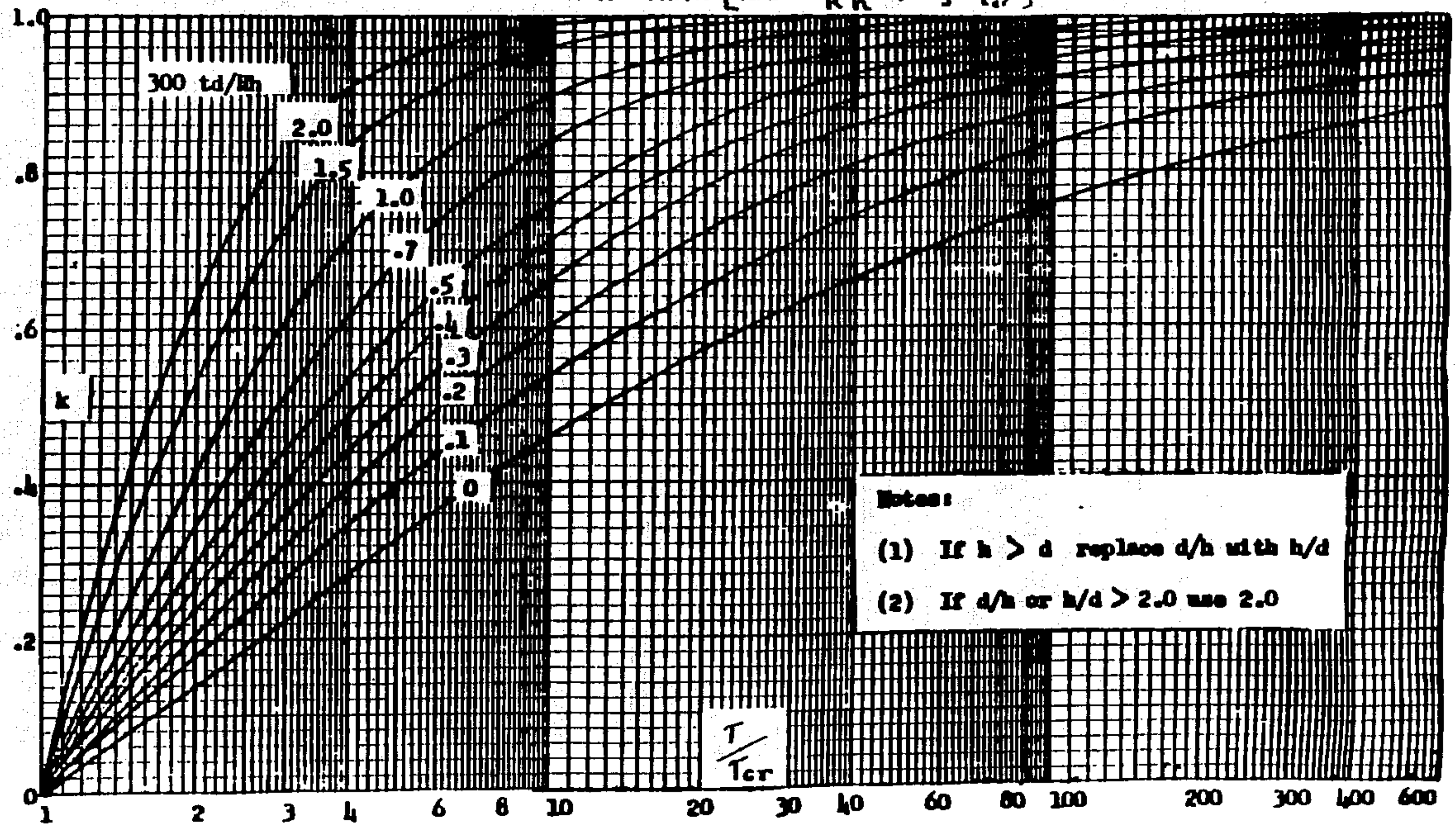


FIGURE 20
CURVED WEB ULTIMATE SHEAR STRENGTH

Notes:

1. For bare or clad materials, $\alpha = 45^{\circ}$
2. For materials other than shown, $\tau_{all}' = \tau_{all_{7075}} \dfrac{F_{tu}'}{F_{tu_{7075}}}$

where F_{tu}' is the ultimate tensile strength of the material under consideration and τ_{all}' is the allowable shear stress of the material

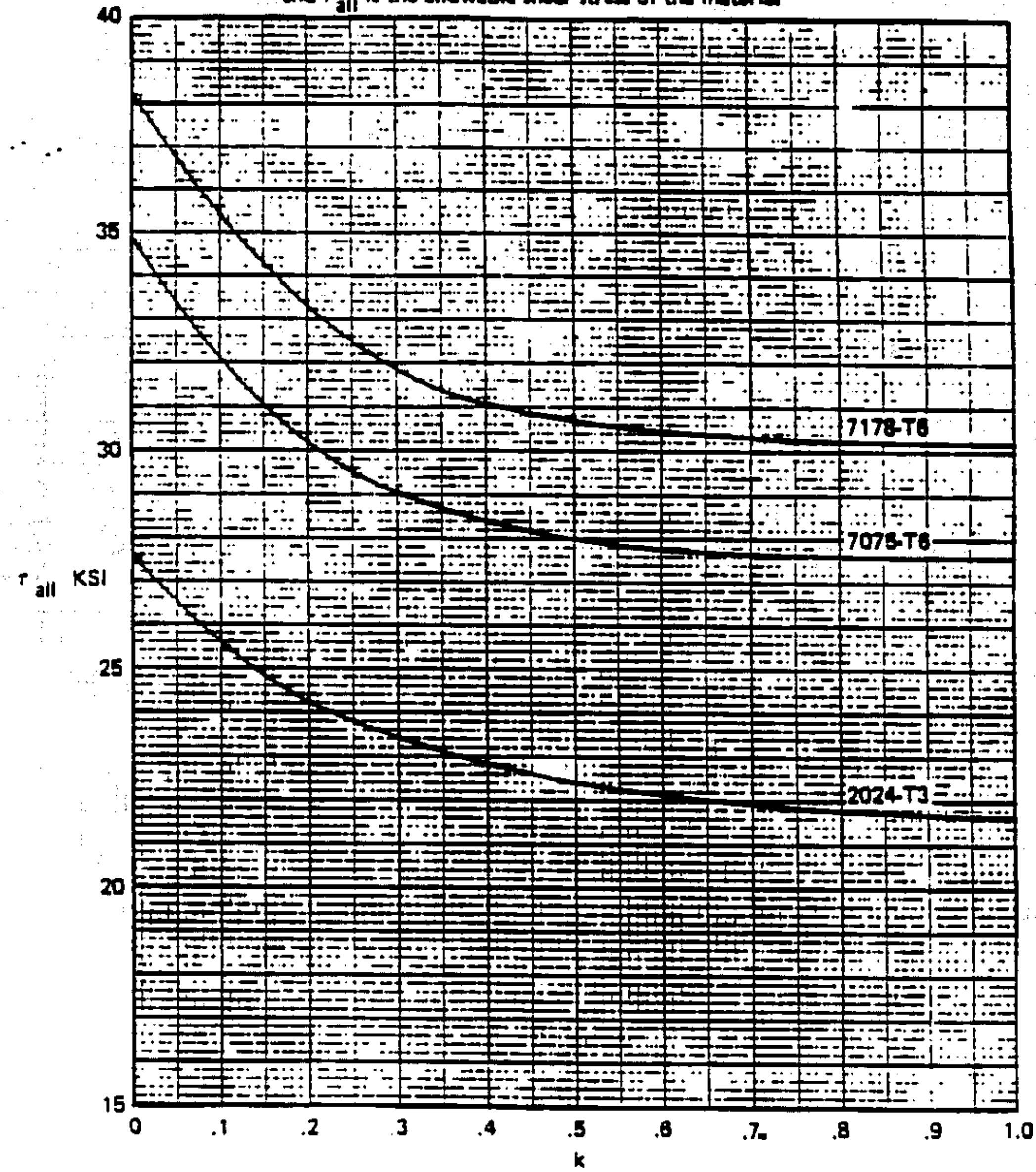
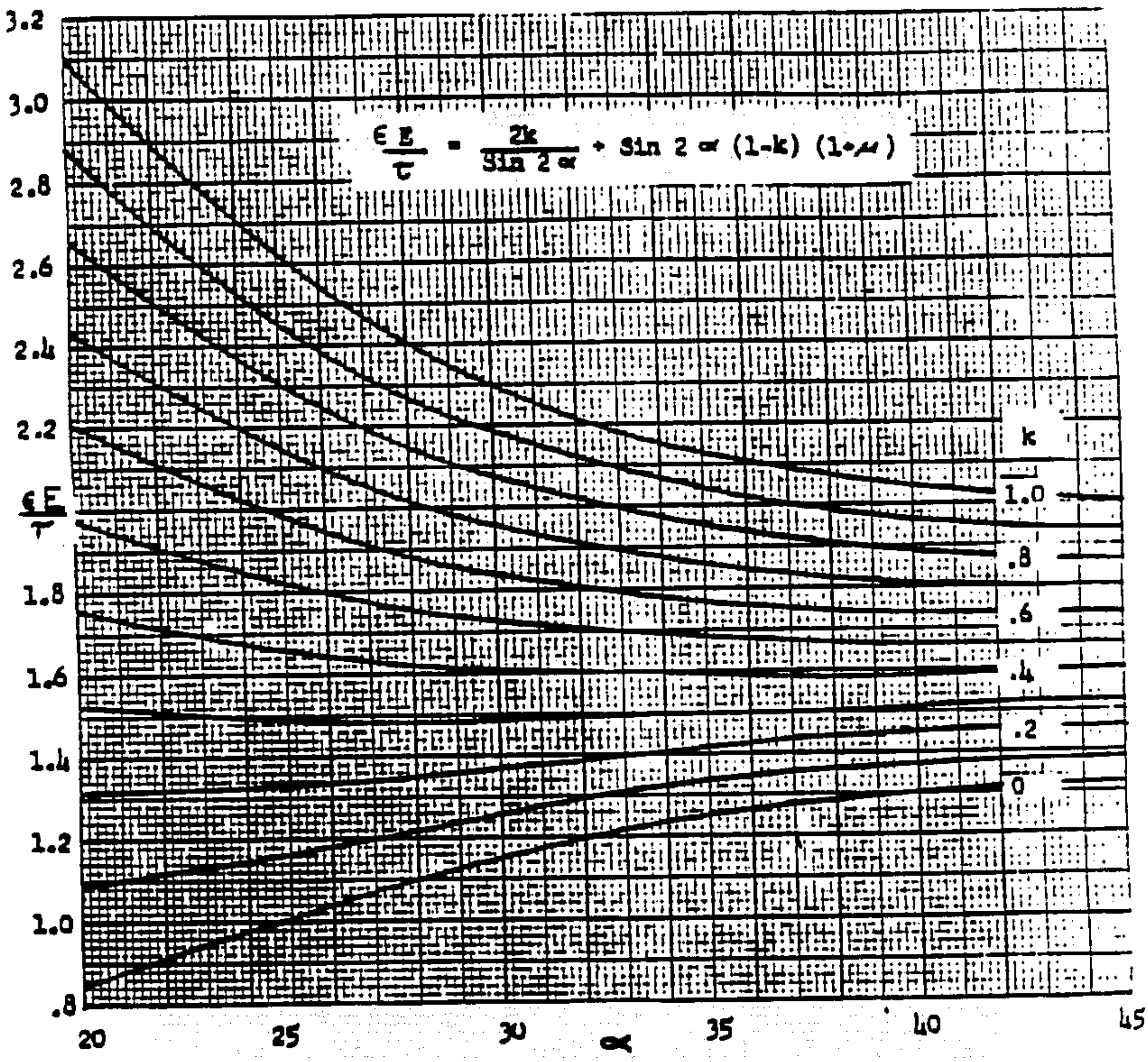


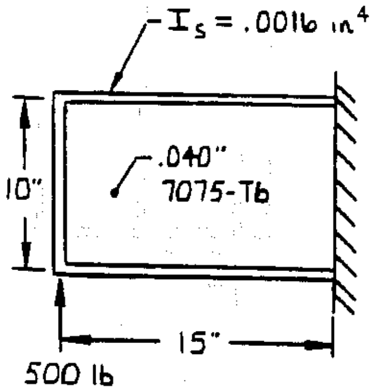
Figure 21

WEB STRAIN

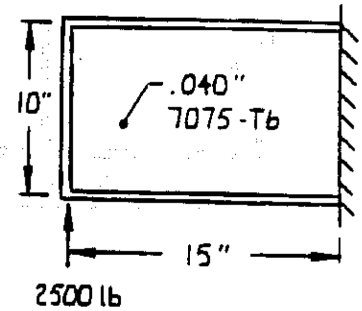


1. Determine the allowable, critical, and applied shear stress for the shear panels below:

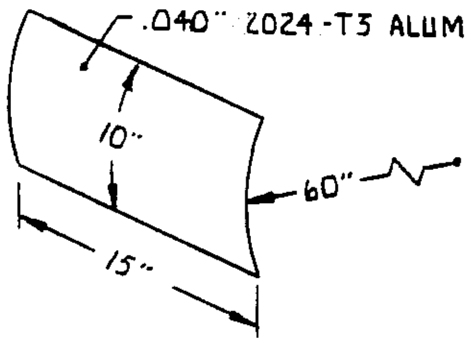
a)



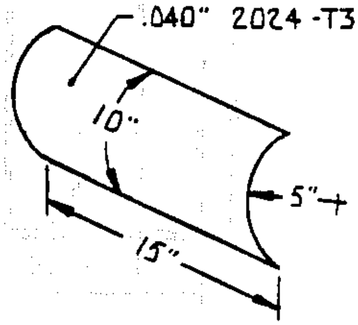
b)



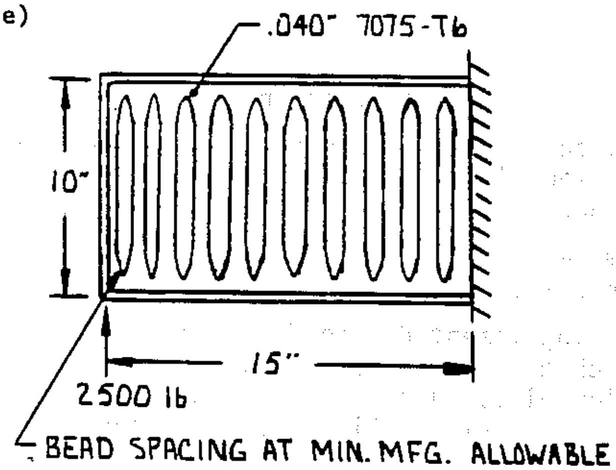
c)



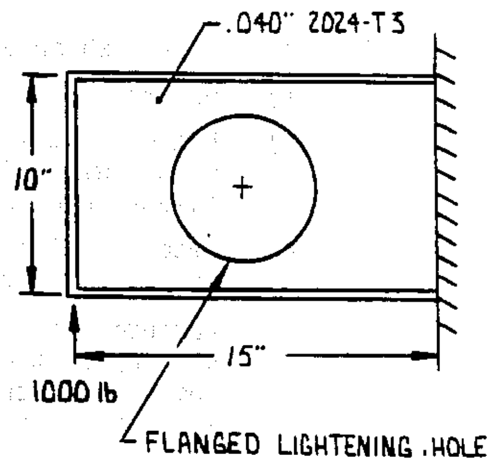
d)



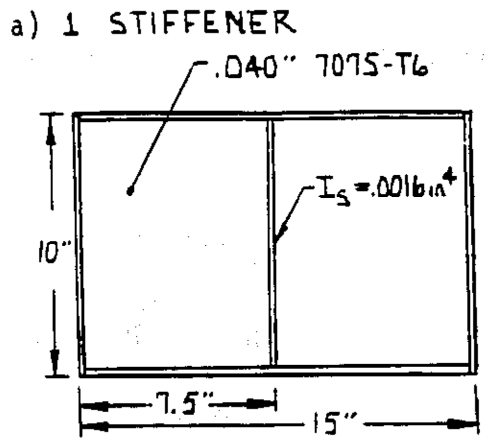
e)

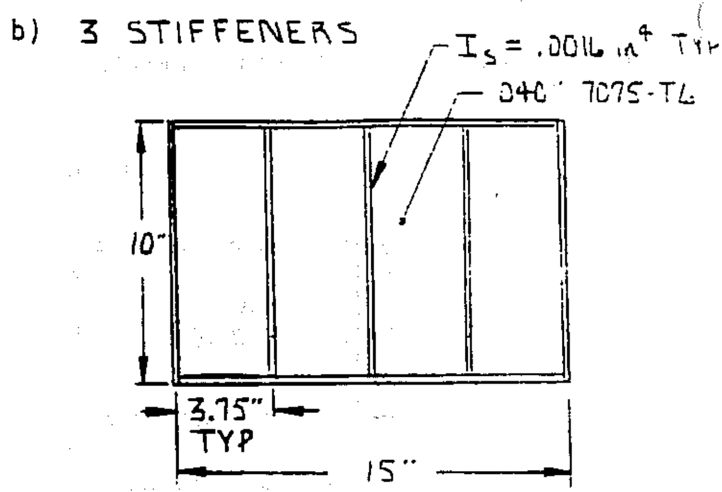


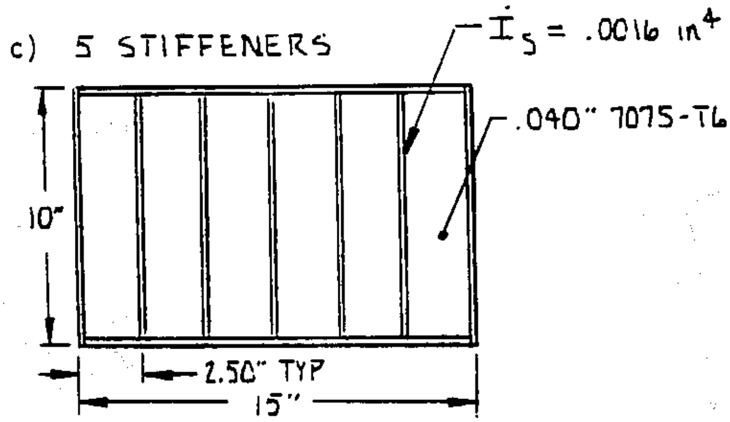
f)



2. Determine the allowable, critical and applied shear stress for the panels below:







3. Analyze the shear panel below

a) Determine the section properties of the caps and stiffeners

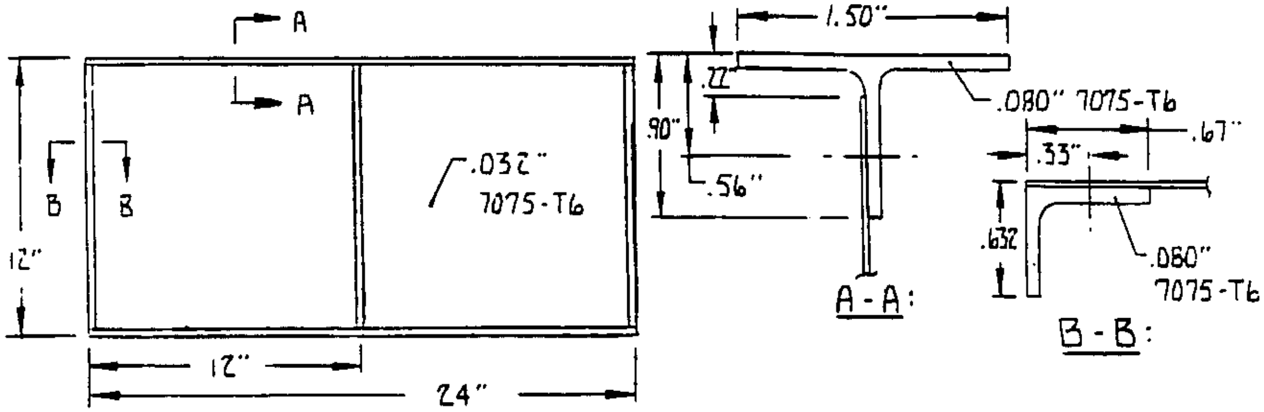
b) Determine the shear flow, shear stress, and critical shear stress of the web. Then compute the allowable shear stress, margin of safety, and net section shear.

c) Determine the stiffener moment of inertia, and compare it to the required moment of inertia.

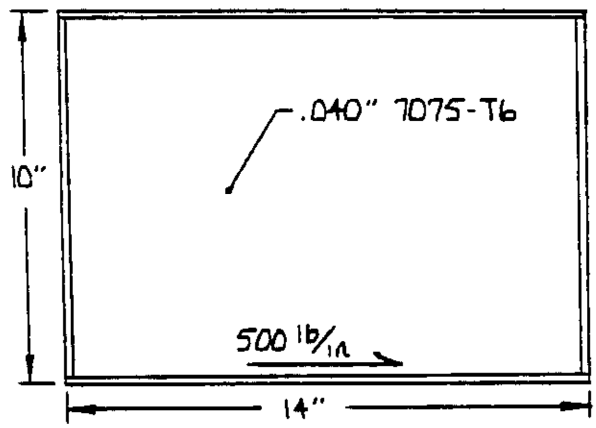
d) Compute the secondary bending moment due to diagonal tension, then use cap-beam analysis to check the cap for tension, compression-crippling allowables. Also, compute the primary cap loads and check tension and crippling allowables as above.

e) Compute the average, maximum, and allowable stiffener stress. Determine the crippling allowable for a single stiffener, and compute the margin of safety.

f) Analyze the web to cap, stiffener to cap, and web to stiffener connections as was done in the lesson.



4. For the shear web below, a one inch diameter hole will be cut at its center. Determine the doubler necessary to provide equivalent strength.



5. For the shear web below: a 1.50 in x 5.00 in slotted hole is to be cut horizontally in the web shown below. Design the doubler which will restore the web to original strength.

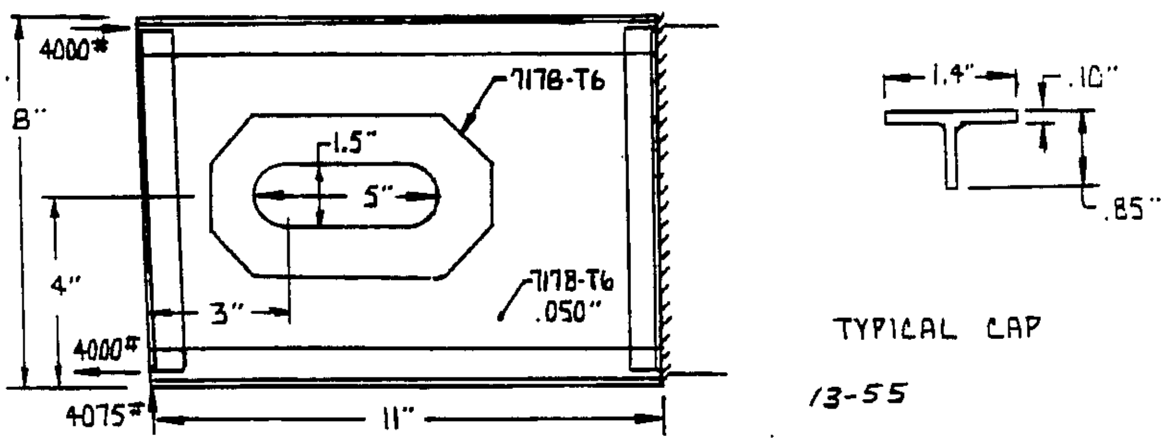
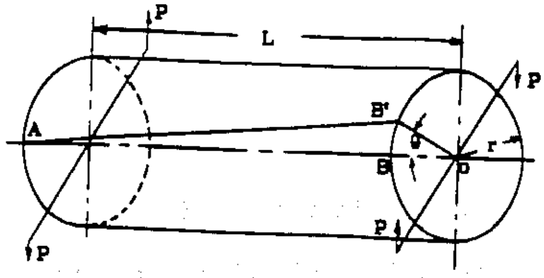


Figure 1



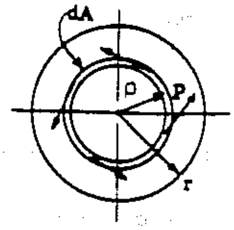


Figure 1 shows a straight circular bar subjected to two equal but opposite torsional couples. When the bar twists, each section is subjected to a shearing stress. Assuming the left end as stationary relative to the rest of the bar, a line AB on the surface will move to AB' under these shearing stresses, and this rotation at any section will be proportional to the distance from the fixed support.

The shearing stress at any point in a circular section is proportional to its distance from the center.

$$\tau = \frac{T\rho}{J}$$

where ρ = radius to point in question

T = externally applied torque

J = polar moment of inertia of the circular cross section; equal to twice the moment of inertia about its diameter. For a solid round section:

$$J = \frac{\pi d^4}{32}$$

Therefore the shearing stress is at its maximum at the outer surface

$$\tau_{max} = \frac{Tr_o}{J}$$

14.2.2 Hollow Tube

The equation given for shearing stress in the previous section is applicable to hollow round sections as well. The only change to the formula is in the polar moment of inertia, J.

So, for a hollow round section with inner radius b, and outer radius c:

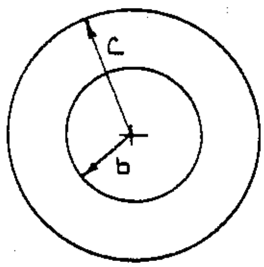


FIGURE 2

$$J = \int_A \rho^2 dA = \int_b^c \rho^2 (2\pi\rho) d\rho = 2\pi \int_b^c \rho^3 d\rho$$

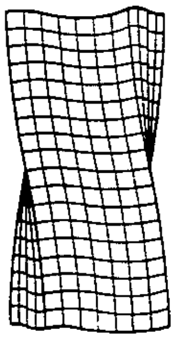
$$J = 2\pi \left[\frac{\rho^4}{4}\right]_b^c = \frac{\pi}{2}(c^4 - b^4)$$

or in terms of the internal and external diameter:

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

14.2.3 Solid Rectangular Section

The exaggerated deformation of a square shaft in torsion is shown in Figure 3. An element at a corner of the cross-section may be compared to the element shown in figure 3b. Since two perpendicular faces of the element can have no shear stresses, all the shear stresses must be zero at a corner of the member.



(a)

(b)

Figure 3

The shear stresses on the cross section have maximum values at the center of each side and have directions approximately as shown in Figure 3b.

One type of member which is frequently used in aircraft has a narrow rectangular cross section. While such sections are inefficient for torsion members, they must frequently resist some torsional stresses. For the cross section shown in figure 4 of length b and width t, the shear stresses must be parallel to the boundary.

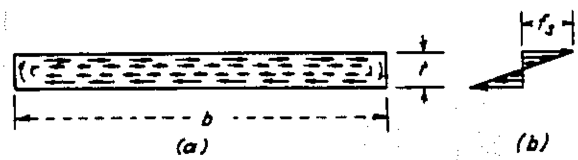


Figure 4

If the length b is large compared to the thickness t (as in sheet metal), the end effects are small, and the shear stresses may be assumed to be distributed as shown in figure 4b for the entire length b. It can be shown that the shear stress is:

$$\tau = \frac{3T}{bt^2}$$

For rectangular cross sections in which the dimension are of the same order, the maximum stress which occurs at the middle of the longest side is found by:

$$\tau = \frac{T}{\alpha b t^2}$$

where α is :

b/t	1.00	1.50	1.75	2.00	2.50	3.00	4	6	8	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.214	0.229	0.249	0.263	0.281	0.299	0.307	0.313	0.333

β in the table above is an angle of twist constant for the equation:

$$\Theta = \frac{TL}{\beta \tau b t^3 G}$$

where the angle of twist θ, is measured in radians.

The torsional properties of a rectangular plate are not appreciably affected if the plate is bent to some cross section, such as those shown in figure 5,

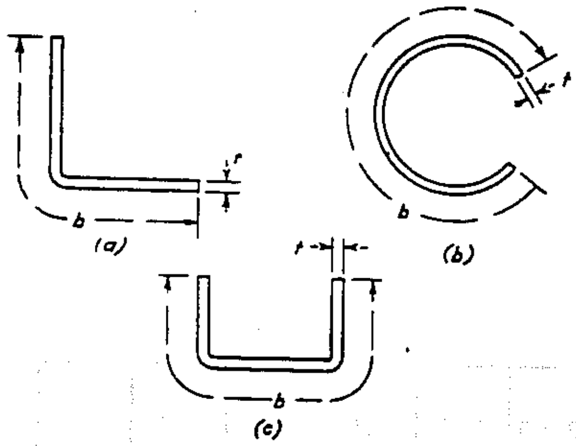


Figure 5

provided that the end cross sections of the member are free to warp.

14.2.4 Thin Web Torque Box

A box beam containing only two stringer areas is shown in Figure 6.

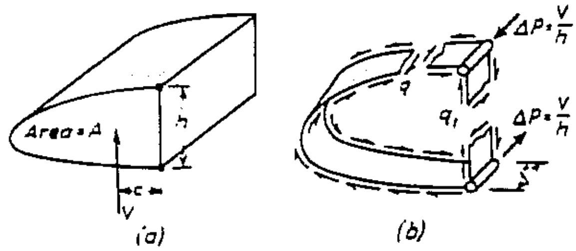


FIGURE 6:

This section is stable for torsional moments, so the vertical shearing force V may be applied at any point in the cross section. If two cross sections one inch apart are considered, the difference in axial load on the stringers between the two cross sections, ΔP, is found by dividing the difference in bending moment, (V)(1"), by the distance between stringers, h. These loads, ΔP, must be balanced by the shear flows shown in Fig. 6b. Considering the equilibrium of the spanwise forces on the upper stringer:

$$q_1 = \frac{V}{h} - q$$

The shear flow, q, may be obtained by equating the moment of the shear flow about any point, to the moment of the resulting shearing force about that point. Taking Moments about the lower stringer:

$$T = Vc = 2qA$$

where A is the total area enclosed by the box.

$$\text{or} \quad q = \frac{Vc}{2A}$$

Thus q_1 can be found:

$$q_1 = \frac{V}{h} - q$$

$$q_1 = \frac{V}{h} - \frac{Vc}{2A}$$

Another way of thinking about this is by using the method of superposition as shown in Figure 7.

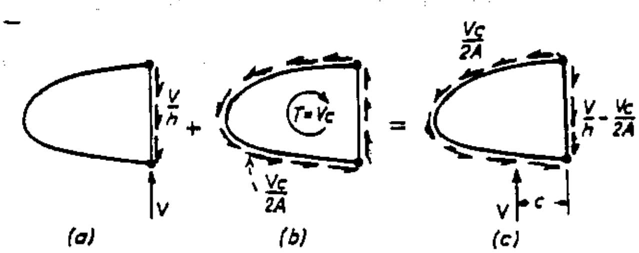
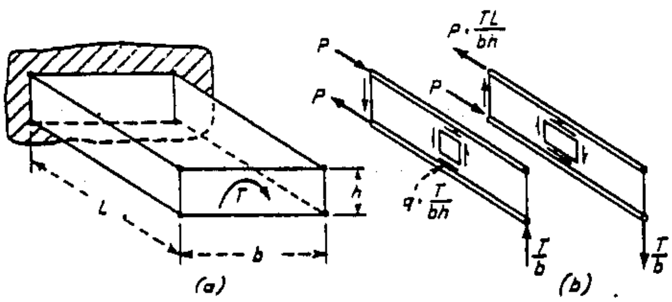


FIGURE 7

14.2.6 Torsion on Open Sections

A simplified sketch of a wing structure with four caps and no lower skin is shown in figure 8. In order for the structure to be stable, it is necessary that one end be built in, so that the torsion may be resisted by the two side webs acting independently as cantilever beams. That is, the spars resist the torsion by differential bending. The flange members resist axial loads, which have the values of $P = \frac{TL}{bh}$ at the support. The shear flows, q, in the vertical webs are double the values obtained in a closed torque box with the same dimensions. The horizontal web does not resist any shear flow in the case of pure torsion loading, but it is necessary for stability in resisting horizontal loads.

Figure 8.



14.3 Plastic Torsion

In some design situations, the ultimate strength of a bar or tube in torsion may be desired. Before a round bar made of ductile material fails in torsion, the shear stresses fall in the inelastic or plastic range. Thus the internal shear distribution is similar to that indicated in Figure 9.

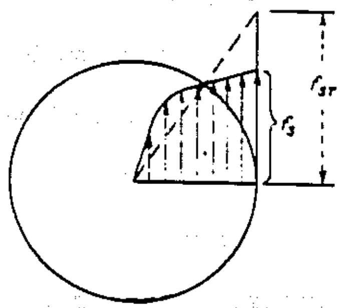


FIGURE 9:

The shear stress-strain curve is similar in shape to the tension stress strain curve, and is approximately 0.6 of the ordinates.

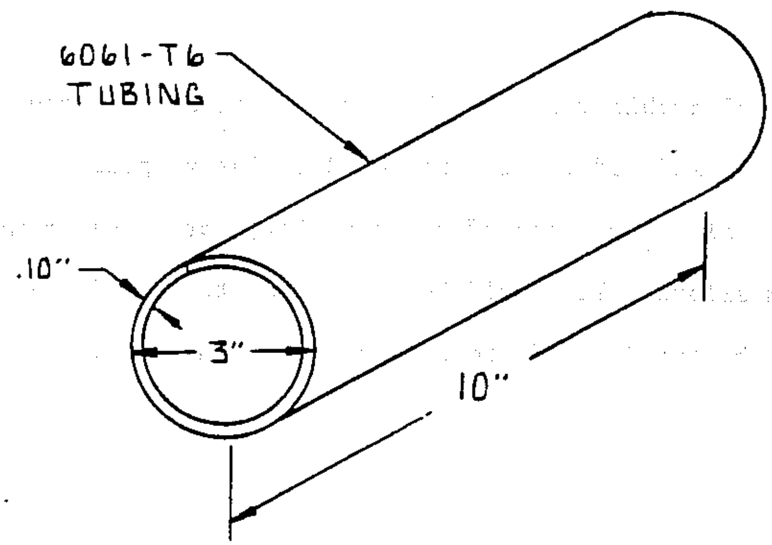
It is convenient to work with a fictitious triangular stress distribution, instead of the exact distribution. This fictitious distribution produces the same torque about its center as does the actual distribution. This stress is designated as the torsional modulus of rupture, Fst, which is determined by the equation below.

$$Fst = \frac{Tr}{J}$$

To obtain the allowable values of F_{st}, much testing has been performed with round tubes and rods. Figures 10-23 inclusive present curves for finding the torsional modules of rupture, F_{st}, for steel, aluminum, and magnesium alloys. It should be noted that the torsional strength is influenced by the D/t and the L/D ratios of the tube.

Homework Problems - Torsion

1. Find the allowable torsion for the hollow tube below assuming an elastic distribution.

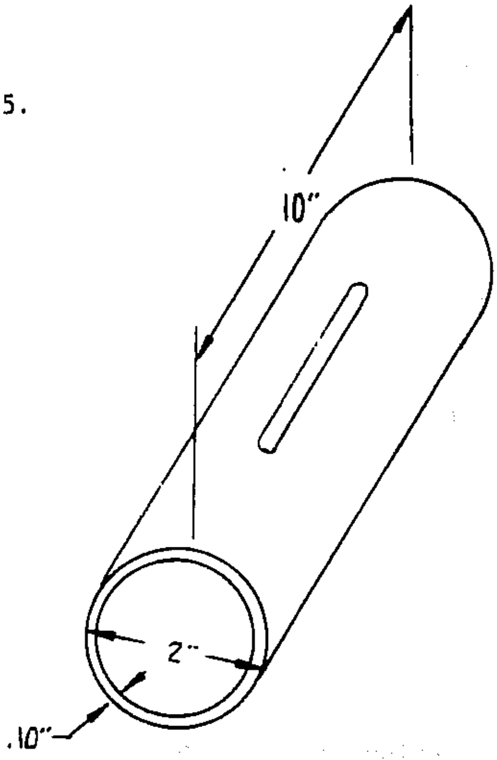


2. Find the maximum allowable torsion for the tube in problem 1.

3. Assume that the tube in problem 1 is solid, determine the allowable torsion for a completely elastic distribution.

4. Determine the maximum allowable torsion for the tube in problem 1, assuming that the tube is solid.

5.



For the tube shown at left, a .25" wide slit, 4 inches long is located halfway through the length.

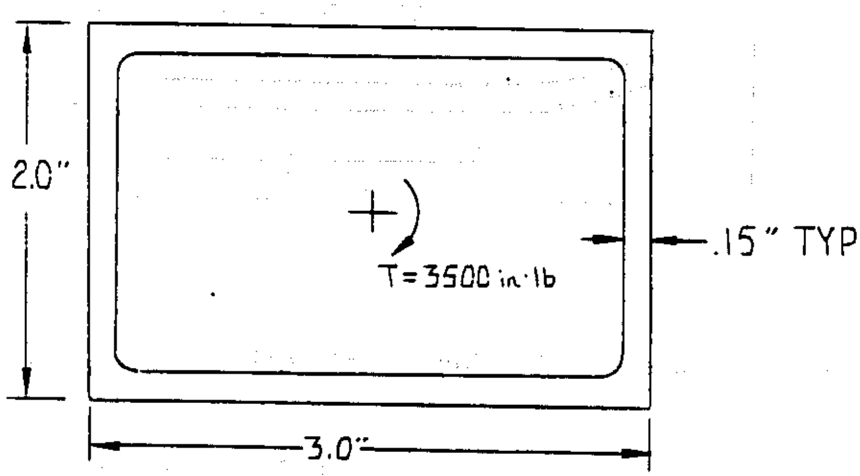
Determine the maximum allowable completely elastic torque:

a) At a section containing the slit

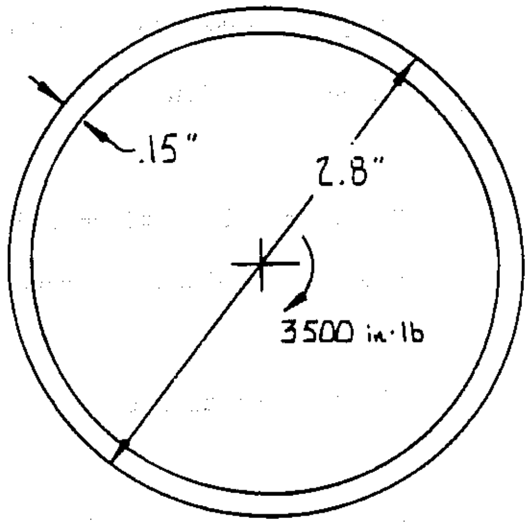
a) At a section without the slit

Which torque value would you use as the maximum design allowable?

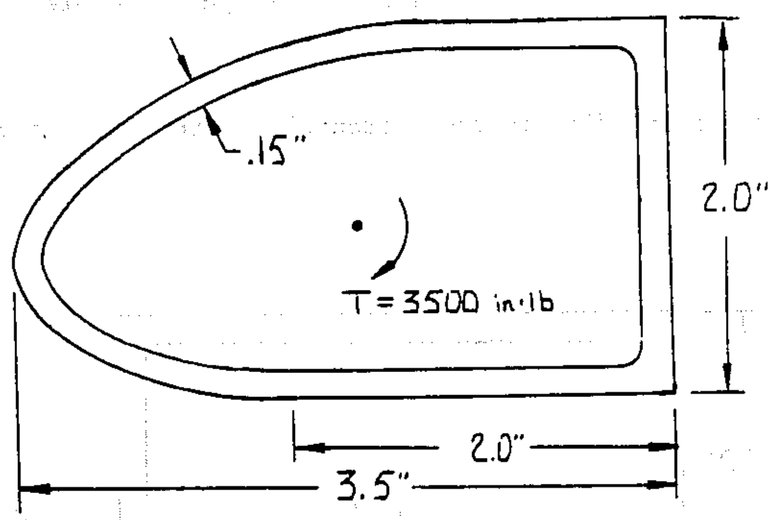
6. Find the shear flow in the rectangular torque box shown below:



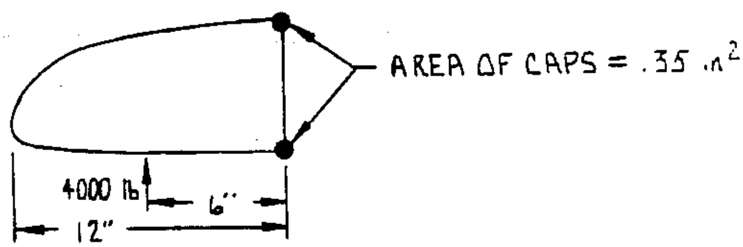
7. Find the shear flow in the circular torque box below:



8. Determine the shear flow for the cross-section shown below:



9. Determine the shear flow for the airfoil with two caps:



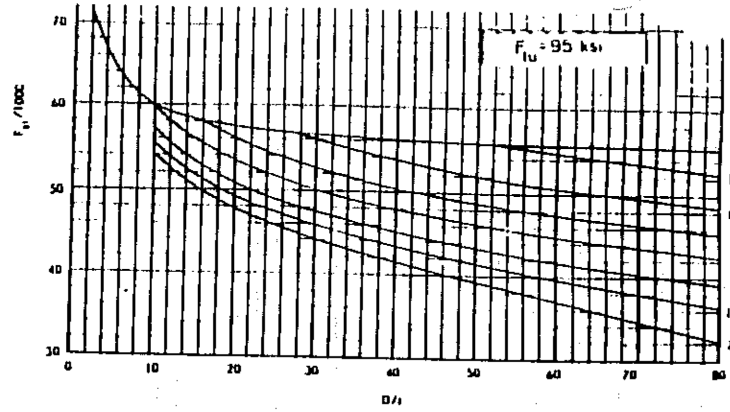


Fig. 10 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 95 ksi.

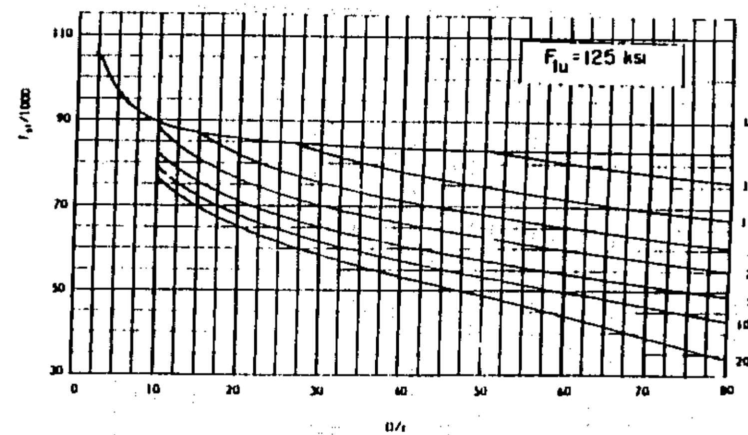


Fig. 11 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 125 ksi.

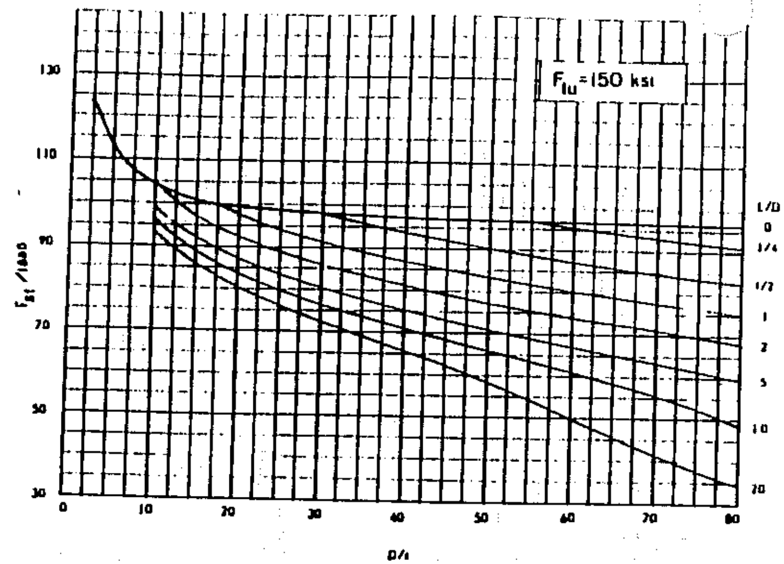


Fig. 12 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 150 ksi.

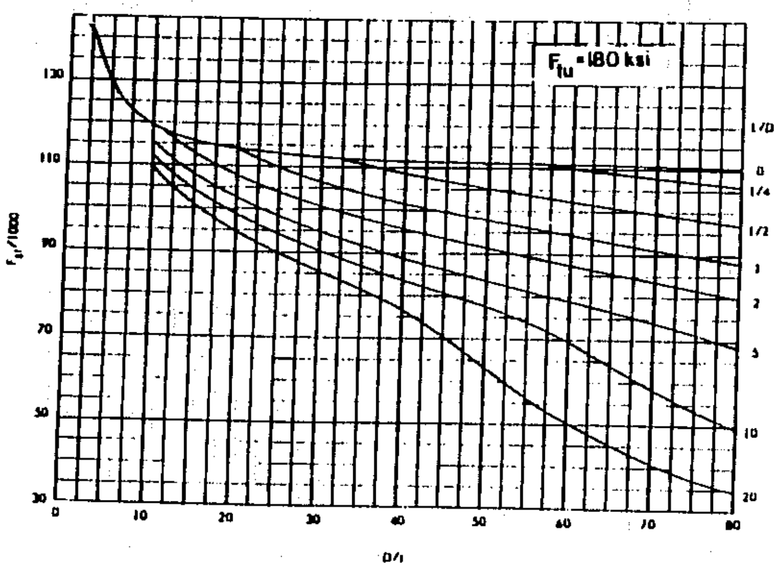


Fig. 13 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 180 ksi.

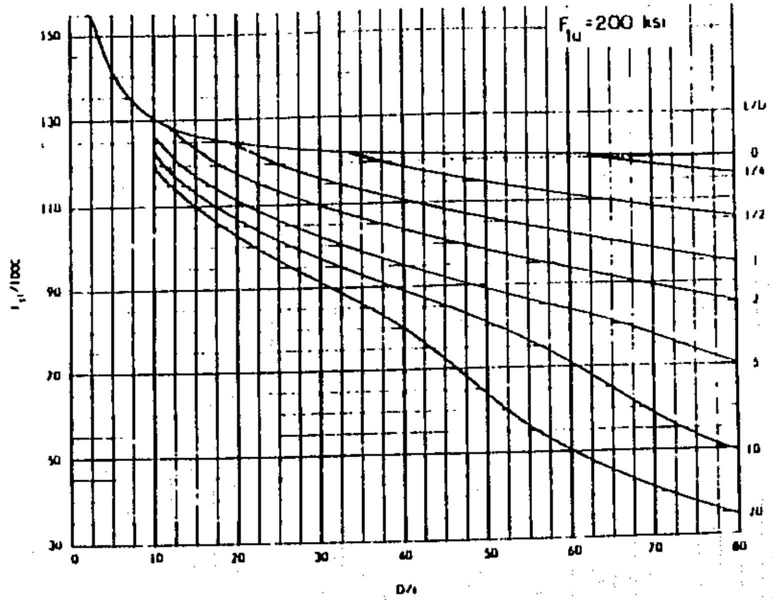


Fig. 14 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 200 ksi.

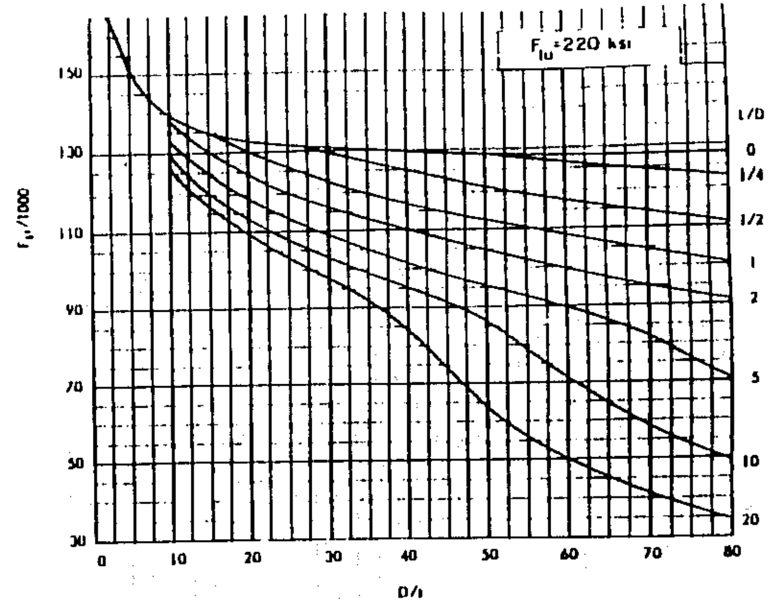


Fig. 15 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 220 ksi.

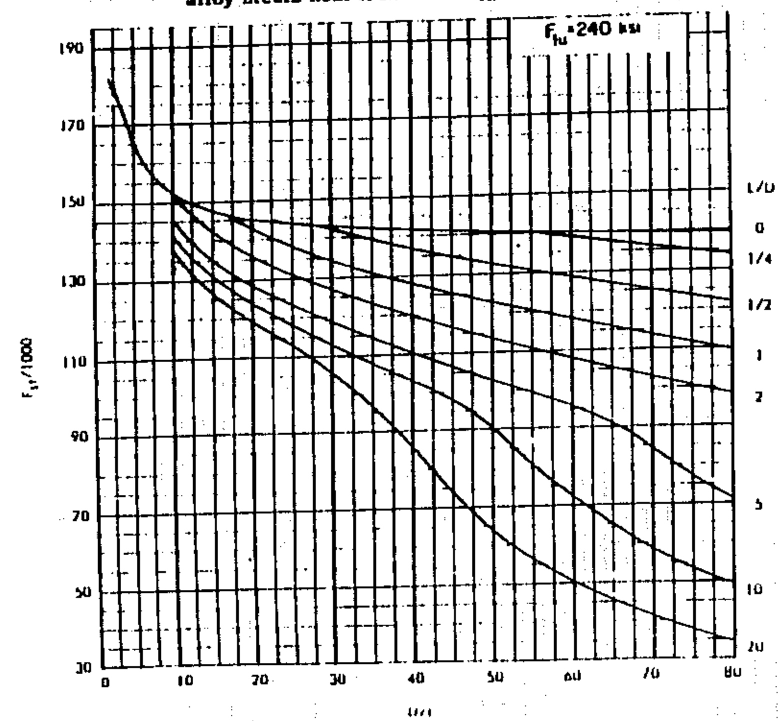


Fig. 16 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 240 ksi

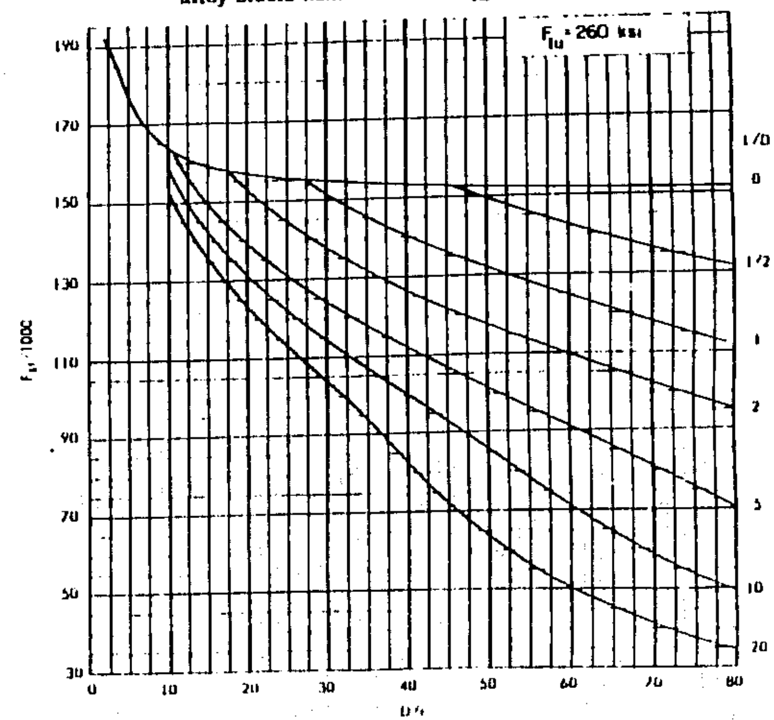


Fig. 17 Torsional modulus of rupture - alloy steels heat treated to F_{tu} = 260 ksi

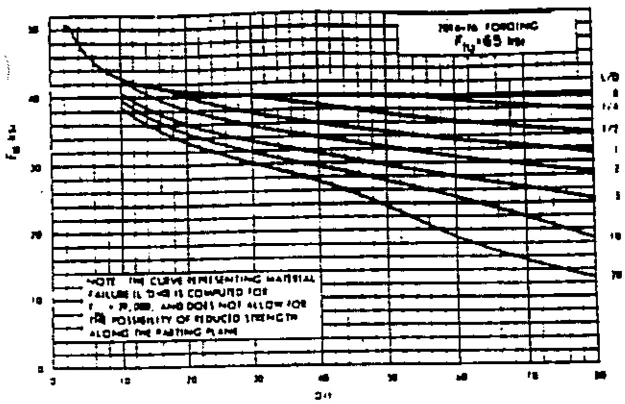


Fig. 18 Torsional modulus of rupture - 2014-T6 aluminum alloy forging.

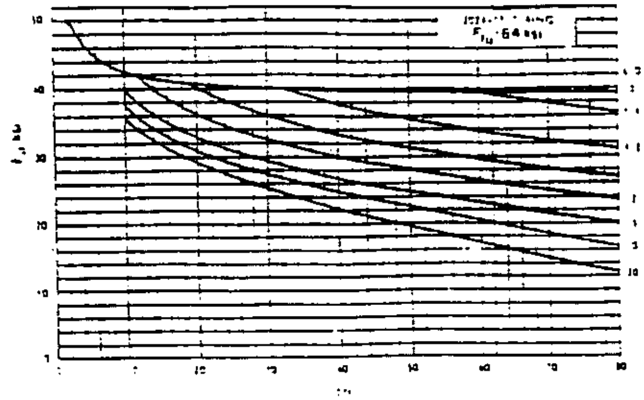


Fig. 19 Torsional modulus of rupture - 2024-T3 aluminum alloy tubing.

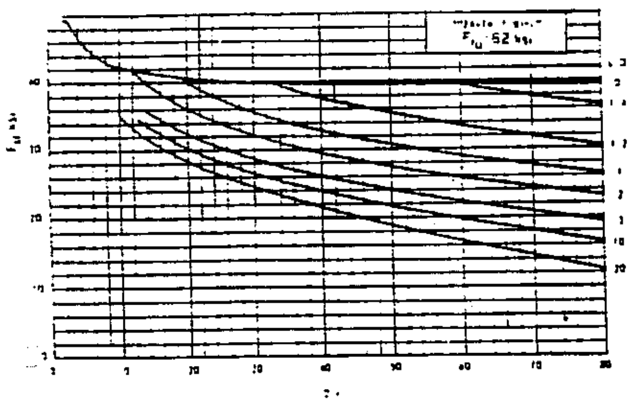


Fig. 20 Torsional modulus of rupture - 2024-T4 aluminum alloy tubing.

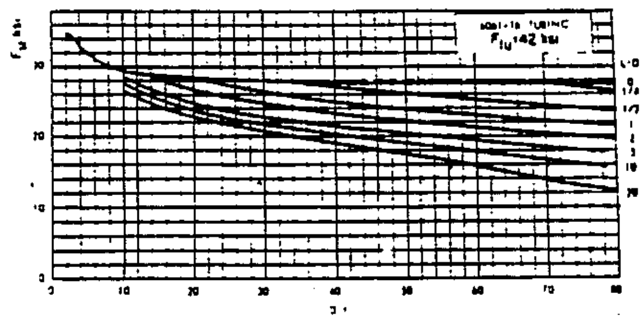


Fig. 21 Torsional modulus of rupture - 6061-T6 aluminum alloy tubing.

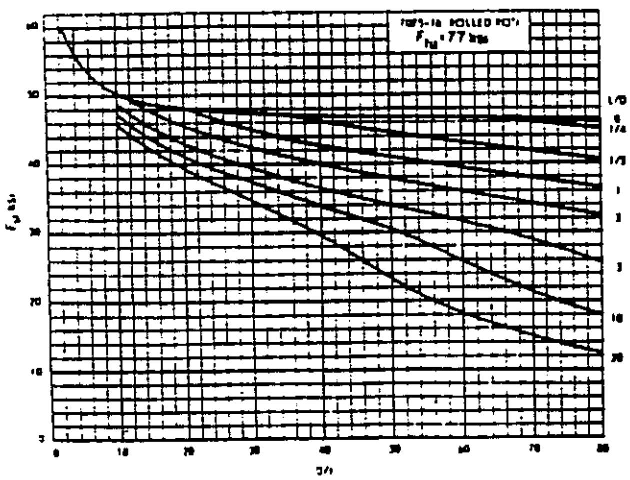


Fig. 22 Torsional modulus of rupture - 7075-T6 aluminum alloy rolled rod.

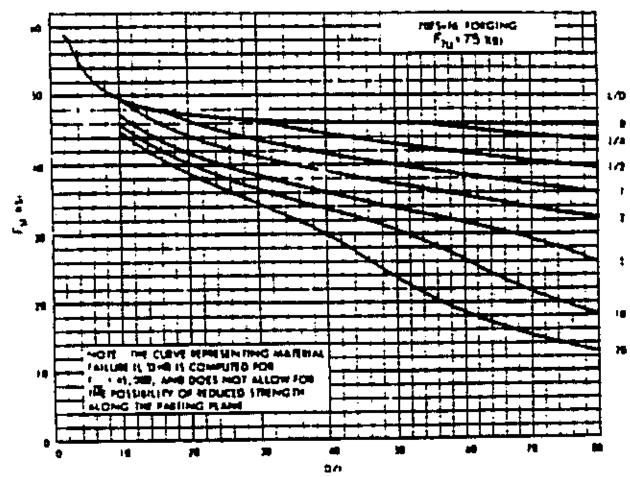
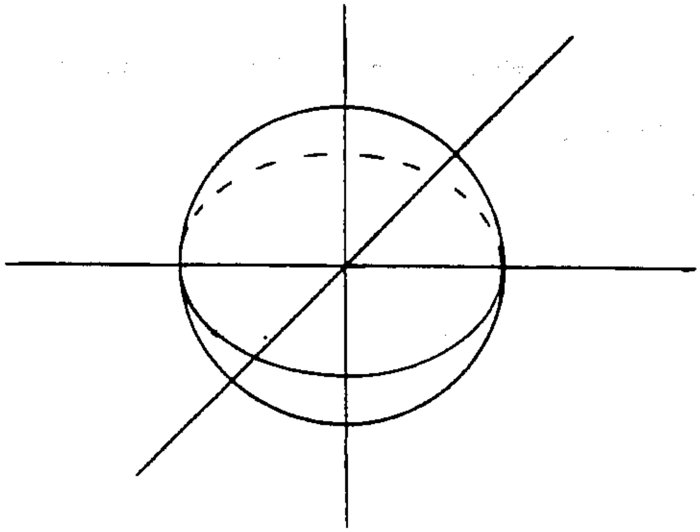


Fig. 23 Torsional modulus of rupture - 7075-T6 aluminum alloy forging.

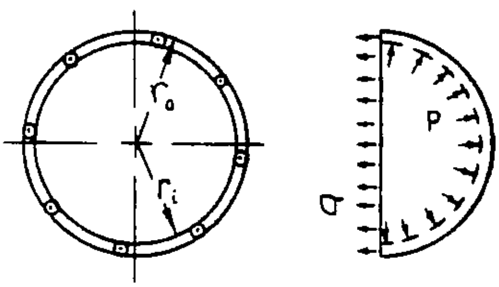
LESSON 15

STRUCTURE LOADED BY PRESSURE

15.1 Spherical Pressure Vessels: The analysis of pressure vessles will begin by studying the thin-walled spherical pressure vessel. An example at this type of structure is shown below.



Denote the internal radius by r_i and the internal pressure in excess of the external pressure (gage pressure) by p. If a cut is taken through the center of the sphere, an expression for the stress can be found by implementing a static balance.



$$\sigma(\pi r_o^2 - \pi r_i^2) = p\pi r_i^2$$

$$\sigma\pi(r_o^2 - r_i^2) = p\pi r_i^2$$

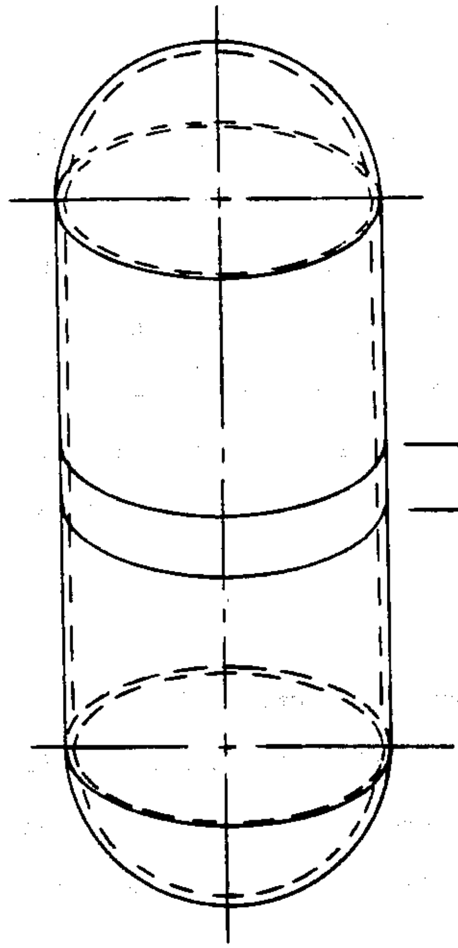
$$\sigma\pi(r_o - r_i)(r_o + r_i) = p\pi r_i^2$$

Note that $r_o - r_i = t$, the thickness of the vessel and since this development is restricted to thin walled pressure vessels, $r_o = r_i = r$. So:

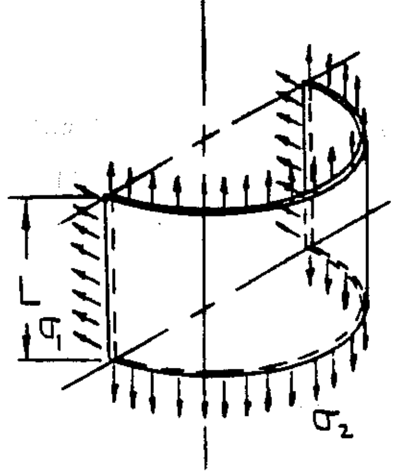
$$\sigma = \frac{Pr^2}{2rt} = \frac{Pr}{2t}$$

Any section passed through the center of the sphere would produce the same balance. Therefore, equal principal stresses act on all of the elements of the sphere.

15.2 <u>Cylindrical Pressure Vessels</u>: The analysis of thin walled cylindrical pressure vessels is similar to that of thin-walled spherical pressure vessels.

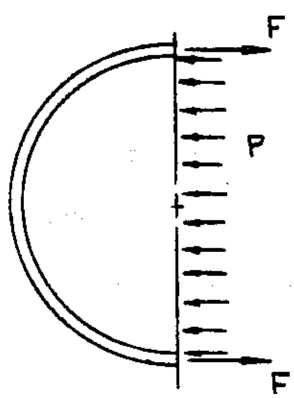


A segment is isolated from this vessel by passing two planes perpendicular to the axis of the cylinder and one additional longitudinal plane through the same axis.



The conditions of symmetry exclude the existence of any shearing stresses in the planes of the sections, as shearing stresses would cause an incompatible distortion of the tube. Therefore, the stresses that exist on the sections of the cylinder can only be the normal stresses, σ_1 and σ_2 shown in the figure above. These stresses are the principal stresses. These stresses, multipled by the respective areas on which they act, maintain the element of the cylinder in equilibrium against the internal pressure.

Again, let the internal pressure be denoted by p, and the internal radius by r. In the figure below, the two forces (F), resist the force developed by the internal pressure which acts perpendicular to the projected area A_1 of the cylindrical segment onto the diametral plane.



This area is $2r_iL$. Hence $P = Ap = 2r_iLp$. This force is resisted by the forces developed in the material in the longitudinal cuts. Since the outside radius of the cylinder is r_o, the area of both longitudinal cuts is $2A = 2L(r_o - r_i)$. If the average normal stress acting or the longitudinal cut is σ_1, the force resisted by the walls of the cylinder is $2L(r_o - r_i)\sigma_1$. So by substituting this expression into the static equilibrium equation for P:

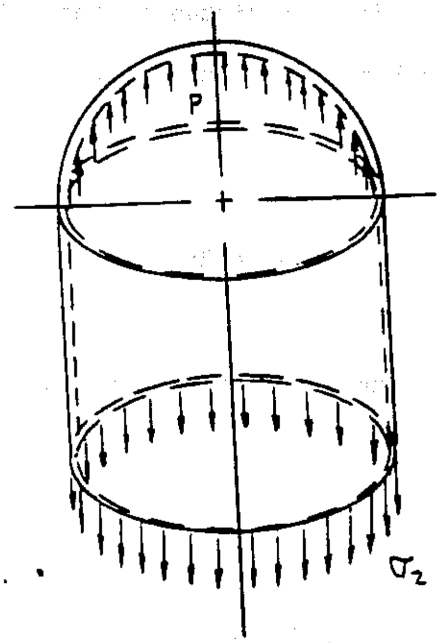
$$2L(r_o - r_i)\sigma_1 = 2r_iLp$$

Since $r_o - r_i = t$, the thickness of the cylinder, the equation above simplifies to:

$$\sigma_1 = \frac{pr_i}{t}$$

The normal stress given by the equation above is referred to as the circumferential or the hoop stress. The expression is valid only for thin walled cylinders, as it gives the average stress in the hoop. An acceptable rule of thumb is that the equations developed for thin-walled cylinders are sufficiently accurate for wall thicknesses up to one-tenth the internal radius. Since this equation is used primarily for thin-walled vessels where $r_i \approx r_o$, the subscript for the radius is often omitted.

The other normal stress, σ_2, developed in a cylindrical pressure vessel acts longitudinally. By passing a section through the vessel perpendicular to its axis, a free body as shown below is obtained.



The force developed by the internal pressure is $p\pi r_i^2$, and the force developed in the walls by the longitudinal stress, σ_2, is $\sigma_2\,(\pi r_o^2 - \pi r_i^2)$. Equating the two forces and solving:

$$p\pi r_i^2 = \sigma_2(\pi r_o^2 - \pi r_i^2)$$

$$p\pi r_i^2 = \sigma_2\pi(r_o - r_i)(r_o + r_i)$$

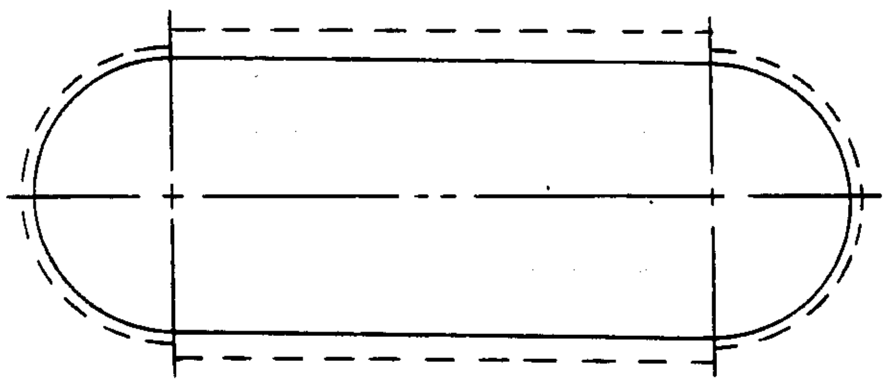
Since $r_o - r_i = t$, and since for thin-walled cylinders $r_o \approx r_i \approx r$, the above expression reduces to:

$$\sigma_2 = \frac{Pr}{2t}$$

Note that for cylindrical pressure vessels:

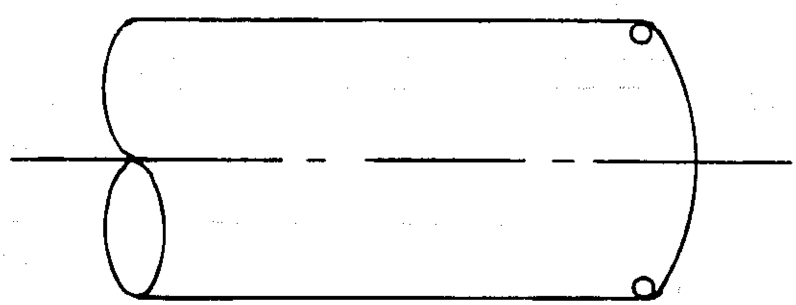
$$\sigma_2 = \frac{1}{2} \sigma_1$$

For a cylindrical pressure vessel with spherical ends, a discontinuity occurs at the juncture of the cylindrical portion with the ends. Under the action of the internal pressure, the cylinder tends to expand as shown by the dashed lines in the figure below.

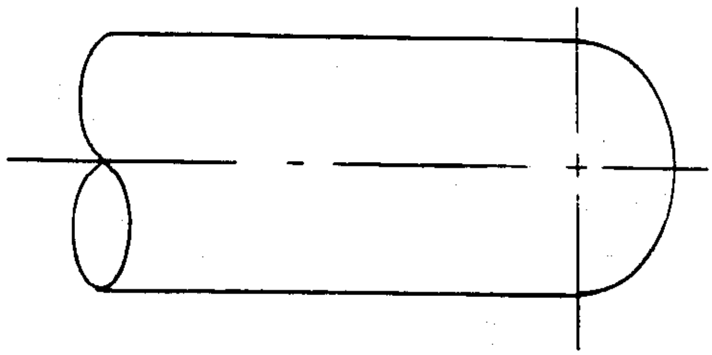


This incompatibility of deformations causes local bending and shearing stresses in the neighborhood of the joint, since there must be physical continuity between the ends and the cylindrical wall. For this reason, properly curved ends must be used for pressure vessels.

Hemispherical ends are highly desirable from a stress standpoint, yet are uneconomical as regards to space utilization. At the other extreme, flat bulkheads, while providing more useful volume, cannot resist the pressure loading by membrane stresses, and hence are structurally ineffecient. A compromise configuration is shown below, in which the bulkhead is a spherical surface of low curvature, ("dished-head") which supports the pressure loading by membrane stresses. A reinforcing ring, placed at the seam, resists the radial component of these stresses.

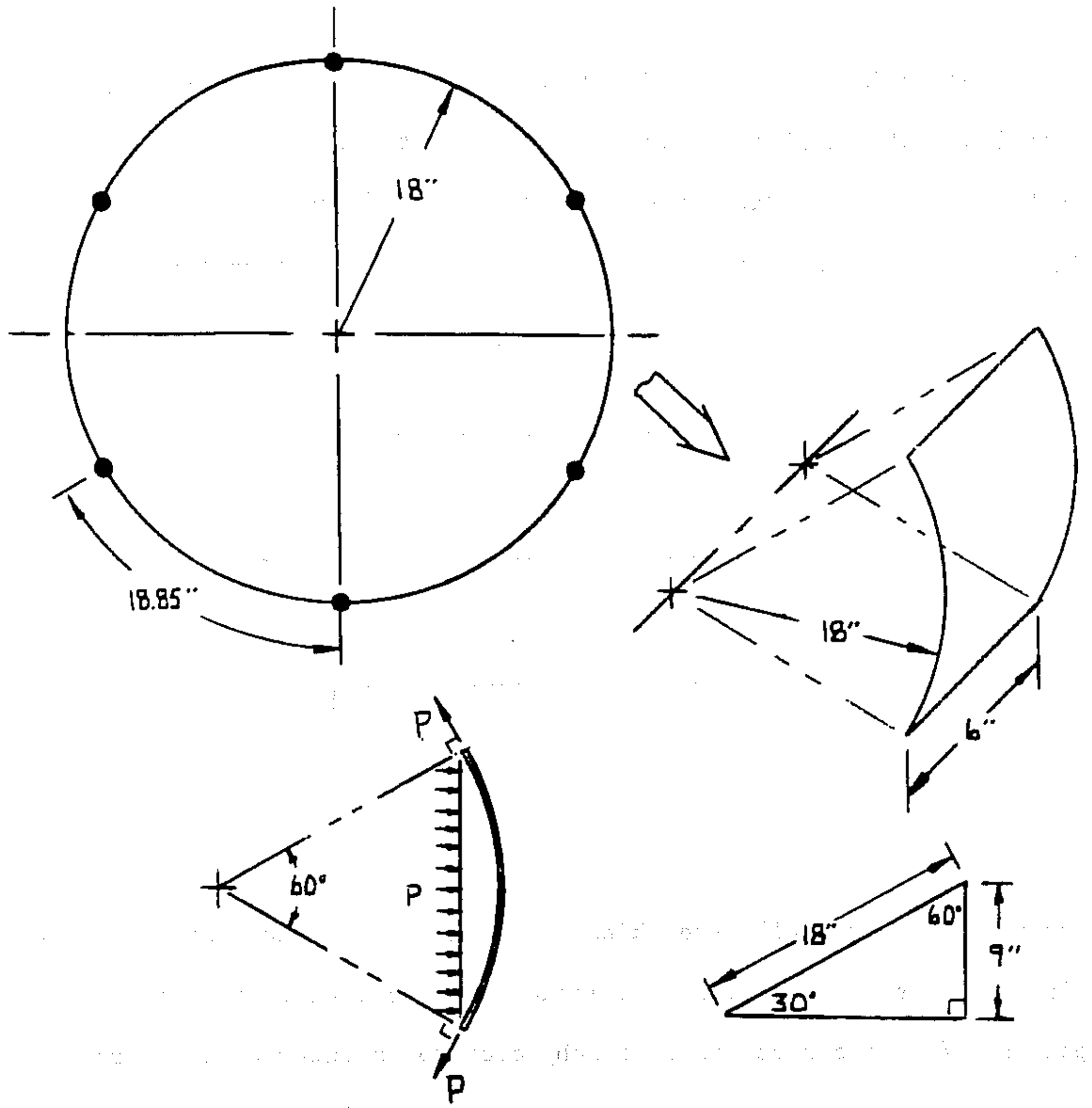


Another form of bulkhead used to close a cylindrical pressure vessel is elliptical in section as shown in the figure below. Such a bulkhead shape provides tangential meridional forces at the seam. Thus, it does not require a reinforcing ring and yet is reasonably space efficient:



15.3 Curved Webs: Typically the skin composing a cylindrical section of an aircraft is not continuous, but instead composed of several curved panels. If the curved panel is subjected to an internal pressure, it reacts the load by hoop tension as shown in the following example.

Example: Given: An .040" x 6.00" x 18.85" flat panel will be formed on a radius of 18 in, and it will be attached to a cylindrical section. This cylinder will react a pressure of 5 psig. Determine the membrane stresses in this panel.



For this panel to be statically balanced, the horizontal components of the forces P must be equal, and opposite to the horizontal component due to the internal pressure. The force due to the horizontal component of the internal pressure can be found by projecting the cylindrical area of the panel onto its chord plane.

$$F_p = pA = (5 \text{ psi})(18 \text{ in})(6 \text{ in}) = 540 \text{ lb}$$

Therefore the horizontal components of the forces in the walls must equal this pressure force:

$$2 P \sin 30° = 540 \text{ lb}$$

$$P = 540 \text{ lb}$$

This force is reacted within the material by developing a hoop tensile stress.

$$\therefore \quad \sigma = \frac{P}{A} = \frac{540\text{lb}}{(.040\text{in})(6\text{in})} = 2250 \text{ psi}$$

Note that this stress is equal to the hoop stress:

$$f_h = \frac{Pr}{t} = \frac{(5 \text{ psig})(18 \text{ in})}{(.040 \text{ in})} = 2250 \text{ psi}$$

This demonstrates that the stress can always be found by the hoop stress equation. Therefore a free body solution is unnecessary.

15.4 Flat Panels:

The stress in a flat panel loaded by a uniform pressure can be determined by using load-deflection equations. The derivation of these equations, however is beyond the scope of this lesson. The reader may refer to Timoshenko's text, "Plates and Shells" for further information on this subject.

Basically, plates subjected to normal loads are classified as

a) Thick plates

This type supports the load by bending.
The stress due to bending is:

$$\sigma_b = \frac{3pb^2}{4t^2} = \frac{750{,}000p}{\left(\frac{1000t}{b}\right)^2}$$

The maximum deflection, w, (at the center) is:

$$\frac{w}{b} = \frac{5(1-\mu^2)pb^3}{32Et^3} = 14.2\,\frac{\frac{10^7 p}{E}}{\left(\frac{1000t}{b}\right)^3}$$

where μ = Poisson's ratio
E = Young's Modulus
t = thickness
b = width
p = pressure

These results are approximately correct for plates simply supported on all four sides if length a is greater than three times width b. For $a < 3b$, results are conservative. For built-in edges, take 2/3 of σ_b (maximum at edge of plate and take $\frac{1}{5}$ of w. Graphs of σ_b vs. p and $\frac{W_{max}}{b}$ vs $\left(\frac{10^7}{E}\right)p$ for various values of $\frac{1000t}{b}$ are presented on pages 17.10 and 17.11 of MAC 339.

b) Membrane plates

This type supports the load by tension. The tension stress in a long membrane of width b and thickness t, subjected to uniform pressure p, and held along the long sides is:

$$\sigma_T = \left[\frac{p^2\ Eb^2}{24\ (1-u^2)\ t^2}\right]^{1/3} = 7700 \left(\frac{\frac{\sqrt{10\ E}}{10^4}\ p}{\frac{1000t}{b}}\right)^{2/3}$$

the maximum deflection (at the center) is:

$$\frac{w}{b} = \frac{pb}{8\sigma_t\ t} = \frac{1}{8}\left(\frac{24(1-u^2)\ pb}{Et}\right)^{1/3} = 0.162 \left(\frac{\left(\frac{10^7}{E}\right)p}{\frac{1000t}{b}}\right)^{1/3}$$

Results are approximately correct for plates held on all four sides if the length a is greater than five times b. ⚠1

Graphs of σ_t vs. $\left(\frac{\sqrt{10E}}{10^4}\right)p$ and $\frac{wmax}{b}$ vs. $\left(\frac{10^7}{E}\right)p$

for various valves of the parameter, $\frac{1000t}{b}$, are

presented in MAC 339 on pages 17.20 and 17.21 respectively.

c) Thin plates

This class represents a middle ground between plates classified as thick, and plates classified as membranes. As such, the load in the plate is taken out by a combination of bending and tensile stresses:

$$\sigma = \sigma_T + \tau_b$$

⚠1 Note: Edges must carry membrane loads.

The tensile stress due to membrane loading must be carried by the edge fasteners.

Graphs of $\frac{10^4}{E}\sigma$ vs. $\frac{10^7}{E}p$ for bending stress, tension stress, and total stress, and graphs of $\frac{W_{max}}{b}$ vs. $\frac{10^7}{E}p$ are presented on pages 17.30 - 17.43 of MAC 339 for plates with simply supported and built-in edges. Results are approximately correct for plates supported on all four sides, if the length a is greater than three times b. For a < 3b, results are conservative. Most thin plates in aircraft however, lie between the extremes of being simply supported and having built-in edges. Therefore it is recommended that an average of the two methods be taken for these cases.

The thickness which separates a plate into one of the 3 classes has been intentionally side-stepped up to this point. This omission was necessary because the categories depend on the deflection-to-thickness ratio. Thus the deflections for each class had to be defined first. Note that the deflection-to-thickness ratio and thus its classification and applicable formulas, will depend on the loading conditions. The approximate criteria for classifying a plate are:

a) Thick plates: $-w < \frac{1}{2}t$

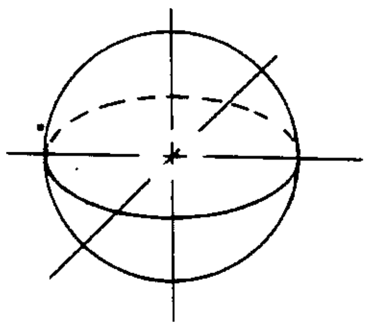
b) Membrane plates: $-w > 5t$

c) Thin plates: $-\frac{1}{2}t < w < 5t$

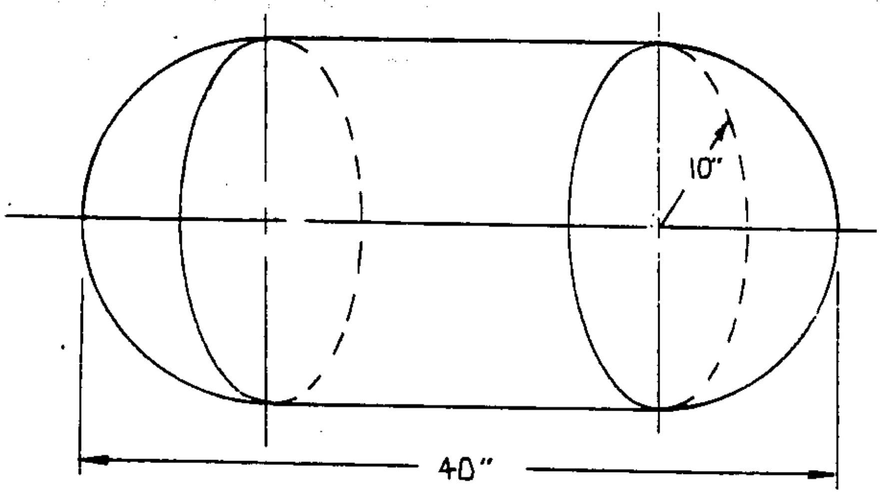
Problems

Lesson 15 - Pressure

1. For the spherical pressure vessel shown, determine the maximum stress if the thickness is .050 in, the radius 10.0 in, and the internal operating pressure is 30 psi. Material of the vessel is 2024-T3. Also determine the Margin of Safety based on a burst pressure factor of 4.



2. Given a cylindrical pressure vessel 40" long, 20" in diameter, and .050" thick with spherical ends.

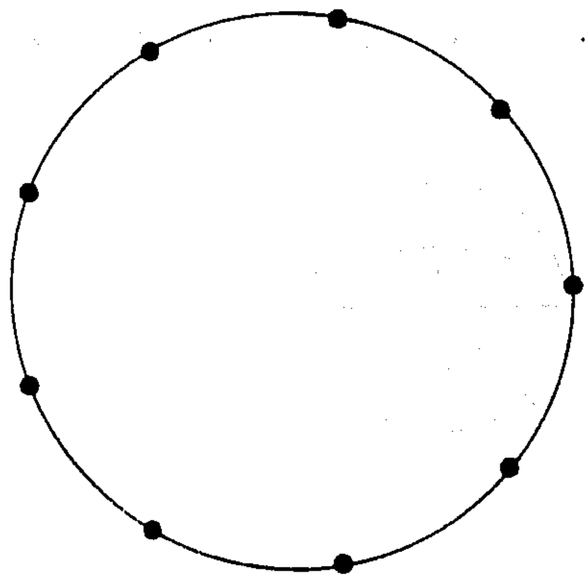


If the operating pressure is 30 psi, determine the circumferential and axial stresses as well as the margin of safety based on burst pressure.

Material: 2024 - T3 Aluminum, F_{TU} = 63 KSI

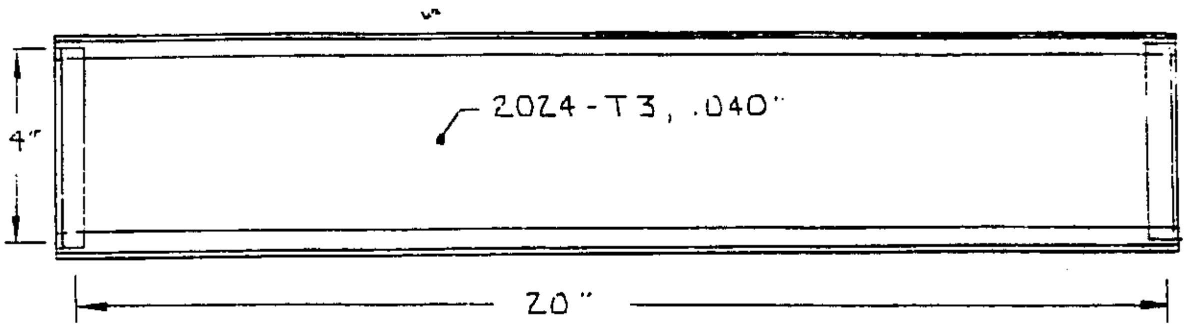
Burst Pressure Factor: 4

3. The cylindrical section shown is made up of .032 in. 7075-T6 panels formed on a 20 in. radius. The length of each panel is 16 inches.



If the panels are loaded with an internal operating pressure of 20 psi, determine the stress at the joints which will maintain equilibrium.

4. Given the flat panel below:



Determine the total stress and maximum deflection if the panel is uniformly loaded with

a) 1 psi

b) 50 psi

c) 10 psi

Assume the fixity of the edging members to be approximately the same as for built in plates.

LESSON 14

TORSION

14.1 GENERAL

Problems involving torsion are common in aircraft structures. The metal covered airplane wing and fuselage are basically thin-walled torque boxes which can be subjected to large torsional moments under many flight and landing conditions. Also various mechanical control systems in an airplane are composed of parts of various cross-sectional shapes which are subjected to torsional forces under operating conditions. Therefore, it is evident that a knowledge of torsional stresses and distortions of members is necessary in aircraft structural design.

14.2.1 Solid Circular Shaft

The following conditions are assumed in the derivation of the equations for torsional stresses and distortions

(1) The member is a circular, solid or hollow round cylinder

(2) Sections remain circular after application of torque

(3) Diameters remain straight after twisting of section

(4) Material is homogenous, isotropic, and elastic

(5) The applied loads lie in a plane or planes perpendendicular to the axis of the shaft or cylinder.