SOLUTIONS TO PROBLEMS

LESSON 1 STRENGTH TERMINOLOGY

1.1 Normal stress and shear stress, or, tension, compression and shear stresses.

1.2 The tension stress (ref. paragraph 1.1.1) is:

$\underline{f_t} = P/A = 186{,}000/(1.50 \times 2.00) = \underline{62{,}000 \text{ psi}}$

1.3 The shear stress (same ref. as above) is:

$\underline{f_s} = V/A = 132{,}000/(3.00) = \underline{44{,}000 \text{ psi}}$

1.4 The limit bearing stress (same ref. as above) is:

$$\underline{f_{br_y}} = \frac{P}{Dt} = \frac{2000}{.250 \times .100} = \underline{80{,}000 \text{ psi}}$$

1.5 The ultimate factor is 1.5. (ref. paragraph 1.1)

$\underline{f_{br_u}} = 1.5 \times f_{br_y} = 1.5 \times 80{,}000 = \underline{120{,}000 \text{ psi}}$

1.6 From paragraph 1.3.1: $E = f/e$ and $E = 10.5 \times 10^6$ psi approximately. Therefore:

$e = f/E$

$$\underline{e} = \frac{35{,}000}{10{,}500{,}000} = \underline{.00333 \text{ in./in.}}$$

1.7 From paragraph 1.2.1, $e = \delta/L$. Therefore:

$\delta = e \times L = .00333 \times 84 = .28$ in.

Then the length with the load applied would be:

$\underline{L} = L_1 - \delta = 84 - .28 = \underline{83.72 \text{ in.}}$

1.8 $E = 10.5 \times 10^6$ psi and $\mu = 0.33$. From paragraph 1.3.3:

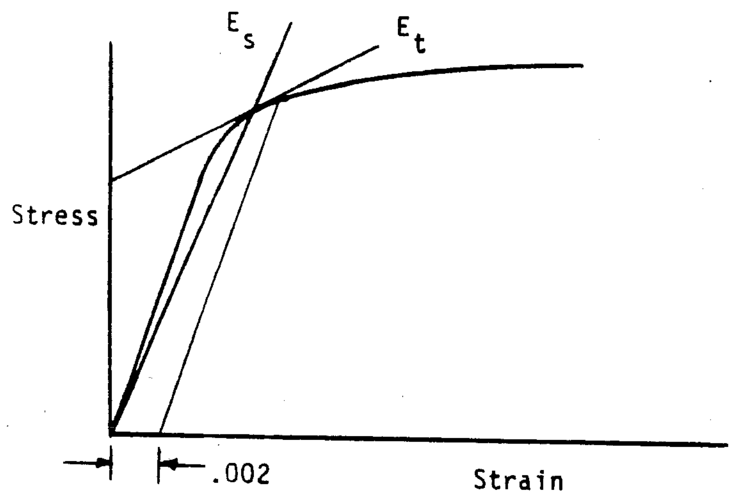
$$\underline{G} = \frac{E}{2(1+\mu)} = \frac{10{,}500{,}000}{2(1.33)} = \underline{3.95 \times 10^6 \text{ psi}}$$

1.9 a) Referring to the definitions of E_t and E_s in paragraph 1.3.4 and the typical stress-strain diagram in paragraph 1.1.2 it can be seen that above the proportional limit the slope of the curve (which is the same as the slope of a tangent to the curve) is smaller than below the proportional limit where the slope is the modulus of elasticity, E. Therefore:

<u>E_t is smaller than E.</u>

b) By drawing a line, imaginary or real, from the point on the curve to the origin it can be seen that E_s is greater than E and:

<u>E_s is greater than E_t.</u>



SOLUTIONS TO PROBLEMS

LESSON 2 REVIEW OF STATICS

2.1 Either reaction can be found by taking moments about the point of action of the other.

$$\Sigma M_B = 0 = 750 \times 20.0 + (50 \times 20.0)\ 20.0 + (250 \times 10.0)\ 5.0 - 25.0\ R_1$$

$$\underline{R_1 = 1900\#}$$

The other reaction can now be found either by taking moments about point A or by summing vertical forces:

$$\Sigma M_A = 0 = 750 \times 5.0 + 50 \times 20.0 \times 5.0 + 250 \times 10.0 \times 20.0 - 25.0\ R_2$$

$\underline{R_2 = 2350\#}$ or:

$$\Sigma F_v = 0 = 750 + 50 \times 20.0 + 250 \times 10.0 - 1900 - R_2$$

$$\underline{R_2 = 2350\#}$$

After finding R_2 by one equation it can be checked by the other equation.

2.2 At joint A:

$$\Sigma F_v = 0 = P_{AB} \sin 45° + P_{AC} \sin 60° - 1000$$

$$.707\ P_{AB} + .866\ P_{AC} = 1000$$

$$\Sigma F_h = 0 = P_{AB} \cos 45° - P_{AC} \cos 60°$$

$$.707\ P_{AB} - .5\ P_{AC} = 0$$

Solving these two equations simultaneously:

$$.707\ P_{AB} = .5\ P_{AC}$$

$$.5\ P_{AC} + .866\ P_{AC} = 1000$$

$$\underline{P_{AC}} = \frac{1000}{(.5 + .866)} = \underline{732\#}$$

$$.707\ P_{AB} - .5\ (732) = 0$$

$$\underline{P_{AB} = 518\#}$$

At joint B:

$\Sigma F_v = 0 = P_{BD} \sin 30° + P_{BE} \sin 30° - P_{AB} \sin 45°$

$.5\ P_{BD} + .5\ P_{BE} = 518 \sin 45° = 366$

$\Sigma F_h = 0 = P_{BD} \cos 30° - P_{BE} \cos 30° - P_{AB} \cos 45°$

$.866\ P_{BD} - .866\ P_{BE} = 518 \cos 45° = 366$

Solving simultaneously:

$.866\ P_{BD} + .866\ P_{BE} = \left(\frac{.866}{.50}\right) 366 = 634$ (Ratioing ΣF_v equation)

$.866\ P_{BD} - .866\ P_{BE} = 366$ (F_h equation)

$.866\ P_{BD} + .866\ P_{BD} = 634 + 366 = 1000$ (Adding the equations)

$\underline{P_{BD} = 577\#}$

$.5 \times 577 + .5\ P_{BE} = 366$

$\underline{P_{BE} = 155\#}$

At joint C:

$\Sigma F_v = 0 = P_{CF} \sin 45° + P_{CG} \sin 45° - P_{AC} \sin 60°$

$.707\ P_{CF} + .707\ P_{CG} = 732 \sin 60° = 634$

$\Sigma F_h = 0 = P_{CF} \cos 45° - P_{CG} \cos 45° + P_{AC} \cos 60°$

$.707\ P_{CF} - .707\ P_{CG} = -732 \cos 60° = -366$

Solving simultaneously:

$2 \times .707\ P_{CF} = 634 - 366 = 268$

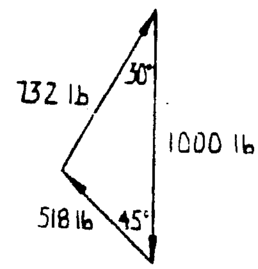
$\underline{P_{CF} = 190\#}$

$.707 \times 190 + .707\ P_{CG} = 634$

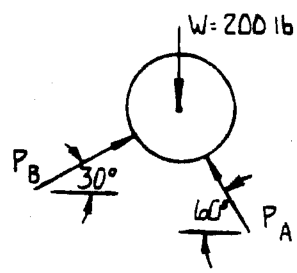
$\underline{P_{CG} = 707\#}$

Note: This cable system has four reactions, which is one more than the three that can normally be found by the three equations of statics and would normally be statically indeterminate. The reason this problem could be solved is that the internal load path was statically determinate.

This problem could also have been solved graphically. This is done by drawing a force diagram at each joint with the lengths of the force vectors proportional to the magnitude of the force and at the angle at which the force acts. The sketch shows the force diagram at point A.



2.3 Look at a free body of the upper cylinder. Only two reactions are available and they act perpendicular to the surface of the cylinder at the points of contact.



$$\Sigma F_h = 0 = P_B \cos 30° - P_A \cos 60°$$

$$P_B = P_A \left(\frac{\cos 60°}{\cos 30°}\right) = .577\ P_A$$

$$\Sigma F_v = 0 = P_B \sin 30° + P_A \sin 60° - 200$$

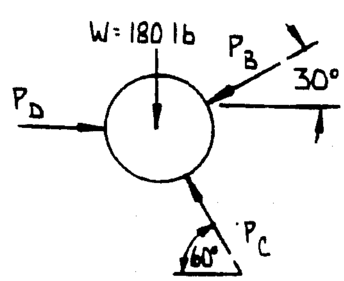
$$.5\ P_B + .866\ P_A = 200$$

$$.5\ (.577\ P_A) + .866\ P_A = 200$$

$$\underline{P_A = 173\#}$$

$$\underline{P_B} = .577\ P_A = .577 \times 173 = \underline{100\#}$$

Now look at a free body of the lower cylinder.



$$\Sigma F_v = 0 = P_C \sin 60° - P_B \sin 30° - 180$$

$$.866\ P_C = 100\ (.50) + 180$$

$$\underline{P_C} = 266\#$$

$$\Sigma F_h = 0 = P_D - P_B \cos 30° - P_C \cos 60°$$

$$P_D = 100\ (.866) + 266\ (.50)$$

$$\underline{P_D = 220\#}$$

2.4 $\Sigma M_B = 0 = 36,000 \times 52 + 18,000 \times 6 - 60\, R_1$

$\underline{R_1 = 33,000\#}$

$\Sigma F_h = 0 = 18,000 - R_3 \cos\Theta$

$\Theta = \arctan 6/60 = 5.71°$

$\underline{R_3} = \dfrac{18000}{\cos 5.71°} = \underline{18,090\#}$

$\Sigma F_v = 0 = R + R_2 - R_3 \sin 5.71° - 36,000$

$33,000 + R_2 - 18,090 \sin 5.71° = 36,000$

$\underline{R_2 = 4800\#}$

2.5 First find the reactions at points E and H to complete the external balance.

$\Sigma M_H = 0 = 4000 \times 9 + 3000 \times 18 + 2000 \times 27 - 27\, R_E$

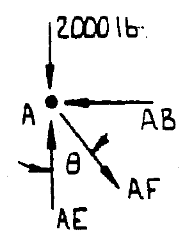
$\underline{R_E = 5333\#}$

$\Sigma F_v = 0 = 2000 + 3000 + 4000 + 5000 - R_E - R_H$

$\underline{R_H} = 14,000 = 5333 = \underline{8667\#}$

Determine the internal loads by balancing each joint, individually.

Joint A: $\Theta = \arctan \dfrac{9}{12} = 36.87°$



$\underline{AE} = R_E = \underline{5333\#}$ comp.

$\underline{AF} = \dfrac{AE-2000}{\cos\ \Theta} = \dfrac{3333}{\cos 36.87°} = 4166\#$ tens

$\underline{AB} = AF \sin\Theta = 4166 \sin 36.87° = \underline{2500\#}$ comp.

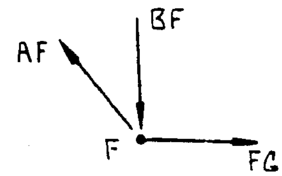
Joint E:

$\Sigma F_h = 0 = EF$

$\underline{EF = 0}$

Joint F:

$\Sigma F_v = 0 = BF - 4F \cos\Theta$



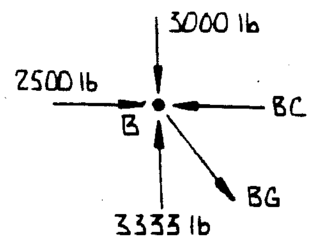
$\underline{BF} = 4166 \cos 36.87° = \underline{3333\#}$ comp.

$\Sigma F_h = 0 = FG - AF \sin\Theta$

$\underline{FG} = 4166 \sin 36.87° = 2500\#$ tens

Joint B:

$\Sigma F_v = 0 = 3333 - 3000 - BG \cos 36.87°$



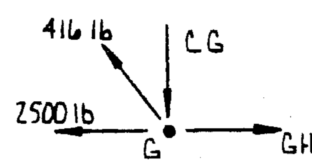
$\underline{BG} = \dfrac{333}{\cos 36.87°} = 416\#$ tens

$\Sigma F_h = 0 = 2500 + BG \sin 36.87° - BC$

$\underline{BC} = 2500 + 416 \sin 36.87° = \underline{2750\#}$ comp.

Joint G:

$\Sigma F_v = 0 = CG - 416 \cos 36.87°$



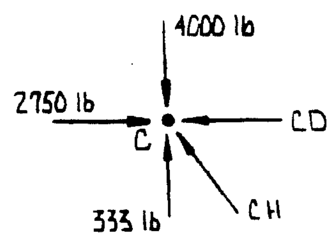
$\underline{CG} = 416 \cos 36.87° = \underline{333\#}$ comp.

$\Sigma F_h = 0 = 2500 + 416 \sin 36.87° - GH$

$\underline{GH} = 2500 + 416 \sin 36.87° = 2750\#$ tens

Joint C:

$\Sigma F_v = 0 = 4000 - 333 - CH \cos 36.87°$



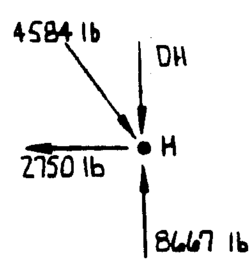
$\underline{CH} = \dfrac{3667}{\cos 36.87°} = \underline{4584\#}$ comp.

$\Sigma F_h = 0 = 2750 - CH \sin 36.87° - CD$

$\underline{CD} = 2750 - 4584 \sin 36.87° = \underline{0}$

Joint H:

$\Sigma F_v = 0 = DH + 4584 \cos 36.87° - 8667$



$\underline{DH} = 8667 - 4584 \cos 36.87° = \underline{5000\#}$

Joint D:

$\Sigma F_v = 0 = 5000 - 5000$

This checks the internal balance.

2.6 $\Sigma M_x = 0 = R_B \times 15 + R_C \times 3 - 600 \times 6$

$$15\ R_B + 3\ R_C = 3600$$

$$\Sigma M_y = 0 = R_B \times 6 + R_C \times 24 - 600 \times 15$$

$$6\ R_B + 24\ R_C = 9000$$

Combine the two equations by multiplying the first equation by a minus 8 and add it to the second equation.

$$\begin{array}{rl} -120\ R_B - 24\ R_C = & -28{,}800 \\ 6\ R_B + 24\ R_C = & 9{,}000 \\ \hline -114\ R_B \qquad\qquad = & -19{,}800 \end{array}$$

$$\underline{R_B = 174\#}$$

$$15\ (174) + 3R_C = 3600$$

$$\underline{R_C = 330\#}$$

$$\Sigma F_v = 0 = R_A + R_B + R_C - 600$$

$$\underline{R_A} = 600 - 174 - 330 = \underline{96\#}$$

2.7 (1) $\Sigma M_y = 0 = 55{,}000 \times 6.0 + 12{,}000 \times 52.0 - (R_{db} \sin 60^\circ)\ 20.0$

$$\underline{R_{db} = 55{,}080\#}$$

(2) $\Sigma F_z = 0 = 55{,}000 - R_{db} \cos 60^\circ - Z_A - Z_B$

$$Z_A + Z_B = 27{,}460\#$$

(3) $\Sigma M_x = 0 = 7000 \times 60.0 + 10.0\ Z_A - 10.0\ Z_B$

$$Z_A - A_B = -42{,}000\#$$

Solving the equations for ΣF_v and ΣM_x simultaneously:

$$Z_A - (27460 - Z_A) = -42{,}000$$

$$2Z_A = -14{,}540$$

$$\underline{Z_A = -7270\#}$$

$$Z_B = 27460 - Z_A = 27460 - (-7270) = \underline{34{,}730\#}$$

Note: The portions of Z_A and Z_B due to the 55,000# vertical load and the 7000# side load could be determined separately and superimposed.

(4) $\Sigma F_x = 0 = 12{,}000 - R_{db} \sin 60° + X_A + X_B$

$X_A + X_B = 35{,}700$

(5) $\Sigma M_z = 0 = 7000 \times 6.0 + 10.0\ X_A - 10.0\ X_B$

$X_A - X_B = -4200$

Solving the equations for ΣF_x and ΣM_z simultaneously:

$X_B = 4200 + X_A$

$X_A + (4200 + X_A) = 35{,}700$

$2X_A = 31{,}500$

$\underline{X_A = 15{,}750\#}$

$\underline{X_B} = 4200 + X_A = 4200 + 12750 = \underline{19{,}950\#}$

Note: The portions of X_A and X_B due to ΣF_x and due to ΣM_z could be determined separately and superimposed.

(6) $\Sigma F_y = 0 = 7000 - Y_B$

$\underline{Y_B = 7000\#}$

Note that in this problem all six equations of statics are required because there are six reactions.

SOLUTIONS TO PROBLEMS

LESSON 3 REVIEW OF STRENGTH OF MATERIALS

3.1 $\delta = \frac{PL}{AE}$

$$= \frac{65{,}000 \times 60}{1.25 \times 10.5 \times 10^6} = \underline{0.297 \text{ in}}$$

3.2 $\delta = \frac{fl}{E} = \frac{-35{,}000 \times 60}{10.5 \times 10^6} = \underline{-0.20 \text{ in}}$

3.3 The reactions are statically indeterminate because there are two reactions and only one equation of statics that applies ($\Sigma F_h = 0$). However, as point B deflects, the length AB must shorten the same amount as length BC elongates. Thus:

$$\delta_{AB} = \delta_{BC}$$

$$\frac{P_{AB} L_{AB}}{AE} = \frac{P_{BC} L_{BC}}{AE}$$

$$P_{AB} = P_{BC}\left(\frac{L_{BC}}{L_{AB}}\right) = P_{BC}\left(\frac{18}{24}\right) = .75 \, P_{BC}$$

This, then, is the second equation that is required, the first being:

$$\Sigma F_h = 0 = R_A + R_B - 30{,}000$$

$$\therefore R_A + R_B = 30{,}000 = P_{AB} + P_{BC}$$

Combining the equations:

$$.75 \, P_{BC} + P_{BC} = 30{,}000$$

$$\underline{P_{BC} = 17{,}140\#}$$

$$\underline{P_{AB}} = 30{,}000 - 17{,}140 = \underline{12{,}860\#}$$

For the deflection at B:

$$\underline{\delta_B} = \frac{P_{AB} L_{AB}}{AE} = \frac{12{,}860 \times 24}{.18 \times 16 \times 10^6} = \underline{0.107 \text{ in.}}$$

or:

$$\underline{\delta_B} = \frac{P_{BC} L_{BC}}{AE} = \frac{17{,}140 \times 18}{.18 \times 16 \times 10^6} = \underline{0.107 \text{ in.}}$$

3.4 The spar cap and strap are forced to elongate the same amount because they are attached along the length. Therefore:

$\delta_A = \delta_S$ where subscripts A and S denote aluminum and steel, respectively.

$$\frac{P_A L}{A_A E_A} = \frac{P_S L}{A_S E_S}$$

$$\frac{P_A \times 84}{1.50 \times 10.4 \times 10^6} = \frac{P_S \times 84}{.75 \times 29.0 \times 10^6}$$

$$P_A = .717\ P_S$$

and by $\Sigma F = 0$:

$$P_A + P_S = 172{,}000$$

$$.717\ P_S + P_S = 172{,}000$$

steel $\underline{P_S = 100{,}200\#}$

$$\underline{f_t} = \frac{P}{A_{net}} = \frac{100{,}200}{.65} = \underline{154{,}200 \text{ psi}}$$

$$\underline{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{160{,}000}{154{,}200} - 1 = \underline{+.04}$$

aluminum $\underline{P_A} = .717\ P_S = .717 \times 100{,}200 = \underline{71{,}800\#}$

$$\underline{f_t} = \frac{P}{A_{net}} = \frac{71{,}800}{1.35} = \underline{53{,}200 \text{ psi}}$$

$$\underline{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{88{,}000}{53{,}200} - 1 = \underline{+.65}$$

Note: The same answers would result from using the equations:

$$e_A = e_S$$

$$\frac{f_A}{E_A} = \frac{f_S}{E_S}$$

3.5 From the equation $\delta = PL/AE$ it can be seen that δ is inversely proportional to AE/PL. P is the load and AE/L is a measure of stiffness. Thus, for the straps to have the same stiffness (that is, the same δ for the same P):

$$\left(\frac{AE}{L}\right)_T = \left(\frac{AE}{L}\right)_S$$ where T denotes titanium and S denotes steel

Since the lengths are the same:

$$(AE)_T = (AE)_S$$

$$\frac{A_T}{A_S} = \frac{E_S}{E_T} = \frac{29 \times 10^6}{16 \times 10^6} = 1.813$$

$$A_S = 3.75 \times .156 = .585 \text{ in}^2$$

$$A_T = 1.813\ A_S = 1.813 \times .585 = \underline{1.061 \text{ in}^2}$$

3.6 Find reactions R_B and R_F

$$\Sigma M_2 = 0 = 500 \times 4 + 2000 \times 10 + 1000 \times 16 + 1000 \times 26 - 20R_1$$

$$\underline{R_B = 3200\#}$$

$$\Sigma F_v = 0 = 1000 + 1000 + 2000 + 500 - R_1 - R_2$$

$$\underline{R_F = 1300\#}$$

The shear at the left end is equal to the applied load of 1000 lb. At each applied load and reaction the shear changes by the magnitude of the load.

The bending moment is zero at the free end. The change in moment in span AB is equal to the area of the shear diagram for that span. Thus,

$$M_B = M_A - 1000 \times 6 = 0 - 6000 = -6000"\#$$

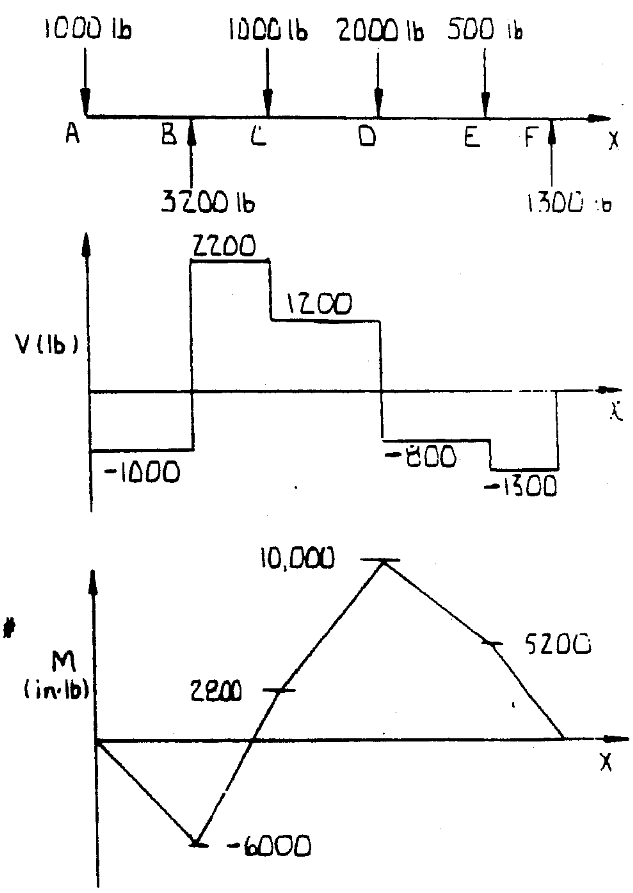
similarly:

$$M_C = M_B + 2200 \times 4 = -6000 + 8800 = 2800"\#$$

$$M_D = M_C + 1200 \times 6 = 2800 + 7200 = 10{,}000"\#$$

$$M_E = M_D - 800 \times 6 = 10{,}000 - 4800 = 5200"\#$$

$$M_F = M_E - 1300 \times 4 = 5200 - 5200 = 0$$



3.7 Find reactions by the equations of statics.

$$\Sigma M_C = 0 = \left(\frac{200 \times 12}{2}\right)\left(\frac{12}{3}\right) + 1000 \times 12 + (100 \times 12)\ 18 - 24R_A$$

$$R_A = \frac{38,400}{24} = 1600\#$$

$$\Sigma F_V = 0 = 100 \times 12 + 1000 + \frac{200 \times 12}{2} - R_A - R_C$$

$$R_C = 1200 + 1000 + 1200 - 1600 = 1800\#$$

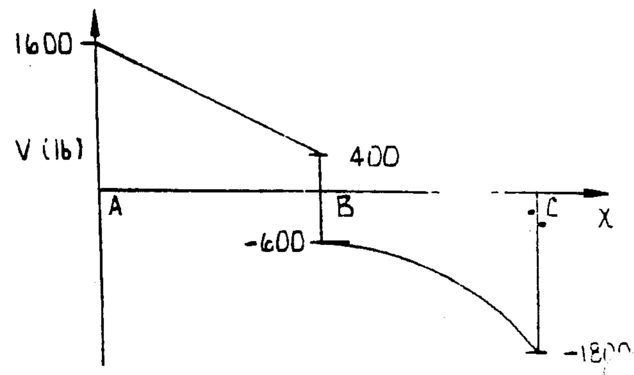
Find shear at A, B and C by summing vertical forces starting at point A:

$$V_A = R_A = 1600\#$$

$$V_{B-} = V_A - 100 \times 12 = 1600 - 1200 = 400\#$$

$$V_{B+} = V_{B-} - 1000 = 400 - 1000 = -600\#$$

$$V_C = V_{B+} - \frac{200}{2} \times 12 = -600 - 1200 = -1800\#$$

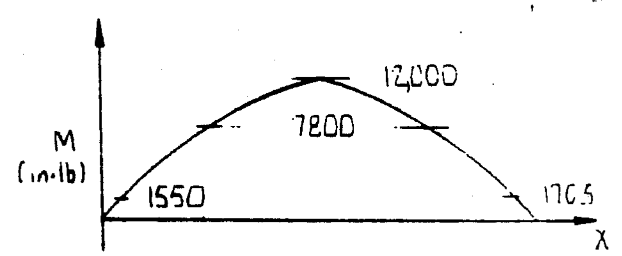


Find bending moment at A, B and C:

$$M_A = 0$$

$$M_B = M_A + \frac{1600 + 400}{2}\ 12 = 0 + 12,000 = 12,000''\#$$

(Using area of shear diagram).



$$M_C = M_B + V_{B+}\ b - \frac{wb^2}{6}$$
$$= 12,000 - 600 \times 12 - \frac{200(12)^2}{6} = 0 \text{ (Using } \Sigma M_C)$$

Find the shear and moment at center of span a, using the equations of statics:

$$\underline{V} = R_A - w\frac{a}{2} = 1600 - 100\left(\frac{12}{2}\right) = \underline{1000\#}$$

$$\underline{M} = R_A\left(\frac{a}{2}\right) - w\left(\frac{a}{2}\right)\frac{a}{4} = 1600 \times 6 - 100(6)\ 3 = \underline{7800''\#}$$

Similarly, at center of span b using the equations of paragraph 3.4.2:

$$\underline{V} = V_{B+} - \frac{w(b/2)^2}{2b} = -600 - \frac{200(6)^2}{2x12} = \underline{-900\#}$$

$$\underline{M} = M_B + \left(V_{B+}\right)(b/2) - \frac{w(b/2)^3}{6b} = 12{,}000-600(6) - \frac{200(6)^3}{6x12} = \underline{7800"\#}$$

Find shear and bending moment one inch from A using the equations of statics:

$$\underline{V} = R_A - w(x) = 1600 - 100(1) = \underline{1500\#}$$

$$\underline{M} = R_A(1) - w(x)\,\frac{x}{2} = 1600 - 100(1)\,\frac{1}{2} = \underline{1550"\#}$$

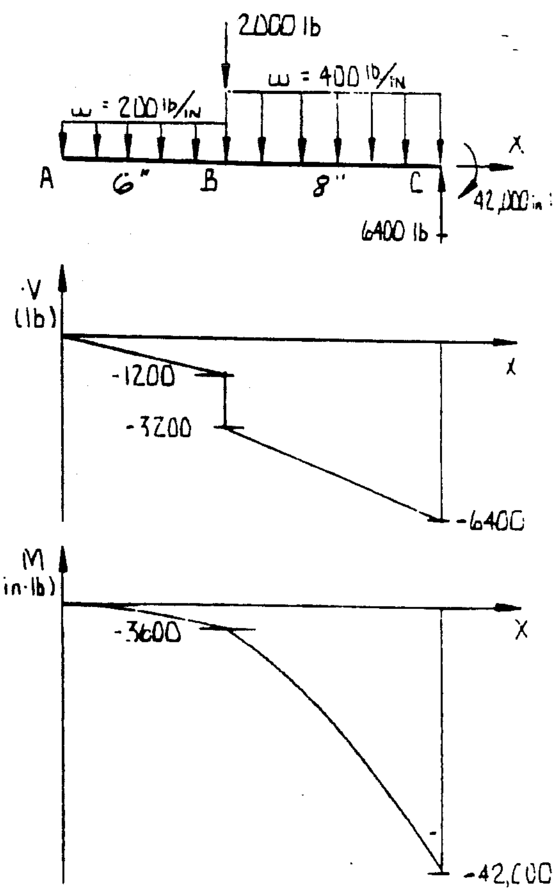
Find shear and bending moment one inch from C using the equations of paragraph 3.4.2:

$$\underline{V} = V_{B+} - \frac{w(x)^2}{2b} = -600 - \frac{200(11)^2}{2x12} = \underline{-1608\#}$$

$$\underline{M} = M_B - \left(V_{B+}\right)(x) - \frac{w(x)^3}{6b} = 12000-600(11) - \frac{200(11)^3}{6x12} = \underline{1703"\#}$$

3.8 Summing vertical loads (shear) and moment, starting at the free end:

$V_A = 0, \; M_A = 0$



$V_{B-} = \left(-w_a\right) a = -200 \times 6 = -1200\#$

$V_{B+} = V_{B-} - P_V = -1200-2000 = -3200\#$

$M_B = w_a\ (a)\ \frac{a}{2} = 200(6)\ 3 = -3600"\#$

$V_C = V_{B+} - \left(w_b\right) b = -3200-400 \times 8 = -6400\#$

$M_C = M_B + \left(V_{B+}\right) b - w_b(b)\left(\frac{b}{2}\right)$

$\quad = -3600-3200 \times 8 - 400 \times 8 \times 4 = -42{,}000"\#$

At center of span a:

$M = w_a\ (x)\left(\frac{x}{2}\right) = 200(3)(1.5) = 900"\#$

At center of span b:

$M = M_b + \left(V_{B+}\right)(x) - w_b\ (x)\left(\frac{x}{2}\right)$

$\quad = -3600-3200(4) - 400(4)(2) = -19{,}600"\#$

3.9 Balance the beam using the equations of statics:

$\Sigma F_h = 0 = R_h - 6000$

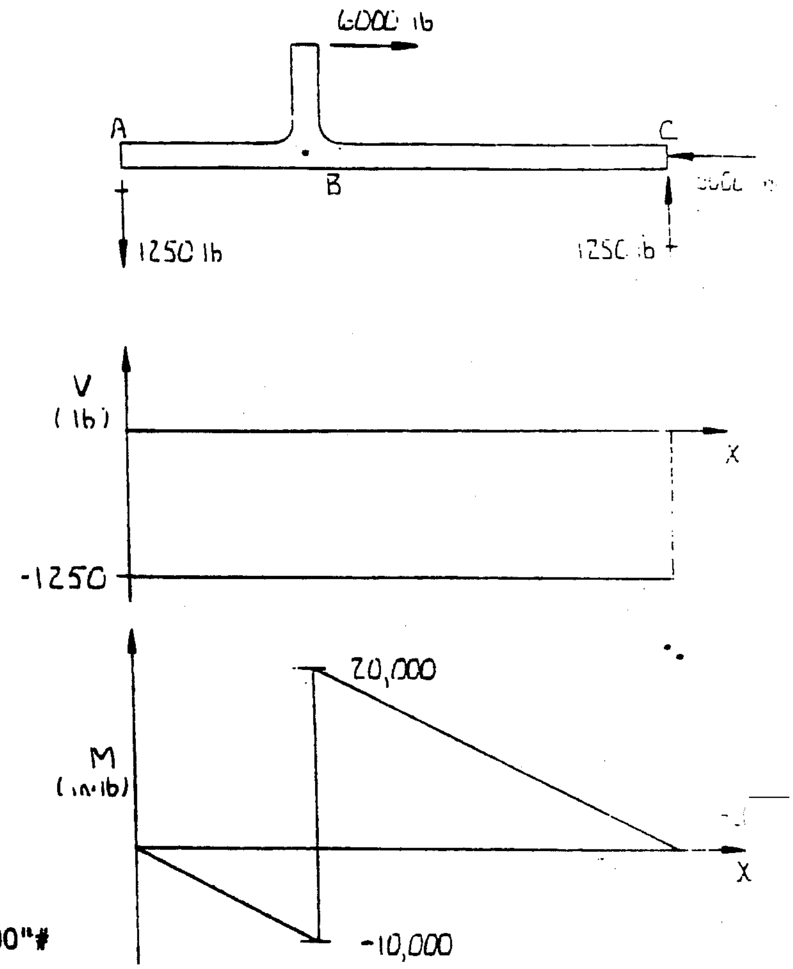
$R_h = 6000\#$

$\Sigma M_C = 0 = 6000 \times 5 - 24 R_A$

$R_A = \dfrac{6000 \times 5}{24} = 1250\#$

$\Sigma F_V = 0 = R_A - R_B$

$R_B = R_A = 1250\#$



At A:

$V = -R_A = -1250\#$

$M = 0$

At B-:

$V = -R_A = -1250\#$

$M = -R_A \times 8 = 8\ (-1250)$
$= -10{,}000''\#$

At B+:

$V = -R_A = -1250\#$

$M = M_{B-} + 6000 \times 5$
$= -10{,}000 + 30{,}000 = 20{,}000''\#$

At C:

$V = -R_A = -1250\#$

$M = M_{B+} + (V_{B+}) \times 16$
$= 20{,}000 - 1250 \times 16 = 0$

SOLUTIONS TO PROBLEMS

LESSON 4 AIRCRAFT EXTERNAL LOADS

4.1 As explained in paragraph 4.1.2:

a) $+N_z$ and loads it produces act <u>down</u>.

b) $+N_x$ and loads it produces act <u>forward</u>.

c) $+N_y$ and loads it produces act <u>left</u>.

4.2 The load factor N_z on the airplane is equal to the total of all vertical forces acting on the aircraft divided by the aircraft weight. Thus:

$$N_z = \frac{252{,}000 + 31{,}750 - 40{,}000}{37{,}500} = \underline{6.5}$$

4.3 Load factors for mass items in the fuselage and wing of the F-4 are found on pages 4.5 and 4.6 respectively.

Prob	$+N_x$	$-N_x$	$+N_y$	$-N_y$	$+N_z$	$-N_z$
a)	6.0	-5.2	0.9	-0.9	8.5	-3.0
b)	6.0	-5.2	2.4	-2.4	9.3	-4.0
c)	6.0	-5.2	5.8	-5.8	12.5	-5.1
d)	6.0	-5.2	4.8*	-4.8*	9.2	-3.4
e)	6.0	-6.1	14.0*	-14.0*	15.5	-9.5

*These values for Ny are from the graph on page 4-6. They are for inertia loads acting outboard. The inboard-acting load would be smaller; however, it is usually assumed to be the same as the outboard-acting load.

4.4 As explained in paragraph 4.2.1, crash load factors apply only to items in, or directly behind, the cockpit. Only the console of problem c) meets this requirement. The crash load factors for design are N_z = 20 ultimate, N_x = 40 ultimate and N = 40 ultimate in a horizontal plane up to 20° either side of forward.

4.5 The highest design ultimate pressure is 48 psi on the engine air duct skins due to engine stalls, as explained in paragraph 4.2.2.

4.6 Control system design limit pilot effort loads are shown on page 4-7. Ultimate loads are determined by multiplying the limit loads by 1.5.

Prob.	Limit Load	Reference Column	Ultimate Load
a)	450 lb	MCAIR	657 lb.
b)	300 lb	Air Force	450 lb.
c)	250 lb	Air Force	375 lb.
d)	100 lb	Air Force	150 lb.

SOLUTIONS

LESSON 5 INTERNAL LOADS

5.1 a) Bending moment and shear

b) Torsion

c) Tension and compression

d) Shear

5.2 a) Beam and torque box

b) Beam, torque box and axial member

c) Beam and torque box

d) Beam

e) Axial member

f) Shear web

g) Beam

h) Beam, axial member

i) Axial member, shear web

5.3 a) Wing shear, bending and torque, caused by air and inertia loads, cause shear and bending in the spar. Fuel pressures cause bending in the spar webs and stiffeners.

b) Wing shear, bending and torque cause shear, bending and axial load in the rib.

c) Wing shear, bending and torque cause axial load and shear. Aerodynamic and fuel pressure cause shear and bending normal to the plane of the skin.

d) Aerodynamic pressures cause shear and bending.

e) Aerodynamic pressures cause shear and bending.

f) Cockpit pressure causes shear and bending.

g) Engine stall pressures cause shear, bending and axial load (tension).

h) Aerodynamic and inertia loads on the fuselage forward of F.S.77.00 are redistributed by the bulkhead causing shear and bending. It distributes nose landing gear loads from the keel webs, causing shear and bending. Cockpit pressure causes bending normal to the plane of the bulkhead.

i) Aerodynamic and engine bay pressurescause shear,bending and axial load.

j) Aerodynamic loads cause shear and bending.

k) Aerodynamic loads cause shear, bending and torsional shear.

l) Stabilator shear, bending and torsion due to aerodynamic loads cause shear and axial load.

SOLUTIONS TO PROBLEMS

LESSON 6 STRESS AND MARGIN OF SAFETY

6.1 Tension, compression, shear and bearing.

6.2 Tension, compression and shear.

6.3 No at limit load; yes at ultimate load.

6.4 Design limit load (DLL) is 1000 lb. Design ultimate load (DUL)=1.50 (DLL) =1500 lb. Failing load (which, by definition, is the allowable load) is 1500 lb.

$\underline{M.S.} = (P_{all}/P_{ult})-1 = (1500/1500)-1 = \underline{0}$

6.5 F_{tu}, F_{ty}, F_{cy}, F_{su}, F_{bry}, F_{bru}, E, G, and μ.

6.6 MIL-HDBK-5 "Military Standardization Handbook - Metallic Materials and Elements for Aerospace Vehicle Structures".

6.7 Compression stresses - due to crippling and/or column buckling.

Shear stresses due to buckling of the shear web.

6.8 None, because the mechanical properties are the maximum allowable stresses for the material.

6.9 McDonnell reports MAC 338 and MAC 339, other company strength manuals, and various college textbooks and handbooks.

6.10 a) $f_t = \frac{P}{A} = \frac{9500}{.188} = 50{,}500$ psi

$F_{tu} = 61{,}000$ psi

$\underline{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{61{,}000}{50{,}500} - 1 = \underline{+.21}$

b) $f_t = \frac{P}{A} = \frac{25000}{.312} = 80{,}100$ psi

$F_{tu} = 86{,}000$ psi

$\underline{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{86{,}000}{80{,}100} - 1 = \underline{+.07}$

c) $f_s = \frac{V}{A} \quad \frac{6500}{.25} = 26{,}000$ psi

$F_{su} = 31{,}000$ psi (using extrusion allowable)

$\underline{M.S.} = \frac{F_{su}}{f_s} - 1 = \frac{31{,}000}{26{,}000} - 1 = \underline{+.19}$

d) $f_{br} = \frac{P}{A} = \frac{P}{Dt} = \frac{3500}{.312 x .050} = 224{,}400$ psi

$F_{bru} = 261{,}000$ psi (using Ti-6Al-4V allowable)

$\underline{M.S.} = \frac{F_{bru}}{f_{br}} - 1 = \frac{261{,}000}{224{,}400} - 1 = \underline{+.16}$

6.11

	Material	F_{tu}	F_{bru}
(1)	AISI 301 half hard	151	304
(2)	125 ksi 4130 steel	125	251

$\frac{F_{tu} \text{ (2)}}{F_{tu} \text{ (1)}} = \frac{125}{151} = .828$

$\frac{F_{bru} \text{ (2)}}{F_{bru} \text{ (1)}} = \frac{251}{304} = .826$

Required $t = \frac{.125}{.826} = .1513$ inch minimum

SOLUTIONS TO PROBLEMS

LESSON 7 SECTION PROPERTIES

7.1 First, find the effective area. Then determine the stress from the equation $f = P/A$. Find the allowable stress from the table in Lesson 6 and determine the margin of safety from the equation $M.S. = (F/f) - 1$.

a) $A_t = .10\left[3.00 - .50 - 2x.25)\right] = .20 \text{ in}^2$

$\underline{f_t} = \frac{P}{A} = \frac{10{,}500}{.20} = \underline{52{,}500 \text{ psi}}$ $\quad F_{tu} = 59 \text{ ksi}$

$\underline{M.S.} = (F_{tu}/f_t) - 1 = (59/52.5) - 1 = \underline{+.12}$

b) $A_c = .10\,(3.00 - .50) = .25 \text{ in}^2$

$\underline{f_c} = \frac{P}{A} = \frac{10{,}500}{.25} = \underline{42{,}000 \text{ psi}}$ $\quad F_c = F_{tu} = 59 \text{ ksi}$

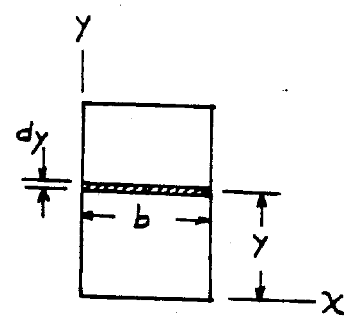
$\underline{M.S.} = (F_c/f_c) - 1 = (59/42) - 1 = \underline{+.40}$

c) $A_s = A_t = .20 \text{ in}^2$

$\underline{f_s} = \frac{V}{A} = \frac{7200}{.20} = \underline{36{,}000 \text{ psi}}$ $\quad F_{su} = 37 \text{ ksi}$

$\underline{M.S.} = (F_{su}/f_s) - 1 = (37/36) - 1 = \underline{+.03}$

7.2 Select a differential area dA having all points equally distant from the base.



$dA = b\,dy$

Then, using the equation $A\bar{y} = \int y\,dA$:

$$\bar{y} = \frac{1}{A}\int_0^h b\,y\,dy = \frac{1}{bh}(b)\left[\frac{y^2}{2}\right]_0^h = \frac{b}{bh}\left[\frac{h^2}{2}\right]$$

$$\underline{\bar{y} = \frac{h}{2}}$$

Using the equation $I_x = \int y^2 dA$:

$$I_x = \int_0^h y^2\,b\,dy = b\int_0^h y^2\,dy = b\left[\frac{y^3}{3}\right]_0^h$$

$$\underline{I_x = \frac{bh^3}{3}}$$

Centroidal $I = I_x - Ad^2 = I_x - A\bar{y}^2 = \frac{bh^3}{3} - (bh)\left(\frac{h^2}{2}\right)$

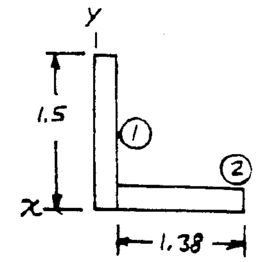
$I = \frac{bh^3}{12}$

7.3 Divide each section into a minimum number of rectangular elements, neglecting the small fillet radius areas, and solve, using the equations given in Lesson 7. This is most conveniently done using a table that combines the calculations of Examples 2 and 4 (paragraphs 7.3.2 and 7.4.3).

a) Angle

(1) $I_h = \frac{bh^3}{12} = \frac{.12(1.5)^3}{12} = .03375 \text{ in}^4$ (2) $I_h = \frac{1.38(.12)^3}{12} = .00020 \text{ in}^4$

Element	b	t	A	y	Ay	Ay^2	I_h
1	1.5	.12	.18	.75	.135	.10125	.03375
2	1.38	.12	.1656	.06	.0099	.00060	.00020
Σ			.3456		.1449	.10185	.03395



$A = .3456 \text{ in}^2$

$\bar{y}$ ($= \bar{x}$ because of symmetry) $= \frac{\Sigma(Ay)}{\Sigma A} = \frac{.1449}{.3456} = .419 \text{ in}$

I_h (= I_v because of symmetry)

$= \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .03395 + .10185 - .3456(.419)^2 = 0751 \text{ in}^4$

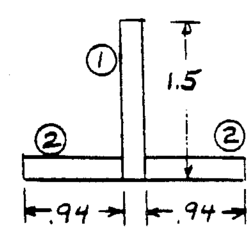
$I_v = I_h$ by symmetry.

$\rho_h = \rho_v = \sqrt{\frac{I}{A}} = \sqrt{\frac{.0751}{.3456}} = .466 \text{ in}$

b) Tee

(1) $I_h = \frac{bh^3}{12} = \frac{.12(1.5)^3}{12} = .03375 \text{ in}^4$ (2) $I_h = \frac{1.88(.12)^3}{12} = .00027 \text{ in}^4$

Element	b	t	A	y	Ay	Ay^2	Ih
1	1.5	.12	.18	.75	.135	.10125	.03375
2	1.88	.12	.2256	.06	.01354	.00081	.00027
Σ			.4056		.14854	.10206	.03402



$A = .4056 \text{ in}^2$

$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{.14854}{.4056} = .366 \text{ in}$

$\bar{x} = 0$ because if there is an axis of symmetry, the centroid lies on that axis.

$\underline{I_h} = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .03402 + .10206 - .4056(.366)^2 = \underline{.0817 \text{ in.}^4}$

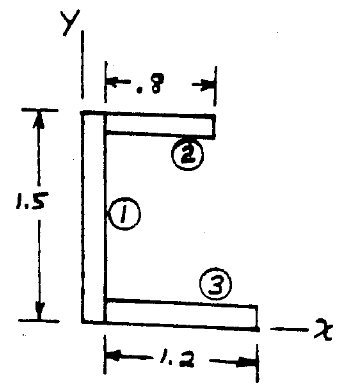
$\underline{I_v} = \Sigma\left(\frac{bh^3}{12}\right) = \frac{.12(2.0)^3}{12} + \frac{1.38(.12)^3}{12} = .08 + .0002 = \underline{.0802 \text{ in.}^4}$

$\underline{\rho_h} = \sqrt{\frac{I_h}{A}} = \sqrt{\frac{.0817}{.4056}} = \underline{.4488 \text{ in.}}$

$\underline{\rho_v} = \sqrt{\frac{I_v}{A}} = \sqrt{\frac{.0802}{.4056}} = \underline{.4447 \text{ in.}}$

c) Channel

Because this section is not symmetrical, it is necessary to tabulate x terms as well as y terms.



(1) $I_h = \frac{bh^3}{12} = \frac{.20(1.5)^3}{12} = .05625 \text{ in}^4$

(2) $I_h = \frac{.80(.20)^3}{12} = .00053 \text{ in}^4$

(3) $I_h = \frac{1.2(.20)^3}{12} = .0008 \text{ in}^4$

Ele.	b	t	A	y	Ay	Ay^2	I_h	x	Ax	Ax^2	I_v
1	1.5	.20	.30	.75	.225	.16875	.05625	.10	.03	.003	.001
2	.80	.20	.16	1.4	.224	.3136	.00053	.60	.096	.0576	.00853
3	1.2	.20	.24	.10	.024	.0024	.00080	.80	.192	.1536	.0288
			.70		.473	.48475	.05758		.318	.2142	.03833

$\underline{A = .70 \text{ in}^2}$

$\underline{\bar{y}} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{.473}{.70} = \underline{.676 \text{ in.}}$

$\underline{\bar{x}} = \frac{\Sigma(Ax)}{\Sigma A} = \frac{.318}{.70} = \underline{.454 \text{ in.}}$

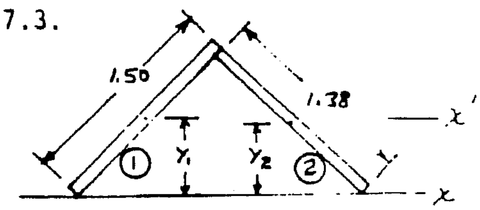
$\underline{I_h} = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .05758 + .48475 - .70(.676)^2 = \underline{.2225} \text{ in}^4$

$\underline{I_v} = \Sigma I_v + \Sigma(Ax^2) - A(\bar{x})^2 = .03833 + .2142 - .70(.454)^2 = \underline{.1082} \text{ in}^4$

$\rho_h = \sqrt{\frac{I_h}{A}} = \sqrt{\frac{.2225}{.70}} = .5638$ in.

$\rho_v = \sqrt{\frac{I_v}{A}} = \sqrt{\frac{.1082}{.70}} = .3932$ in.

7.4 This problem is solved the same way as problem 7.3.



$y_1 = \frac{1.5}{2} \sin 45° + \frac{.12}{2} \sin 45° = .573$ in.

$y_2 = \frac{1.38}{2} \sin 45° + \frac{.12}{2} \sin 45° = .530$ in.

Find I_h of the two elements from the equation of paragraph 7.4.4.

Element ① $I_{max} = \frac{tb^3}{12} = \frac{.12(1.5)^3}{12} = .03375 \text{ in}^4$

$I_{min} = \frac{bt^3}{12} = \frac{1.5(.12)^3}{12} = .000216 \text{ in}^4$

$I_{45°} = I_{max} \cos^2 45° + I_{min} \sin^2 45°$

$= .016875 + .000108 = .016983 \text{ in}^4$

Element ② $I_{max} = \frac{tb^3}{12} = \frac{.12(1.38)^3}{12} = .026280 \text{ in}^4$

$I_{min} = \frac{bt^3}{12} = \frac{1.38(.12)^3}{12} = .000199 \text{ in}^4$

$I_{45°} = I_{max} \cos^2 45° + I_{min} \sin^2 45°$

$= .013140 + .000099 = .013239 \text{ in}^4$

Ele.	b	t	A	y	Ay	Ay^2	I_h
1	1.50	.12	.18	.573	.10314	.05910	.016983
2	1.38	.12	.1656	.530	.08777	.04652	.013239
			.3456		.19091	.10562	.030222

$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{.19091}{.3456} = .552$ in

$I_{x'} = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .030222 + .10562 - .3456(.552)^2$

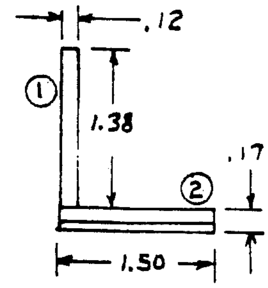
$= .030536 \text{ in}^4$

$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{.030536}{.3456}} = .297$ in.

7.5 This problem is solved the same way as problem 7.3 The area may be divided into elements in any of several ways but the answers will be the same.

a) Angle

(1) $I_h = \dfrac{.12(1.38)^3}{12} = .026281 \text{ in}^4$ (2) $I_h = \dfrac{1.50(.17)^3}{12} = .000614 \text{ in}^4$



Ele.	b	t	A	y	Ay	Ay^2	I_h
1	1.38	.12	.1656	.86	.142416	.122478	.026281
2	1.50	.17	.255	.085	.021675	.001842	.000614
			.4206		.164091	.124320	.026895

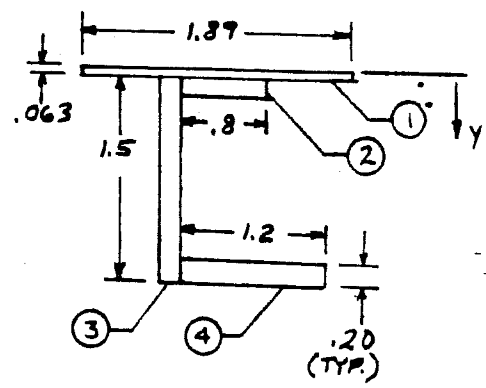
A = .4206 in²

$\bar{y} = \dfrac{\Sigma(Ay)}{\Sigma A} = \dfrac{.164091}{.4206} = .39 \text{ in}$

$I = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .026895 + .124320 - .4206(.39)^2$

$= .08724 \text{ in}^4$

b) Channel



(1) $I_h = \dfrac{1.89(.063)^3}{12} = .000039 \text{ in}^4$

(2) $I_h = \dfrac{.80(.20)^3}{12} = .000533 \text{ in}^4$

(3) $I_h = \dfrac{.20(1.50)^3}{12} = .05625 \text{ in}^4$

(4) $I_h = \dfrac{1.20(.20)^3}{12} = .0008 \text{ in}^4$

Ele	b	t	A	y	Ay	AY^2	I_h
1	1.89	.063	.119	-.0315	-.003749	.000118	.000039
2	.80	.20	.16	-.163	-.02608	.004251	.000533
3	1.50	.20	.30	-.813	-.2439	.198291	.05625
4	1.20	.20	.24	-1.463	-.35112	.513337	.0008
			.819		-.624849	.715997	.057622

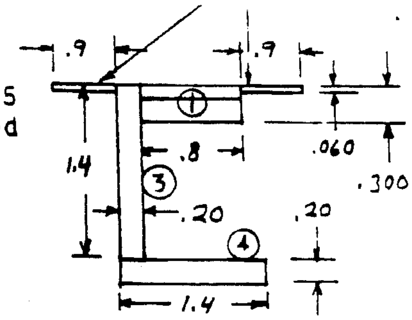
A = .819 in²

$\bar{y} = \dfrac{\Sigma(Ay)}{\Sigma A} = \dfrac{-.624849}{.819} = -.763 \text{ in}$

$I = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .057622 + .715997 - .819(.763)^2$

$= .7972 \text{ in}^4$

7.6 This problem is solved the same way as problem 7.5 Again, there are many ways the area can be divided into elements.



(1) $I_h = .0018 \text{ in}^4$
(2) $I_h = .000032 \text{ in}^4$
(3) $I_h = .045733 \text{ in}^4$
(4) $I_h = .000933 \text{ in}^4$

Ele	b	t	A	y	Ay	AY^2	I_h
1	.80	.30	.24	-.15	-.036	.0054	.0018
2	1.80	.060	.108	-.03	-.00324	.000097	.000032
3	1.40	.20	.28	-.70	-.196	.1372	.045733
4	1.40	.20	.28	-1.50	-.42	.63	.000933
			.908		-.65524	.772697	.048498

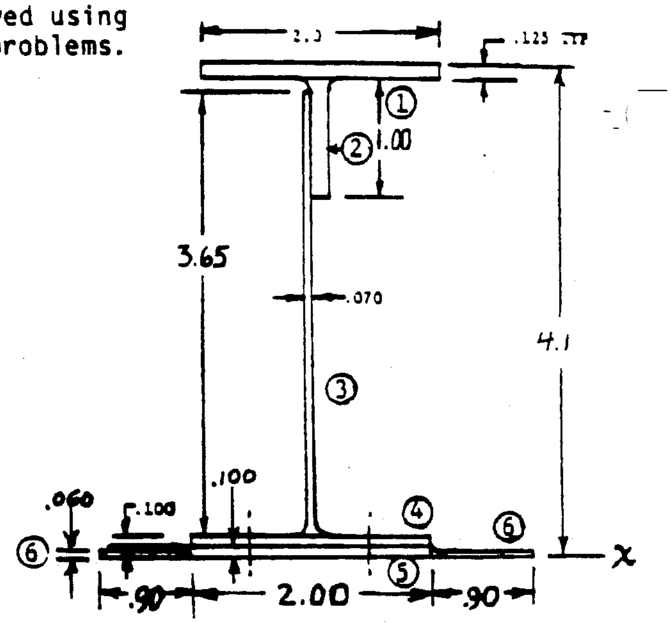
$\underline{A = .908 \text{ in}^2}$

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{-.65524}{.908} = \underline{-.722 \text{ in}}$$

$$I = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2 = .048498 + .772697 - .908(.722)^2$$

$$= \underline{.3479 \text{ in}^4}$$

7.7 This problem is most conveniently solved using a table similar to those of previous problems.



(1) $I_h = \frac{2.0(.125)^3}{12} = .0003 \text{ in}^4$

(2) $I_h = \frac{.125(1.0)^3}{12} = .0104 \text{ in}^4$

(3) $I_h = \frac{.070(3.65)^3}{12} = .2837 \text{ in}^4$

(4) $I_h = \frac{2.0(.100)^3}{12} = .0002 \text{ in}^4$

(5) $I_h = \frac{2.0(.100)^3}{12} = .0002 \text{ in}^4$

(6) $I_h = \frac{1.8(.060)^3}{12} = .0000 \text{ in}^4$

Ele	b	t	Area	y	Ay	AY^2	I_h
1	2.0	.125	.25	4.0375	1.0094	4.0754	.0003
2	1.0	.125	.125	3.475	.4344	1.5095	.0104
3	3.65	.070	.2555	2.025	.5174	1.0477	.2837
4	2.0	.100	.20	.15	.0300	.0045	.0002
5	2.0	.100	.20	.05	.0100	.0005	.0002
6	1.8	.060	.108	.03	.0032	.0001	.---
			1.1385		2.0044	6.6377	.2948

a) Area $\underline{A = 1.1385 \text{ in}^2}$

b) Centroid $\bar{y} = \frac{\Sigma (Ay)}{\Sigma A} = \frac{2.0044}{1.1385} = \underline{1.76 \text{ in.}}$

c) Moment of inertia $\underline{I} = \Sigma I_h + \Sigma(Ay^2) - A(\bar{y})^2$

$= .2948 + 6.6377 - 1.1385(1.76)^2$

$= \underline{3.406 \text{ in}^4}$

d) First moment of area of the skin, $Q = Ay$.

$\underline{Q_{5,6}} = (Ay)_5 + (Ay)_6$ where y (1) is measured from the centroid.

$= .20(1.76 - .05) + .108(1.76 - .03)$

$= \underline{.5288 \text{ in}^3}$

e) First moment of area of inner tee cap $Q = Ay$.

$Q_{1,2} = (Ay)_1 + (Ay)_2$

$= .25(4.0375 - 1.76) + .125(3.475-1.76)$

$= \underline{.7838 \text{ in}^3}$

Note (1): For calculating first moment of area, y is either $(\bar{y} - y_{table})$ or $(y_{table} - \bar{y})$.

SOLUTIONS TO PROBLEMS

LESSON #8 FASTENERS AND JOINTS

Problem 8.1

	Member	P_{all}	Source (page no. are in MAC 339)	Joint P_{all}
a	1 2	1099 927	Pg. 1.13 bottom table for e/D = 2 Pg. 1.13 bottom table for e/D = 1.5	927#
b	1 2	470 551	Pg. 1.14 table Pg. 1.13 bottom table for e/D = 2	470#
c	1 2	778 -	Pg. 1.15 bottom table Not required per Note 2 on pg. 1.15	778#
d	1 2	520 575	Pg. 1.15.01 Air Force value Pg. 1.13 bottom table, e/D = 2	520#
e	1 2	2690 2750	Pg. 1.23 shear strength table (1) $P_{br_{all}} = DtF_{bru} = .1875x.090x163,000$ (2)	2690#
f	1 2	2380 2447	Pg. 1.38 for a #10 bolt $P_{br_{all}} = DtF_{bru} = .1875x.050.x261,000$ (2)	2380#
g	1 2	1384 1238	Pg. 1.39.01 for a 5/32 in. hi-lok $P_{br_{all}} = DtF_{bru} = .156x.063x126,000$ (2)	1238#
h	1 2	1765 2053	Pg. 1.45 for a 3/16" steel jo-bolt $P_{br_{all}} = DtF_{bru} = .200x.063x163,000$ (2) (Note that the dia. of a 3/16 jo-bolt is .200")	1765#
i	1 2	1550 2053	Pg. 1.45 for a 3/16" aluminum jo-bolt (Same as problem 8.1h)	1550#
j	1 2	8620 8120	$P_{br_{all}} = DtF_{bru} = .375x.190x121,000$ (2) $P_{br_{all}} = DtF_{bru} = .375x.190x114,000$ (2)	8120#
	Fastener	9740	Pg. 1.45.01 shear strength	

(1) The value 2920# in the upper table is below the small letter "a" which indicates that it is higher than the strength for a 160 ksi fastener as explained in Note 2 of MAC 339, pg. 1.23.

(2) F_{bru} from the table in Lesson No. 6.

Problem 8.2

	Member	P_{all}	Source (page no. are in MAC 339)	Joint P_{all}
a	1	676	Pg. 1.12	676
	2	1099	Pg. 1.13	+1099
	3	1815 △1	.080 x 1875 x 121,000 △2	1775#
	Fastener	1180	Pg. 1.12 or 1.13 (per shear face)	
b	1	2035	.050 x .156 x 261,000 △3	
	2	1591	.050 x .156 x 204,000 △2	2212#
	3	2212	.100 x .156 x 141,800 △3	
	Fastener	1820	Pg. 1.22 (per shear face)	
c	1	3780	.125 x .250 x 121,000 △2	3780 △4
	2	5748	.190 x .250 x 121,000 △2	+4660
	3	9813	.250 x .250 x 157,000 △2	8440#
	Fastener	4660	Pg. 1.39	

△1 MAC 339 gives a value of 1180# for a DD6 rivet in .080, but that is for a single shear joint. Because this joint is loaded in double shear, the load in member 3 is not limited to the single shear value.

△2 F_{bru} from the table in Lesson No. 6.

△3 Because e/D is less than 1.5, the F_{bru} must be calculated. On pg. 12.12 of MAC 339, enter the chart with e/D = 1.2. Move horizontally to the R/D curve and from that point move vertically to read F_{br}/F_{tu} = 1.02. From the table in Lesson No. 6, F_{tu} = 139,000 psi. Therefore:

$$F_{br} = 1.02 \times 139,000 = 141,800 \text{ psi}$$

△4 Note that member 2 can pick up 5748# but the fastener is only capable of transferring 4660#.

Problem 8.3

	Member	At Room Temperature			At Elevated Temperature			
		P_{all} (1)	Source	Joint P_{all}	Factor (2)	Source	P_{all} (1)x(2)	Joint P_{all}
a	1	470	Pg 1.14	470#	.912	Pg 1.83, 2024	429	429#
	2	575	Pg 1.13		.776	Pg 1.83, 7178	446	
	Rivet	596	Pg 1.13		.885	Pg 1.81, 2017	527	
b	1	470	Pg 1.14	470#	.848	Pg 1.83, 2024	399	
	2	575	Pg 1.13		.450	Pg 1.83, 7178	259	259#
	Rivet	596	Pg 1.13		.650	Pg 1.81, 2017	387	
c	1	4290	Pg 1.39.06	4290	.753	Pg 1.84	3230	3230
	2	4788	.063x.25x304,000	+4660	.65	Lesson #8 △1	3112	+3112
	3	10040	.160x.25x251,000	8950#	.875	Lesson #8 △2	8785	6342#
	Hi-lok	4660			.84		3914	

Page numbers are from MAC 339.

△1 For 301 st. st. 1/2 hard the temp. factors are .67 for 600° and .59 for 800°. For 650°, the factor is $.67 - \left(\frac{650-600}{800-600}\right)(.67-.59) = .65$.

△2 For alloy steel the temp factors are .92 for 600° and .74 for 800°. For 650° the factor is $.92 - \left(\frac{650-600}{800-600}\right)(.92-.74) = .875$.

8.4 a) The loads in the fasteners are proportional to the fastener cross-sectional areas which are proportional to the squares of the diameters.

Fastener no.	D	D^2
1	.1875	.0352
2	.25	.0625
		.0977

Fastener #1 $\underline{P_1} = \left(\frac{D^2}{\Sigma(D^2)}\right)P = \frac{.0352}{.0977} \times 5000 = \underline{1801\#}$

Fastener #2 $\underline{P_2} = \left(\frac{D^2}{\Sigma(D^2)}\right)P = \frac{.0625}{.0977} \times 5000 = \underline{3199\#}$

b) Following the same procedure as for problem a) above:

Fastener no.	D	D^2
1	.156	.0243
2	.156	.0243
3	.1875	.0352
4	.1875	.0352
		.1190

Rivets #1 & #2 $\underline{P_1 = P_2} = \left(\frac{D^2}{\Sigma(D^2)}\right)P = \frac{.0243}{.1190} \times 2750 = \underline{562\#}$

Rivets #3 & #4 $\underline{P_3 = P_4} = \left(\frac{D^2}{\Sigma(D^2)}\right)P = \frac{.0352}{.1190} \times 2750 = \underline{813\#}$

c) Again, the load in the fasteners is proportional to the diameters squared. Find the proportion of the load P in each fastener and then find the location of P by the laws of statics.

Fastener no.	D	D^2	$D^2/\Sigma(D^2)$
1	.260	.0676	.5026
2	.164	.0269	.2000
3	.200	.0400	.2974
		.1345	

$\Sigma M_3 = Pa = .5026P \times 2.0 + .2000P \times 1.00 = 1.205\ P$

$\underline{a = 1.205 \text{ in.}}$

d) The allowable load for the joint is the lowest load that produces a zero margin of safety on the critical fastener. For each fastener find the total load P that would cause failure of that fastener. The lowest of these three loads is P_{all}.

Fastener no.	(1) $D^2/\Sigma(D^2)$	(2) $P_{s_{all}}$*	(2) / (1)
1	.5026	4500	8953
2	.2000	1680	8400
3	.2974	2620	8910

* From MAC 339 pages 1.45.01

$\underline{P_{all} = 8400\#}$ based on fastener #2

$\underline{P_1} = .5026\ P = .5026 \times 8400 = \underline{4222\#}$

$\underline{P_2} = .2000\ P = .2000 \times 8400 = \underline{1680\#}$

$\underline{P_3} = .2974\ P = .2974 \times 8400 = \underline{2498\#}$

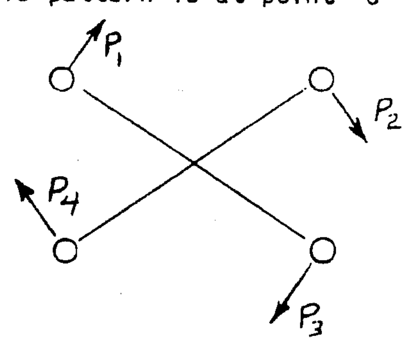
8.5 Find the centroid of the fastener pattern and the moment about that centroid.

a) Because of symmetry, the centroid of this pattern is at point "C" as shown. The distance from the centroid to each fastener is:

$$e = \left(2.0^2 + 1.5^2\right)^{1/2} = 2.5 \text{ in.}$$

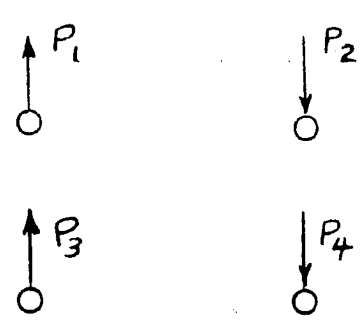
$$P = \frac{MeA}{\Sigma(e^2A)} = \frac{M}{\Sigma e}$$ because all fasteners have the same A and E.

$$\underline{P_1=P_2=P_3=P_4} = \frac{M}{\Sigma e} = \frac{8000}{4\text{x}2.5} = \underline{800\#}$$

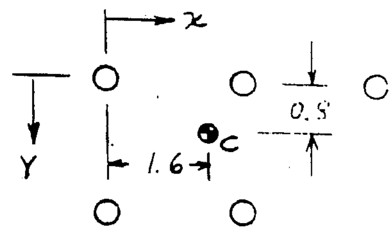


b) Because the structure can only carry axial loads, the moment must be reacted by a couple as shown. Then:

$$\underline{P_1=P_2=P_3=P_4} = \frac{8000}{2\text{x}4.0} = \underline{1000\#}$$



c) Because the pattern is not symmetrical, the centroid must be calculated. The value 1.0 can be used for rivet areas because all rivets are the same size.



$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{2(1.0\text{x}2.0)}{5.0} = 0.8"$$

$$\bar{x} = \frac{\Sigma(Ax)}{\Sigma A} = \frac{2(1.0\text{x}2.0)+1.0\text{x}4.0}{5.0} = 1.60"$$

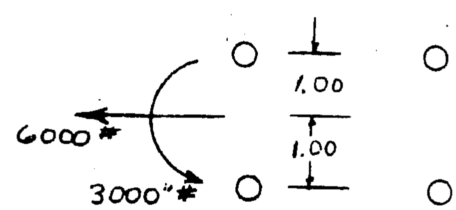
$$e = \left[(x-\bar{x})^2 + (y-\bar{y})^2\right]^{1/2}$$

$$p = \frac{MeA}{\Sigma(e^2A)} = \frac{Me}{\Sigma(e^2)}$$

For each fastener, the values of e and p are calculated in the following table.

Fastener	e^2	e	Me	P
1	$1.60^2 + .80^2 = 3.20$	1.789	17,890	1118#
2	$.40^2 + .80^2 = .80$	.894	8,940	559#
3	$2.40^2 + .80^2 = 6.40$	2.530	25,300	1581#
4	$1.60^2 + 1.20^2 = 4.00$	2.000	20,000	1250#
5	$.40^2 + 1.20^2 = 1.60$	1.265	12,650	701#
	16.00			

d) Separate the unsymmetrical applied load into its symmetric and antisymmetric components. The symmetric load is the 6000# load and the antisymmetric load is the moment caused by the load not being at the centroid.



M = 6000 x .50 = 3000#

Find the fastener loads due to the symmetric load.

$$P_p = \frac{6000}{4} = 1500\#$$

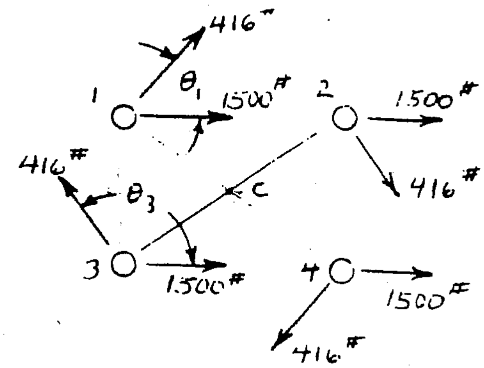
Find the fastener loads due to the antisymmetric load.

$$P_m = \frac{MeA}{\Sigma(e^2A)} = \frac{M}{\Sigma e}$$

$$e = \left(1.5^2 + 1.0^2\right)^{1/2} = 1.803 \text{ in.}$$

$$P_m = \frac{3000}{4\text{x}1.803} = 416\#$$

Combine symmetrical and antisymmetrical loads.



For fasteners no. 1 and 2:

$\theta = 90 - \arctan(1.0/1.5) = 56.31°$

By the law of cosines:

$P^2 = P_p^2 + P_m^2 + 2P_p P_m \cos\theta$

$= 1500^2 + 416^2 + 2\,(1500)(416)\cos 56.31°$
$= 3{,}115{,}300$

$P_1 = P_2 = 1765\#$

For fasteners no. 3 and 4:

$\theta = 90° + \arctan(1.0/1.5) = 123.69°$

By the law of cosines:

$P^2 = P_p^2 + P_m^2 + 2P_p P_m \cos\theta$

$= 1500^2 + 416^2 + 2\,(1500)(416)\cos 123.69° = 1{,}730{,}800$

$P_3 = P_4 = 1316\#$

8.6 a) For infinitely stiff fasteners, the elongation of both straps between fasteners no. 1 and 2 must be the same. The equation for elongation is:

$$\delta = \frac{PL}{AE}$$

Because the L, A, E and δ of both straps are the same between fasteners no. 1 and 2, the P must also be the same. This is only possible if 50% of the load is transferred by the end fastener. The same is true for the other end of the splice. Therefore, the load distribution is as shown in the table.

Fastener	Load
1	.5P
2	0
3	0
4	.5P

b) The referenced table shows that for four fasteners, each end fastener will transfer 37.5% of the load. Therefore, the load distribution is as shown in the table.

Fastener	Load
1	.375P
2	.125P
3	.125P
4	.375P

c) For infinitely stiff fasteners:

$$\delta a = \delta b$$

and $\delta = \frac{PL}{AE}$ as shown in Lesson #3.

$$\therefore \left(\frac{PL}{AE}\right)_a = \left(\frac{PL}{AE}\right)_b$$

$$\frac{.20PL}{wt_aE} = \frac{.80PL}{wt_bE}$$

$$\frac{.20}{t_a} = \frac{.80}{t_b}$$

$$\underline{\frac{t_b}{t_a}} = \frac{.80}{.20} = \underline{4}$$

d) The end fasteners carry all the load, as the load increases, until the bearing yield stress (F_{bry}) is reached. As the load continues to increase, the holes in the bearing - critical part will elongate slightly. This yielding adds to the deflection $= \frac{PL}{AE}$ of the strap, allowing the total deflection of the two straps to remain equal even though the loads are not equal. This permits the middle fasteners to load up until they, also, reach the yield point. As the load increases further, the fastener loads increase equally until the failure point is reached. Therefore, just before failure:

$$\underline{P_1 = P_2 = P_3 = P_4}$$

8.7 a) The load on the clip is the reaction at one end of the beam.

$$P = \frac{wL}{2} = \frac{80 \times 20.0}{2} = 800\#$$

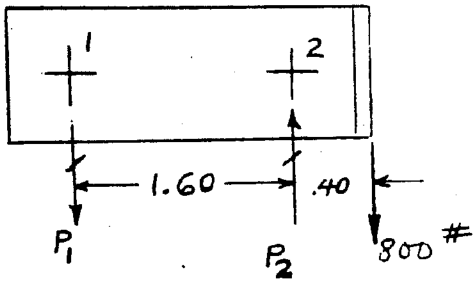
The rivet loads are found by the laws of statics with the load applied at the apex of the clip angle.

$$\Sigma M_2 = 0 = .40 \times 800 - 1.6P_1$$

$$\underline{P_1 = 200\#}$$

$$\Sigma F_v = 0 = P_2 - 800 - 200$$

$$\underline{P_2 = 1000\#}$$

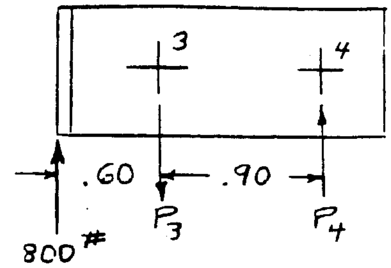


$\Sigma M_3 = 0 = .60 \times 800 - .90\ P_4$

$P_4 = 533\#$

$F_v = 0 = P_3 - 800 - 533$

$P_3 = 1333\#$



For each rivet location, find the smallest rivet that is strong enough in shear and check the allowable load in the clip and in the mating structure.

Rivet No.	P	Trial Rivet	P_{all} (1) in Clip	Mating Structure	P_{all} (1) in Structure	Fastener P_{all}	M.S. (3)
1	200	AD4 (BJ4)	388	.125 7178-76	388	388	+.94
2	1000	DD6 (CX6)	1116	.125 7178-76	1180	1116	+.12
3	1333	DD8 (CX8)	1501	.040 7178-76	1630 (2)	1501	+.13
4	533	AD5 (BJ5)	596	.040 7178-76	575	575	+.08

(1) From MAC 339 Pg. 1.12 and 1.13 except as shown.

(2) $P_{all} = tDF_{bru} = .040 \times .250 \times 163{,}000 = 1630\#$ where F_{bru} is from the table in Lesson No. 6.

(3) $M.S. = (P_{all}/P) - 1$

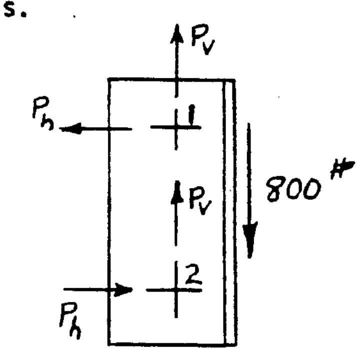
b) The load is 800# as in Problem a). The rivet loads are found as follows. The vertical applied load is divided equally between the two rivets in a flange because they are of equal stiffness. The moment is reacted as a couple on the two rivets. The total load on each rivet is the resultant of these two loads.

For rivets #1 and #2:

$P_v = \frac{P}{2} = \frac{800}{2} = 400\#$

$\Sigma M_2 = 0 = 800 \times .40 - \frac{P_h}{1.6}$

$P_h = 200\#$



Resultant load:

$P_1 = P_2 = (400^2 + 200^2)^{1/2} = 447\#$

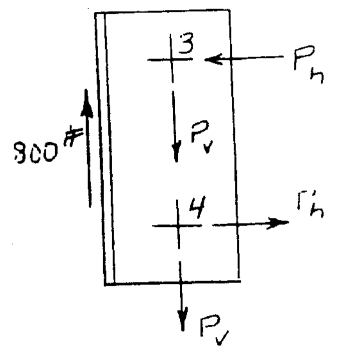
For rivets #3 and #4:

$P_v = \frac{P}{2} = \frac{800}{2} = 400\#$

$M_4 = 0 = 800 \times .60 - \frac{P_h}{.90}$

$P_h = 300\#$

$\underline{P_3 = P_4} = (400^2 + 300^2)^{1/2} = \underline{500\#}$



Rivet No.	P	Trial Rivet	P_{all} (1) in Clip	Mating Structure	P_{all} (1) in Structure	Fastener P_{all}	M.S. (2)
1 & 2	447	AD5 (BJ5)	594	.125 7178-76	596	594	+.33
3 & 4	500	AD5 (BJ5)	594	.040 7178-76	575	575	+.15

(1) From MAC 339 Pg. 1.12 and 1.13.

(2) M.S. = $(P_{all}/P) - 1$

c) The vertical load is divided equally between the three rivets because they are of equal stiffness.

$P_v = \frac{1500}{3} = 500\#$

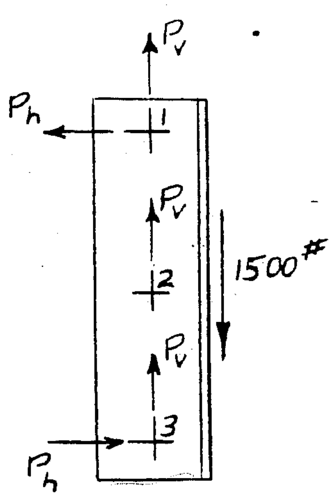
$\Sigma M_2 = 0 = .30 \times 1500 - 2.0\ P_h$

$P_h = 225\#$

Resultant rivet loads are:

$\underline{P_1 = P_3} = (500^2 + 225^2)^{1/2} = \underline{548\#}$

$\underline{P_2} = P_v = \underline{500\#}$



8.8 a) Because the clip has only a single bolt rather than a row of bolts, the effective bolt spacing is the clip width which is 1 inch. Bolt head clearance is 0.18 in. and thickness is .090.

Enter the chart on MAC 339 page 12.01 with t = .090. Go up vertically to the curve for bolt spacing = 1.0" and from there to the right horizontally to the curve for bolt head clearance = .18 (interpolating between the .15 and .20 curves). Move vertically downward from this point and read yield load per bolt of 600 lbs.

$\underline{P_{all_{ult}}} = 1.5\ P_{all_{yield}} = 1.5 \times 600 = \underline{900\#}$

b) This problem is solved using MAC 339 page 12.02. The parameters are: $T_1 = T_2 = .090$, W=1.0 inch and e=.435 inch.

Enter the chart with $T_1 = .090$, go up to the $T_2 = .090$ curve, right to the D=.25 curve and down to read ultimate load of 1.5 kips. This is the allowable load only if the clip has the following geometric proportions shown on MAC 339 page 12.02: W=4D and e=1.75D. For this clip, W=1.0=4 X .25=4D and e=.438=1.75 x .25=1.75D. Therefore, it is not necessary to use the factors K_1 and K_2 of MAC 339 page 12.00.

$\underline{P_{all_{ult}}} = 1.5 \text{ kips} = \underline{1500\#}$

c) This problem is solved using MAC 339 page 12.01.01. The parameters are thickness = .090, bolt spacing = 1" and bolt head clearance = 0.18 inch. Enter the chart with t=.090, go up to the bolt spacing = 1" curve, right to the bolt head clearance = .18 (interpolated) curve and down to read yield load per bolt of 760 lbs. The double clip has two bolts and the chart gives the yield load per bolt. ∴

$\underline{P_{all_{ult}}} = 1.5 \times 2 \times 760 = \underline{2280\#}$

d) This problem is solved using MAC 339 page 12.02.02. The parameters are $\frac{T_1}{H-R/2-D/2}$, $\frac{T_1}{D}$ and $\frac{T_1}{R}$.

H = Distance from bolt centerline to edge of flange = .435"

T_1 = Thickness of flange in bending = .090"

R = Radius of tee fillet = .180"

D = Diameter of bolt shank = .25"

Compute the values of:

$$\frac{T_1}{H - R/2 - D/2} = \frac{.090}{.435 - .180/2 - .25/2} = .409$$

$$\frac{T_1}{D} = \frac{.090}{.25} = .36$$

$$\frac{T_1}{R} = \frac{.090}{.180} = .50$$

Enter the curve in MAC 339 on page 12.02.02 with $\frac{T_1}{H - R/2 - D/2}$ = .409. Go up vertically to the T_1/D = .36 line, then left horizontally to the T_1/R = .50 line. Move down from this point and read the value of P/D^2 = 53 ksi. Solve for $P_{all} = 53(D)^2 = 53(.25)^2$ = 3.31 kips = 3310 lbs. Bolt spacing = 4D = 4 x .25 = 1.0.

$P_{allow_{ult}}$ = 3310 lbs.

e) Enter the chart on MAC 339 page 12.02 with T_1 = .25, go up to the interpolated curve for T_2 = .125, right to the D= .38 curve and down to read ultimate load of 4.6 kips. As in problem b), this is the allowable load only if W=4D=4x.375=1.5 inch and if e=1.75D=1.75x.375=.656 inch.

The actual values are W=1.25 and e=.55. Therefore, using the equations on MAC 339 page 12.00:

$$n = W/D = (1.25/.375) = 3.333$$

$$K_1 = (n-1)/3 = (3.33-1)/3 = .778$$

$$m = e/D = (.55/.375) = 1.467$$

$$K_2 = 1.75/m = (1.75/1.467) = 1.193$$

$P_{all_{ult}} = P_{chart} \times K_1 \times K_2$ = 4600 x .778 x 1.193 = 4270#

3.9 a) The allowable load is the allowable bearing load:

$P_{br_{all}} = DtF_{br}$

F_{br} is the allowable bearing stress based on R/D or W/D, whichever is critical.

R/D = 1.00/1.00 = 1.0

F_{br}/F_{tu} = .78 (from MAC 339 pg. 12.12)

F_{tu} = 61,000 psi (from table in Lesson No. 6)

F_{br} = .78 x 61,000 = 47,600 psi (based on R/D)

W/D = 2.00/1.00 = 2.0 (W = 2R ∴ not critical)

$\underline{P_{br_{all}}} = DtF_{br}$ = 1.00 x .750 x 47,600 = $\underline{35,700\#}$

b) The allowable load is the allowable bearing load in member (1) or (2) or the fastener shear strength, whichever is the lowest.

Member (1) allowable bearing load:

e/D = .35/.25 = 1.40 = R/D

F_{br}/F_{tu} = 1.23 (from MAC 339 pg. 12.12)

F_{tu} = 86,000 psi (from table in Lesson No. 6)

$F_{br} = 1.23\ F_{tu}$ = 1.23 x 86,000 = 105,800 psi

Member (1) $P_{br_{all}} = DtF_{br}$ = .250 x .10 x 105,800 = 2645#

Member (2) allowable bearing load:

e/D = .30/.25 = 1.20 = R/D

F_{br}/F_{tu} = 1.02 (from MAC 339 pg. 12.12)

F_{tu} = 59,000 psi (from table in Lesson No. 6)

$F_{br} = 1.02\ F_{tu}$ = 1.02 x 59,000 = 60,200 psi

Member (2) $P_{br_{all}} = DtF_{br}$ = .250 x .160 x 60,200 = 2408#

Check the bolt:

$P_{s_{all}}$ = 4650# per MAC 339 pg. 1.33

Thus, the joint strength is:

$\underline{P_{all} = 2408\#}$

c) The allowable load is the allowable bearing load on the lug.

$R/D = .65/.50 = 1.30$

F_{tu} = 88,000 psi per table in Lesson No. 6

$F_{br} = 1.13\ F_{tu} = 1.13 \times 88{,}000 = 99{,}400$ psi (from MAC 339 pg. 12.12)

For W/D the grain direction is long transverse, as shown on MAC 339 page 12.12. The table on MAC 339 page 12.13 specifies the use of lug curve #2 for a 7178-T6 extrusion with LT grain direction. Therefore, on MAC 339 page 12.12:

$W/D = 1.20/.50 = 2.40$

$F_{br} = 1.34\ F_{tu} = 1.34 \times 88{,}000 = 117{,}900$ psi

If θ was equal to zero the allowable load would be:

$P_o = DtF_{br_{min}} = .50 \times .375 \times 99{,}400 = 13{,}640\#$

However:

$\theta = 90 - 35 = 55°$

$P_\theta/P_o = .828$ per MAC 339 pg. 12.16.

$P_\theta = .828\ P_o = .828 \times 18{,}640 = \underline{15{,}430\#}$

8.10 a) The allowable load is found by using the chart on page 12.15 of MAC 339. The parameters are F_{br_1}, F_{br_2}, e/D and the degree of clamp-up.

For lugs (1), the female or outside lugs:

$R/D = .90/.50 = 1.8$

$F_{br_1} = 1.58\ F_{tu}$ from MAC 339 pg. 12.12

F_{tu} = 61,000 psi from table in Lesson No. 6

$F_{br_1} = 1.58 \times 61{,}000 = \underline{96{,}400 \text{ psi}}$

$W = 1.80 = 2R$ $\therefore$ W/D is not critical.

For lug (2), the center or male lug:

$R/D = .60/.50 = 1.20$

$\underline{F_{br_2}} = 1.02\ F_{tu} = 1.02 \times 61.00 = \underline{62,200\ psi}$

W/D is not critical as for lugs (1) .

The maximum gap at worst tolerance is:

$x\text{-}d = (1.56+.03) - (1.50-.03) = .12$

Because the male lug can be on one side of the slot the entire gap can be on one side. This would create a more critical bending moment on the bolt than for the male lug being centered, so use:

$e = .12$

$\underline{e/D} = .12/.50 = \underline{.24}$

Enter the chart on page 12.15 of MAC 339 with F_{br_2} = 62.2 ksi and move upward to an interpolated curve for F_{br_1} = 96.4 ksi. From this point, move horizontally to the curve labeled "160 ksi unclamped" (the second curve from the right) and then downward from that point to an interpolated curve for e/D = .24. From this point move horizontally to the curve labeled "unclamped" and from there downward to read K = .88. Then:

Bolt $P_{all} = KV_{all}$

V_{all} = 18,650# from MAC 339 pg. 1.33

Bolt P_{all} = .88 x 18,650 = 16,400#

Check lug (1) :

Lug (1) $P_{all} = 2DtF_{br_1} = 2 \times .500 \times .42 \times 96,400 = 40,500\#$

Check lug (2) :

Lug (2) $P_{all} = DtF_{br_2} = .50 \times 1.50 \times 62,200 = 46,650\#$

The allowable load is the lowest of these three allowable loads. ∴

$\underline{P_{all} = 16,400\#}$

b) The torque is correct, as shown on MAC 339 page 1.34, for clamping-up with a tension nut. Follow the same procedure as above except when going to the left from the F_{br_1}, curve, go to the "160 ksi clamped tension nut" curve (the third from last curve). From that point move downward to the same interpolated e/D = .24 curve and from there horizontally to the "clamped tension nut" curve and then downward to read K = 1.34. Then:

$$\text{Bolt } P_{all} = 1.34\ V_{all} = 1.34 \times 18{,}650 = 24{,}990\#$$

Because the allowable lug bearing loads are larger than this allowable bolt load:

$$\underline{P_{all} = 24{,}990\#}$$

c) Permissible clamp-up (gap) is determined from MAC 339 page 12.14. The parameters are L/t, material and grain direction. The table on MAC 339 page 12.13 tells which curve to use on page 12.14 for various alloys and grain directions. For 2024-T4 plate, curve number 5 is used regardless of grain direction.

$$L/t = 2.5/.42 = 5.95$$

Enter the chart on MAC 339 page 12.14 with L/t = 5.95. Move upwards to curve 5 and, from there, move to the left and read:

$$\frac{x-d}{t} = .10$$

Allowable gap = (x-d) = .10t = .10 x .38 = .038 inch. The nominal gap is .06 and the maximum gap is .12.

$\therefore$ <u>Clamp-up is not permissible</u>.

d) The gap in the sketch is .06 + .06, so the maximum gap is .12 inch. The required lug length for this gap is found by reversing the procedure of the preceding problem.

$$x-d = .12$$

$$\frac{x-d}{t} = .12/.42 = .286$$

Enter the chart on MAC 339 page 12.14 with (x-d)/t = .286 and move horizontally to curve 5. From that point move downward to read:

$L/t = 10.1$ Then:

$$\underline{L} = 10.1\ t = 10.1 \times .42 = \underline{4.24 \text{ inches}}$$

SOLUTION TO PROBLEMS

LESSON 9 CRIPPLING

PROBLEM 9.1

- First find the critical crippling load and stress for the angle if it is alclad 7075-T6 sheet metal.

Element	b	t (2)	b/t	EDGE COND.	Fcc (1)	bt	btFcc
(1)	1.45	.1	14.5	1 e.f.	31,000	.145	4495
(2)	.95	.1	9.5	1 e.f.	43,000	.095	4085
						.240	8580

(1) Ref: MAC 339 Pg. 16.16 (2) Cladding = 2.5% per side

$P_{cc} = \Sigma \text{ bt } F_{cc} = 8580 \text{ lbs.}$

$F_{cc} = \frac{\Sigma \text{ bt } F_{cc}}{\Sigma \text{ bt}} = \frac{8580}{.24} = 35{,}750 \text{ psi}$

- Next, find the critical crippling load and stress for the angle if it is bare 7075-T6 sheet metal

$F_{cy} = 80{,}000$ psi
$E = 10.3 \times 10^6$ psi
} from table in lesson 6

F_{cc} will be found for each flange using the curve on page 16.14 of MAC 339:

Element	b	t	$\frac{F_{cy}}{E}(\frac{b}{t})$	EDGE COND.	$\frac{F_{cc}}{F_{cy}}$ ⚠1	Fcc	bt	btFcc
① ②	1.45 .95	.1 .1	1.27 .84	1 e.f. 1 e.f.	.47 .65	37,600 52,000	.145 .095 .240	5452 4940 10392

⚠1 Ref: MAC 339, Pg. 16.14

$P_{cc} = \Sigma\ bt\ F_{cc} = 10392$ lbs.

$$F_{cc} = \frac{\Sigma\ bt\ F_{cc}}{\Sigma\ bt} = \frac{10392}{.24} = 43{,}300 \text{ psi}$$

- Finally find the critical crippling load and stress for the angle if it is a 7075-T6 extrusion.

Element	b	t	b/t	EDGE COND.	Fcc ⚠1	bt	btFcc
① ②	1.45 .95	.1 .1	14.5 9.5	1 e.f. 1 e.f.	34,800 49,000	.145 .095 .240	5046 4655 9701

⚠1 Ref: MAC 339, Pg. 16.15

$P_{cc} = \Sigma\ bt\ F_{cc} = 9701$ lbs.

$$F_{cc} = \frac{\Sigma\ bt\ F_{cc}}{\Sigma\ bt} = \frac{9701}{.24} = 40{,}420 \text{ psi}$$

PROBLEM 9.2

- First, find the critical crippling load and stress for the 7075-T6 extruded tee.

Element	b	t	b/t	EDGE COND.	Fcc (1)	bt	btFcc
(1)	1.469	.15	9.8	1 e.f.	47,000	.220	10,340
(2)	1.0	.063	15.9	1 e.f.	32,000	.063	2,016
(3)	1.0	.063	15.9	1 e.f.	32,000	.063	2,016
						.346	14,372

(1) Ref: MAC 339, Pg. 16.15

$P_{cc} = \Sigma \, bt \, F_{cc} = 14{,}372$ lbs.

$$F_{cc} = \frac{\Sigma \, bt \, F_{cc}}{\Sigma \, bt} = \frac{14{,}372}{.346} = 41{,}537 \text{ psi}$$

- Next, find the critical crippling load and stress for the 7075-T6 extruded channel.

Element	b	t	b/t	EDGE COND.	Fcc (1)	bt	btFcc
(1)	.825	.080	10.3	1 e.f.	46,000	.066	3,036
(2)	1.43	.15	9.5	No e.f.	78,000	.214	16,692
(3)	.825	.080	10.3	1 e.f.	46,000	.066	3,036
						.346	22,764

(1) Ref: MAC 339, Pg. 16.15

$P_{cc} = \Sigma \, bt \, F_{cc} = 22{,}764$ lbs.

$$F_{cc} = \frac{\Sigma \, bt \, F_{cc}}{\Sigma \, bt} = \frac{22{,}764}{.346} = 65{,}792 \text{ psi}$$

PROBLEM 9.3

- Find the Fcy and E for extruded 7075-T6 material at 300°F (10 hr. exposure)

  1) Room temperature properties from table in lesson 6

     Fcy = 61,000 psi   Ref: MIL-HDBK-5, Pg. 3-287

     $E = 10.4 \times 10^6$ psi

  2) Temperature reduction factors

     $K_{Fcy} = .77$ (MIL-HDBK-5, Pg. 3-291)

     $K_E = .90$ (MIL-HDBK-5, Pg. 3-293)

  $Fcy = Fcy_{RT}\, K_{cy} = 61{,}000\,(.77) = 46{,}970$ psi

  $E = E_{RT}\, K_E = 10.4 \times 10^6\,(.90) = 9.36 \times 10^6$ psi

Fcc will be found for each flange using the curve on page 16.14 of MAC 339.

Element	b	t	$\frac{Fcy}{E}(\frac{b}{t})$	EDGE COND.	$\frac{Fcc}{Fcy}$ (1)	Fcc	bt	btFcc
(1)	1.469	.15	.69	1 e.f.	.76	35,697	.220	7,853
(2)	1.0	.063	1.13	1 e.f.	.51	23,955	.063	1,509
(3)	1.0	.063	1.13	1 e.f.	.51	23,955	.063	1,509
							.346	10,871

(1) Ref: MAC 339, Pg. 16.14

$P_{cc} = \Sigma\, bt\, F_{cc} = 10871$ lbs.

$$F_{cc} = \frac{\Sigma\, bt\, F_{cc}}{\Sigma\, bt} = \frac{10871}{.346} = 31{,}419 \text{ psi}$$

9.4 Use p. 16.28.09 of MAC 339:

Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
①	1.469	.15	9.8	1 e.f.	.220	83,000	18,260
②	1.0	.063	15.9	1 e.f.	.063	56,000	3,528
③	1.0	.063	15.9	1 e.f.	.063	56,000	3,528
					.346		25,316

$P_{cc} = \Sigma\ bt\ F_{cc} = 25{,}316$ lbs.

$F_{cc} = \dfrac{\Sigma\ bt\ F_{cc}}{\Sigma\ bt} = \dfrac{25{,}316}{.346} = 73{,}170$ psi

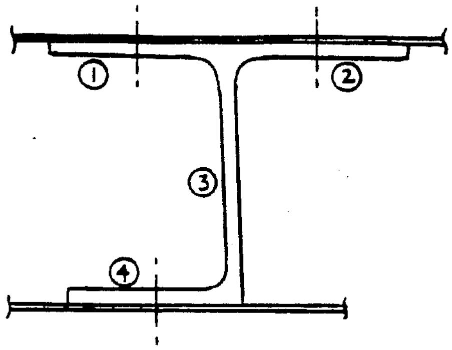
9.5 Use p. 16.28.09 of MAC 339:

Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
①	1.469	.150	9.8	1 e.f.	.220	72,000	15,840
②	1.0	.063	15.9	1 e.f.	.063	49,000	3,087
③	1.0	.063	15.9	1 e.f.	.063	49,000	3,087
					.346		22,014

$P_{cc} = \Sigma\ bt\ F_{cc} = 22{,}014$ lbs.

$F_{cc} = \dfrac{\Sigma\ bt\ F_{cc}}{\Sigma\ bt} = \dfrac{22{,}014}{.346} = 63{,}620$ psi

9.6



Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
①	1.00	.080	12.5	1 e.f.	.080	39,000	3,120
②	1.00	.080	12.5	1 e.f.	.080	39,000	3,120
③	1.425	.070	20.4	No e.f.	.100	65,000	6,500
④	.965	.070	13.8	1 e.f.	.068	36,000	2,448
					.328		15,188

$$F_{cc} = \frac{\Sigma\ bt\ F_{cc}}{\Sigma\ bt} = \frac{15,188\ \#}{.328\ in^2} = 46,300\ psi$$

Determine the effective width for the upper skins:

$$\frac{(\eta E)\ skin}{\sqrt{(\eta E)\ stiff}} = \frac{10.0 \times 10^3 KSI}{\sqrt{10.7 \times 10^3 KSI}} = 96.7 \approx 100$$

Using p. 16.13 of MAC 339:

$$\frac{2w_e}{t} = 35.5$$

For the upper skins which are one edge free:

$$\frac{w_e}{t} = .35\ \left(\frac{2w_e}{t}\right) = .35\ (35.5) = 12.4$$

$$w_e = (12.4)(.050\ in) = .62\ in.$$

The total width of skin to use for the upper skins is:

$w = 2(.62 \text{ in}) + 1.00 \text{ in.} = 2.24 \text{ in.}$

$A_5 = wt = (2.24 \text{ in})(.050 \text{ in.}) = .112 \text{ in}^2$

Determine the effective width for the lower skin:

$$\frac{(\eta E) \text{ skin}}{\sqrt{(\eta E) \text{ stiff}}} = \frac{10.5 \times 10^3 \text{KSI}}{\sqrt{10.7 \times 10^3 \text{KSI}}} = 101.5 \approx 100$$

Using p. 16.13 of MAC 339:

$$\frac{2w_e}{t} = 35.5$$

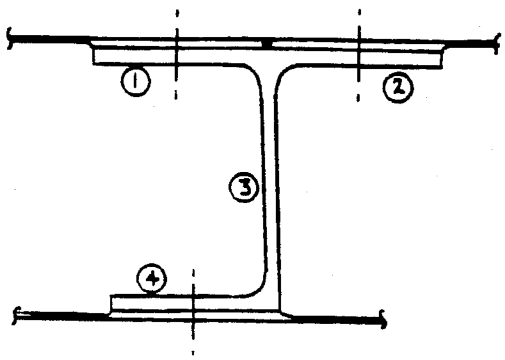
$$w_e = \frac{1}{2}(35.5)(.050 \text{ in}) = .89 \text{ in.}$$

$A_6 = 2(w_e)t = 2(.89 \text{ in})(.050 \text{ in}) = .089 \text{ in}^2$

So the total allowable crippling load is:

$P_{cc} = (46300 \text{ psi})(.328 \text{ in}^2 + .112 \text{ in}^2 + .089 \text{ in}^2) = \underline{24490 \text{ lb}}$

9.7



Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
1	1.00	.080	12.5	1 e.f.	.080	39,000	3,120
2	1.00	.080	12.5	1 e.f.	.080	39,000	3,120
3	1.425	.070	20.4	No e.f.	.100	65,000	6,500
4	.965	.070	13.8	1 e.f.	.068	36,000	2,448
					.328		15,188

$$F_{cc} = \frac{\Sigma F_{cc} bt}{\Sigma bt} = \frac{15{,}188 \ \#}{.328 \ in^2} = 46{,}300 \text{ psi}$$

Determine the effective width for the upper skins:

$$\frac{(\eta E) \text{ skin}}{\sqrt{(\eta E) \text{ stiff}}} = \frac{10.0 \times 10^3 KSI}{\sqrt{10.7 \times 10^3 KSI}} = 96.7 \approx 100$$

Using p. 16.13 of MAC 339:

$$\frac{2w_e}{t} = 35.5$$

For the upper skins which are one edge free:

$$\frac{w_e}{t} = .35 \left(\frac{2w_e}{t}\right) = 12.4$$

Since these are chem-milled skins, determine from what point the effective width should be measured.

(1) $\left(\frac{w_e}{t}\right)(.050 \text{ in}) = (12.4)(.050 \text{ in}) = .62 \text{ in.}$

(2) $\left(\frac{w_e}{t}\right)(.020 \text{ in}) + .50 \text{ in.} = (12.4)(.070 \text{ in}) + .50 \text{ in} = .748 \text{ in.}$

Since the effective width of criteria (1) is less than (2), measure the effective width off of the fastener.

So: $A_5 = 2[(.50 \text{ in})(.050 \text{ in}) + (.12 \text{ in})(.020 \text{ in})] + (.050 \text{ "})(1.0\text{"})$

$= .105 \text{ in}^2$

Determine the effective width for the lower skin:

from problem 9.6 :

$$2 \frac{w_e}{t} = 35.5$$

$$\frac{w_e}{t} = 17.75$$

Check criteria:

(1) $(\frac{w_e}{t})(.050 \text{ in}) = (17.75)(.050 \text{ in}) = .89 \text{ in.}$

(2) $(\frac{w_e}{t})(.020 \text{ in}) + .50 \text{ in.} = (17.75)(.020 \text{ in}) + .50 \text{ in} = .86 \text{ in.}$

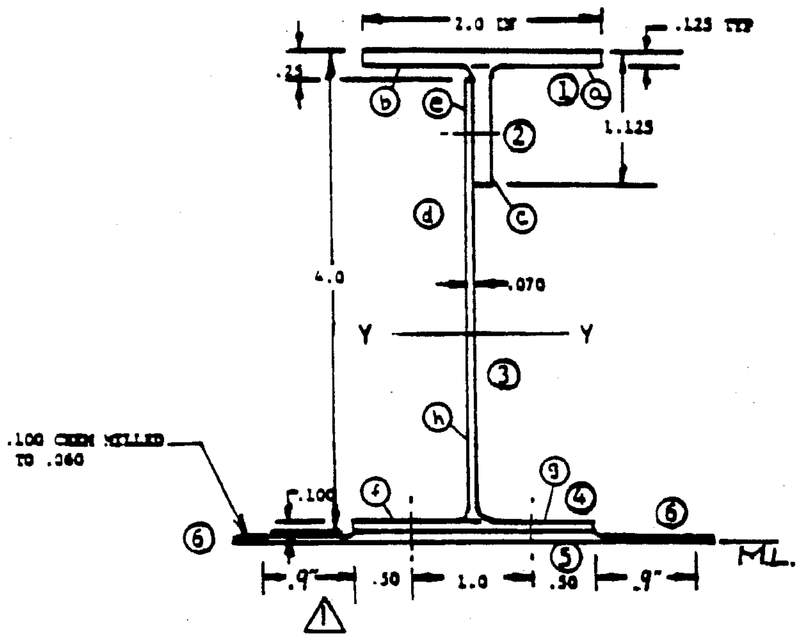
Since criteria (2) is less than (1), measure the effective width off of the end of the chem-mill land.

$A_6 = 2[(.50 \text{ in})(.050 \text{ in}) + (17.75)(.020 \text{ in})(.020 \text{ in})] = .064 \text{ in}^2$

The total allowable crippling load is:

$P_{cc} = (46300 \text{ psi})(.328 \text{ in}^2 + .105 \text{ in}^2 + .064 \text{ in}^2) = \underline{23010 \text{ lb}}$

9.8 Determine the centroid of the section:



Element	b	t	Area	y	Ay	AY^2	I_h
①	2.0	.125	.25	4.0375	1.0094	4.0754	.0003
②	1.0	.125	.125	3.475	.4344	1.5095	.0104
③	3.65	.070	.2555	2.025	.5174	1.0477	.2837
④	2.0	.100	.20	.15	.0300	.0045	.0002
⑤	2.0	.100	.20	.05	.0100	.0005	.0002
⑥	1.8	.060	.108	.03	.0032	.0001	.---
			1.1385		2.0044	6.6377	.2948

a) Area A = 1.1385 in^2

b) Centroid $\bar{y} = \frac{\Sigma\,(Ay)}{\Sigma A} = \frac{2.0044}{1.1385} = 1.76$ in.

⚠ .90" = 15 (thickness)

For the upper side of the section:

Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
a	1.00	.125	8	1 e.f.	.125	36,500	4,563
b	1.00	.125	8	1 e.f.	.125	36,500	4,563
c	1.063	.125	8.5	1 e.f.	.133	35,000	4,655
d	1.69	.070	24.1	No e.f.	.118	57,500	6,785
e	.40	.070	5.7	1 e.f.	.028	74,000	2,072
					.529		22,638

$$F_{cc} = \frac{\Sigma F_{cc}bt}{\Sigma bt} = \frac{22,638}{.529} = 42,790 \text{ psi}$$

For the lower side of the section:

Element	b	t	b/t	EDGE COND.	bt	Fcc	Fccbt
h	1.53	.070	21.9	No e.f.	.107	62,000	6,634
f	1.00	.10	10	1 e.f.	.10	47,000	4,700
g	1.00	.10	10	1 e.f.	.10	47,000	4,700
					.307		16,034

$$F_{cc} = \frac{\Sigma F_{cc}bt}{\Sigma bt} = \frac{16,034\#}{.307 \text{ in}^2} = 52,230 \text{ psi}$$

$$\frac{(\eta E) \text{ skin}}{\sqrt{(\eta E) \text{ stiff}}} \approx 100$$

Use p. 16.13 of MAC 339 to determine effective width of skin:

$$\frac{2w_e}{t} = 33.5$$

$$\frac{w_e}{t} = .35\ (33.5) = 11.7$$

Determine whether to measure off of fastener or chem-mill land:

(1) (11.7)(.100 in) = 1.17 in.

(2) (11.7)(.060 in) + .50 in. = 1.20 in.

Since (1) < (2), measure effective width off of fastener.

A_{skin} = (2.00 in)(.100 in) + 2(.67 in)(.060 in) = .280 in^2

This area of the skin is assumed to act at the average stiffener stress:

Fcc = 52230 psi

Pcc = (52230 psi)(.307 in^2 + .280 in^2) = 30659 lb

LESSON 10 - PROBLEM SOLUTIONS

1. First determine ρ

$$\rho = \sqrt{\frac{I}{A}}$$

$$A = 2\pi R_{AVG}\, t = 2\pi\ (.234\ in)(.032\ in) = .047\ in^2$$

$$I = \pi R_{AVG}^3\, t = \pi(.234\ in)^3(.032\ in) = .0013\ in^4$$

$$\rho = \sqrt{\frac{.0013\ in^4}{.047\ in^2}} = .166\ in.$$

This strut will be treated as a pin-ended column.

$$L' = L = 5\ in$$

$$\frac{L'}{\rho} = \frac{5\ in}{.166\ in} = 30.1$$

Using p. 16.31 of MAC 339, note that this value of L'/ρ falls in the short column region.

To determine the allowable crippling stress for this section, refer to the inset of the upper figure or p. 3-404 of MIL-HDBK-5D:

$$\frac{D}{t} = \frac{.50\ in}{.032\ in} = 15.6$$

so $F_{cc} = 49$ KSI

Using p. 16.31 of MAC 339, begin on the vertical axis at F_{cc} = 48 KSI. Follow a parabolic curve, interpolating between the plotted curves, to the value of $\frac{L'}{\rho}$ = 30.1 on the horizontal axis. Following a horizontal line back to the vertical axis from this point, gives F_c = 42.5 KSI.

Alternatively, the equation for the Johnson curve could be used to get the same result:

$$F_c = F_{cc} - \frac{(F_{cc})^2}{4\pi^2 E}\left(\frac{L'}{\rho}\right)^2 = 48\text{ KSI} - \frac{(48\text{ KSI})^2}{4(3.14)^2(10.2\text{x}10^6\text{psi})}(30.1)^2$$

$\underline{F_c = 42.8\text{ KSI}}$

Determine Allowable Load:

$\underline{P_{cr}}$ = (42.5 KSI)(.047 in^2) = $\underline{2000\text{ lb}}$

2. ρ will be the same as in problem 1.

$L' = L = 15$ in

$\frac{L'}{\rho} = 90.3$

The allowable crippling stress, F_{cc} = 48 KSI (from problem 1)

Using p. 16.31 of MAC 339, begin at F_{cc} = 48 KSI, read over on the Johnson curve, follow it along the Euler curve until the value of $\frac{L'}{\rho}$ = 90.3 is reached. Reading over horizontally from this point gives an F_c = 12 KSI.

Alternatively the Euler curve equation can be used to determine F_c:

$$Fc = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2} = \frac{(3.14)^2(10.2 \times 10^6 \text{ psi})}{(90.3)^2} = 12.3 \text{ KSI}$$

The allowable load is

$$P_{cr} = (12 \text{ KSI})(.047 \text{ in}^2) = 564 \text{ lb}$$

3. a) Determine section properties

$$A = (.399")(.399") - (.335")(.335") = .047 \text{ in}^2$$

$$I = \frac{(.399")^4}{12} - \frac{(.335")^4}{12} = .00106 \text{ in}^4$$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{.00106}{.047}} = .150 \text{ in}$$

b) Determine the allowable crippling stress:

Each of the four sides of the section can be considered to be a no-edge-free element with b = .378" and t = .032".

This gives $\frac{b}{t} = 11.8$. Entering the crippling curve or page 16.19 of MAC 339 gives

$$F_{cc} = F_{cy} = 57.0 \text{ KSI}$$

Using the curve on p. 16.31 at MAC 339:

$$\frac{L'}{\rho} = \frac{5.0''}{.150''} = 33.3$$

$$F_c = \underline{48.0 \text{ KSI}}$$

$$P_{cr} = (48.0 \text{ KSI})(.047 \text{ in}^2) = \underline{2256 \text{ lb}} \text{ ultimate}$$

4. **Solution:**

$\rho = .166$ in (See problem 1)

$$L' = \frac{L}{\sqrt{C}} = \frac{5 \text{ in}}{\sqrt{2.05}} = 3.49 \text{ in}$$

$$\frac{L'}{\rho} = \frac{3.49 \text{ in}}{.166 \text{ in}} = 21.0$$

referring to the inset of the upper figure on p. 3-404 of MIL-5D:

$$\frac{D}{t} = \frac{.50 \text{ in}}{.032 \text{ in}} = 15.6$$

$$F_{cc} = 48 \text{ KSI}$$

Using p. 16.31 of MAC 339,

$F_c = 45.8$ KSI

$P_{cr} = (45.8 \text{ ksi})(.047 \text{ in}^2) = \underline{2153 \text{ lb}}$

5. Solution: $\rho = .166$ in (See problem 1)

$$L' = \frac{L}{\sqrt{C}} = \frac{5 \text{ in}}{\sqrt{4}} = 2.5''$$

$$\frac{L'}{\rho} = \frac{2.50 \text{ in}}{.166 \text{ in}} = 15.1$$

referring to the inset of the upper figure on p. 3-404 of MIL-5D:

$$\frac{D}{t} = \frac{.50 \text{ in}}{.032 \text{ in}} = 15.6$$

$F_{cc} = 48$ KSI

using p. 16.31 of MAC 339:

$F_c = \underline{47 \text{ KSI}}$

$P_{cr} = (47 \text{ KSI})(.047 \text{ in}^2) = 2209$ lb

6. **Solution**: $\rho = .166$ in (See problem 1)

$$L' = \frac{L}{\sqrt{C}} = \frac{5 \text{ in}}{\sqrt{.25}} = 10 \text{ in}$$

$$\frac{L'}{\rho} = \frac{10 \text{ in}}{.166 \text{ in}} = 60.2$$

referring to the inset of the upper figure on p. 3-404 of MIL-5D:

$$\frac{D}{t} = \frac{.50 \text{ in}}{.032 \text{ in}} = 15.6$$

$$F_{cc} = 48 \text{ KSI}$$

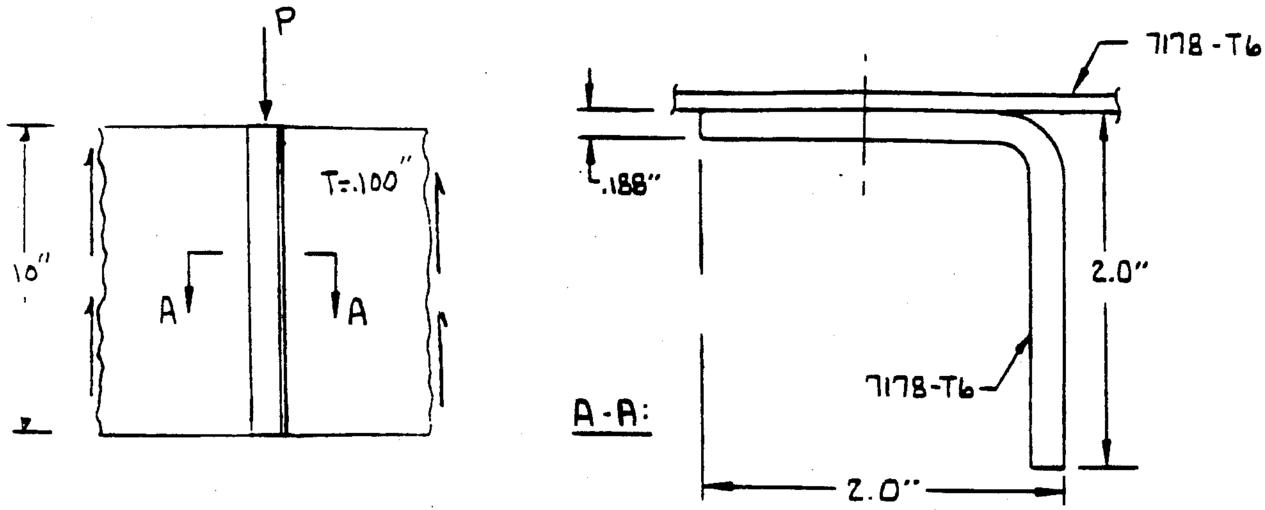
Using p. 16.31 of MAC 339:

$$F_c = 27 \text{ KSI}$$

The ultimate allowable load is:

$$P_{cr} = (27 \text{ KSI})(.047 \text{ in}^2) = 1269 \text{ lb}$$

7. Solution:



Determine the allowable crippling stress for the section:

$$\frac{b}{t} = \frac{1.906''}{.1875''} = 10.2$$

Since this is an equal leg angle stiffener, the average stress for the total section is equal to F_{cc} computed for any leg. From p. 16.23 of MAC 339:

$$F_{cc} = 48.0 \text{ KSI}$$

Determine ρ :

First determine the moment of inertia and area of the stiffener-web combination:

$\eta = 1$ $\quad E = 10.7 \times 10^6$ psi

Using p. 16.13 of MAC 339

$$\frac{2\,w_e}{t} = 35.5$$

$$W_e = 1/2\ (35.5)\ (.10\ \text{in}) = 1.78\ \text{in}$$

Ele	b	t	A	y	Ay	Ay^2	I
1	2.00	.188	.376	1.10	.414	.4554	.1253
2	1.81	.188	.340	.194	.066	.0128	.0010
3	3.56	.100	.356	.050	.018	.0009	.0003
			1.072		.498	.4691	.1266

$$\bar{y} = \frac{Ay}{A} = \frac{.498}{1.072} = .465''$$

$$I_{NA} = I + Ay^2 - \bar{y}^2 A = .1266\ \text{in}^4 + .4691\ \text{in}^4 - (1.072\ \text{in}^2)\ (.465\ \text{in})^2 = .364\ \text{in}^4$$

$$= \frac{I}{A} = \frac{.364\ \text{in}^4}{1.072\ \text{in}^2} = .582\ \text{in}$$

So $\frac{L'}{\rho} = \frac{10 \text{ in}}{.582 \text{ in}} = 17.2$

from figure on p. 16.31 at MAC 339:

and using $F_{cc} = 48.0$ KSI and $\frac{L'}{\rho} = 17.2$

gives F_c = 46.5 KSI

$P_{cr} = (46.5 \text{ KSI})(1.072 \text{ in}^2) = 49{,}850 \text{ lb} = P_J$ (Johnson critical column load)

Using fig. 16.38 with $\frac{P_1}{P_2} = 0$ gives $\frac{P_2}{P_E} = 1.92$

Using the Johnson critical column load to determine the allowable column load for a uniformly unloaded column, P_2:

$P_2 = \left(\frac{P_2}{P_E}\right)(P_J)$, $P_2 = (1.92)(49850 \text{ lb}) = 95710 \text{ lb}$

The stress produced by this load is well beyond the elastic limit as well, and much higher than the crippling load. So the allowable load for this section is the crippling load:

$P_c = (48.0 \text{ KSI})(1.073 \text{ in}^2) = \underline{51.5 \text{ Kips}}$

8. Solution: For 2024-T3

$F_{cy} = 34.0$ KSI

$A_2 = 2\pi r_{avg} t = (2\pi)(.451 \text{ in})(.049 \text{ in}) = .139 \text{ in}^2$

$I_2 = \pi R_{AVG}{}^3 t = \pi(.451 \text{ in})^3(.049 \text{ in}) = .014 \text{ in}^4$

$E_2 = 10.2 \times 10^6$ psi

For 4340 Steel End Fitting:

$F_{cy} = 240$ KSI

$A_1 = \pi\left(\frac{.375 \text{ in}}{2}\right)^2 = .110 \text{ in}^2$

$I_1 = \frac{\pi\left(\frac{.375''}{2}\right)^4}{4} = 9.71 \times 10^{-4} \text{ in}^4$

$E_1 = 29.0 \times 10^6$ psi

$\frac{EI_1}{EI_2} \quad \frac{(29.0 \times 10^6 \text{ psi})(9.71 \times 10^{-4} \text{ in}^4)}{(10.2 \times 10^6 \text{ psi})(.014 \text{ in}^4)} = .197$

Using the figure on page 16.42 of MAC 339:

with: $\frac{A}{L} = \frac{14 \text{ in}}{18 \text{ in}} = .78$

$\frac{P_{cr}}{P_E} = .92$

$P_E = \frac{\pi^2 (EI)_2}{(L)^2} = \frac{(3.14)^2(10.2 \times 10^6 \text{ psi})(0.014 \text{ in}^4)}{(18 \text{ in})^2} = 4350 \text{ lb}$

$P_{cr} = (.92)(4350 \text{ lb}) = \underline{4000 \text{ lb}}$

9. Solution: Using the beam column analysis p. 16.50 and 16.51 of MAC 339:

$P_{ALL} = \left(\frac{P_{ALL}}{Pcr}\right) P_{cr}$

where $\frac{P_{all}}{Pcr} = -\frac{b - \sqrt{b^2 - 4ac}}{2a}$

where $a = \frac{P_{cr}}{Fcc\,A}$

$b = -\left(\frac{P_{cr}}{Fcc\,A} + \frac{P_{cr}\,Y_o}{M_{ALL}} - \frac{M_o}{M_{ALL}} + 1\right)$

$c = -\frac{M_o}{Mall} + 1$

From p. 18.23 of MAC 339

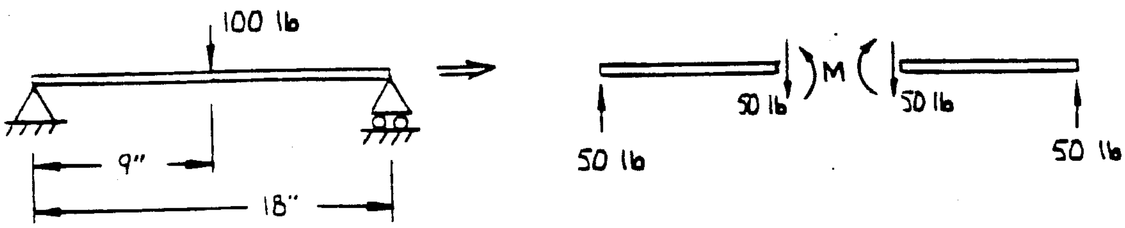
$M_{allow} = 2215 \text{ in} \cdot \text{lb}$

$F_{cc}A = 6610 \text{ lb}$

From problem 8:

$$P_{cr} = 4000 \text{ lb}$$

For a simply supported beam



$$Y_o = \frac{(100 \text{ lb})(18 \text{ in})^3}{48(10.2 \times 10^6 \text{ psi})(.014 \text{ in}^4)} = .0851 \text{ in (Ref: Roark p. 106)}$$

$$M_o = \underline{(50 \text{ lb})(9 \text{ in}) = 450 \text{ in lb}}$$

Determine the quadratic formula parameters:

$$a = \frac{P_{cr}}{F_{cc}A} = \frac{4000 \text{ lb}}{6610 \text{ lb}} = .605$$

$$b = -\left[\frac{P_{cr}}{F_{cc}A} + \frac{P_{cr}Y_o}{M_{all}} - \frac{M_o}{M_{all}} + 1\right] = -\left[\frac{4000 \text{ lb}}{6610 \text{ lb}} + \frac{(4000 \text{ lb})(.0851 \text{ in})}{2215 \text{ in lb}} - \frac{450 \text{ in lb}}{2215 \text{ in lb}} + 1\right]$$

$$b = -[.605 + .154 - .203 + 1] = -1.556$$

$$c = -\frac{M_o}{M_{all}} + 1 = -\frac{450 \text{ in lb}}{2215 \text{ in lb}} + 1 = .797$$

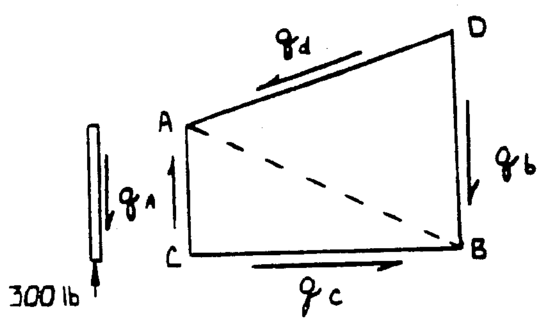
$$\frac{P_{all}}{P_{cr}} = \frac{1.556 - \sqrt{(-1.556)^2 - 4(.605)(.797)}}{2(.605)} = .706$$

$$P_{all} = (.706)(4000 \text{ lb}) = \underline{2824 \text{ lb}}$$

SHEAR FLOW HOMEWORK PROBLEMS

Solutions

11.1 a)



Solution: $q_a = \dfrac{300 \text{ lb}}{3.0 \text{ in}} = 100 \text{ lb/in}$

Take Moments about Pt. B:

$\Sigma M_B = 2A_{ADB}\, q_d - 2A_{ACB} q_a = 0$

$(6 \text{ in})(5 \text{ in}) q_d = (6 \text{ in})(3 \text{ in})(100 \text{ lb/in})$

$q_d = 60 \text{ lb/in}$

$q_c = 60 \text{ lb/in}$

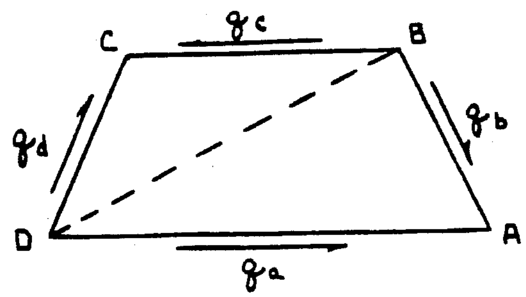
Sum moments about Pt. A:

$\Sigma M_a = 2A_{ADB} q_b - 2A_{ABC} q_c = 0$

$(5 \text{ in})(6 \text{ in}) q_b = (3 \text{ in})(6 \text{ in})(60 \text{ lb/in})$

$q_b = 36 \text{ lb/in}$

11.1 b)



Solution:

Since shear flows on non-parallel sides of a trapezoid are equal.

$$q_d = 750 \text{ lb/in}$$

Sum Moments about D:

$$\Sigma M_D = 2A_{DCB}q_c - 2A_{DBA}q_b = 0$$

$$(4 \text{ in})(6 \text{ in})q_c = (4 \text{ in})(10 \text{ in})(750 \text{ lb/in})$$

$$q_c = 1250 \text{ lb/in}$$

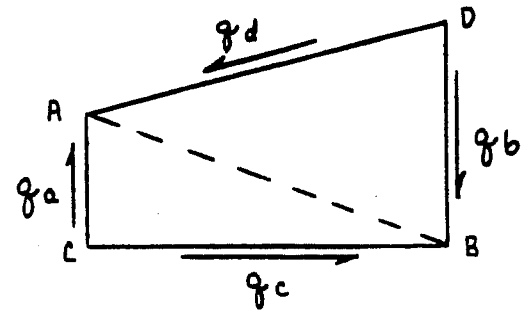
Sum Moments about B:

$$\Sigma M_B = 2A_{DCB}q_d - 2A_{DBA}q_a = 0$$

$$(4 \text{ in})(6 \text{ in})\left(750 \frac{\text{lb}}{\text{in}}\right) = (10 \text{ in})(4 \text{ in})q_a$$

$$q_a = 450 \text{ lb/in}$$

11.1 c)



Solution:

Note that this shear panel has identical heights to those in 11.1 a) but with a different base.

Sum Moments about B:

$$\Sigma M_B = 2A_{ADB}q_d - 2A_{ACB}q_a = 0$$

$$(5\text{ in})(8\text{ in})(q_d) = (8\text{ in})(3\text{ in})(100\text{ lb/in})$$

$$q_d = 60\text{ lb/in}$$

for horizontal equilibrium

$$q_c = 60\text{ lb/in}$$

Sum Moments about A:

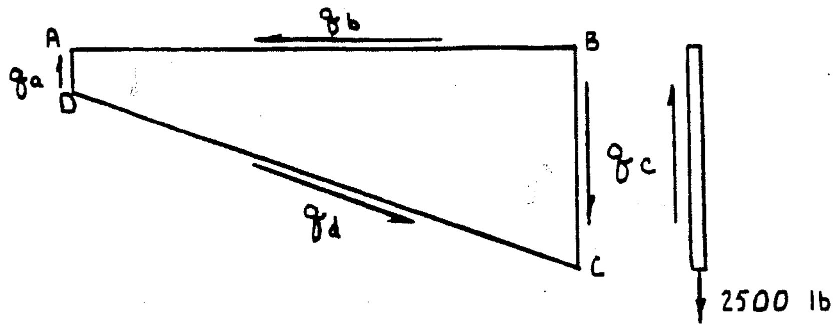
$$\Sigma M_A = 2A_{ADB}q_b - 2A_{ACB}q_c = 0$$

$$(5\text{ in})(8\text{ in})q_b = (3\text{ in})(8\text{ in})(60\text{ lb/in})$$

$$q_b = 36\text{ lb/in}$$

Recall that the values of the shear flows found in 11.1 a) are exactly the same as found here. This demonstrates that the value of the shear flows are dependent <u>only</u> on the ratio of the heights on opposite ends of the trapezoid.

11.1 d)



Solution:

$$q_c = \frac{2500\ \text{lb}}{5.00\ \text{in}} = 500\ \text{lb/in}$$

Sum Moments about B:

$$\Sigma M_B = 2A_{ADB}q_d - 2A_{BDC}q_c = 0$$

$$(11\ \text{in})(1\ \text{in})q_d = (5\ \text{in})(11\ \text{in})(500\ \text{lb/in})$$

$$q_d = 2500\ \text{lb/in}$$
$$q_b = 2500\ \text{lb/in}$$

Sum Moments about D:

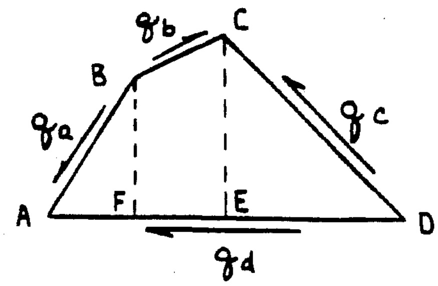
$$\Sigma M_D = 2A_{ABD}q_a - 2A_{BDC}q_b = 0$$

$$(11\ \text{in})(1\ \text{in})q_a = (11\ \text{in})(5\ \text{in})(2500\ \text{lb/in})$$

$$q_a = 12{,}500\ \text{lb/in}$$

Notice that $q_a = 25\ q_c$! This example shows that shear panels with one side very short should be avoided.

11.1 e)



Solution:

1) Find the total area of the panel

$A = A_{AFB} + A_{BCEF} + A_{CDE}$

$A = \frac{(3.0\ in)(2.0\ in)}{2} + (2.0\ in)(1/2)(3.0\ in + 4.0\ in) + \frac{(4.0\ in)(4.0\ in)}{2} = 18.0\ in^2$

2) Find A_{ABD}:

$A_{ABD} = \frac{(3.0\ in)(8.0\ in)}{2} = 12.0\ in^2$

3) Find A_{BCD}:

$A_{BCD} = A - A_{ABD} = 18.0\ in^2 - 12.0\ in^2 = 6.0\ in^2$

4) Sum Moments about D:

$\Sigma M_D = q_a(2A_{ABD}) - q_b(2A_{BCD}) = 0$

$q_a = (200\ lb/in)\ \frac{12.0\ in^2}{24.0\ in^2} = \underline{100\ lb/in}$

5) Find A_{ACD}:

$A_{ACD} = (1/2)(4.0\ in)(8.0\ in) = 16.0\ in^2$

6) Find A_{ABC}:

$A_{ABC} = A - A_{ACD} = 18.0\ in^2 - 16.0\ in^2 = 2.0\ in^2$

7) Sum Moments about A:

$\Sigma M_A = q_b(2A_{ABC}) - q_c(2A_{ACD}) = 0$

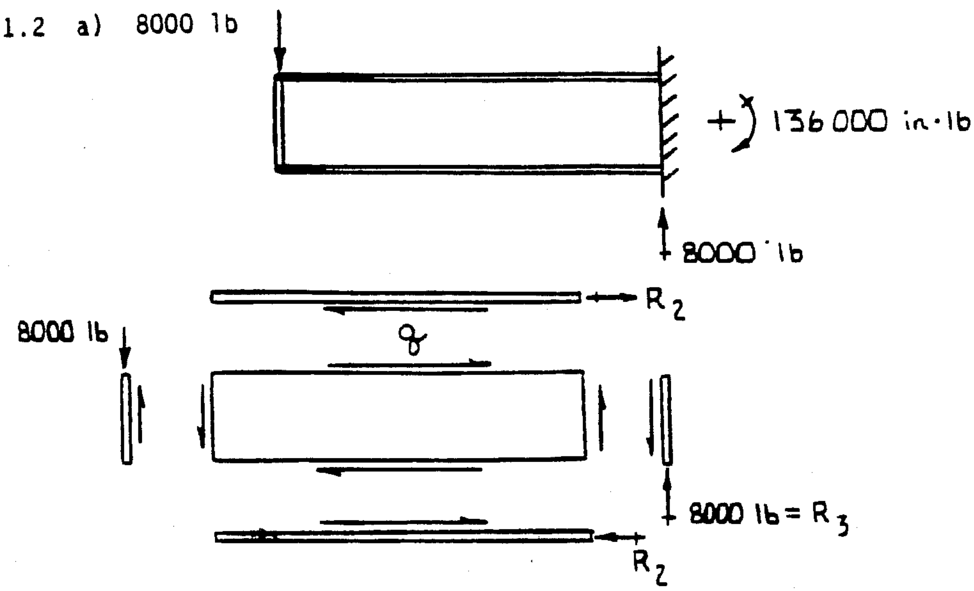
$q_c = (200\ lb/in)\ \frac{4.0\ in^2}{32.0\ in^2} = \underline{25\ lb/in}$

8) Sum Moments about B:

$\Sigma M_B = q_d\ 2A_{ABD} - q_c\ 2A_{BCD} = 0$

$q_d = (25\ lb/in)\ \frac{12.0\ in^2}{24.0\ in^2} = \underline{12.5\ lb/in}$

11.2 a)



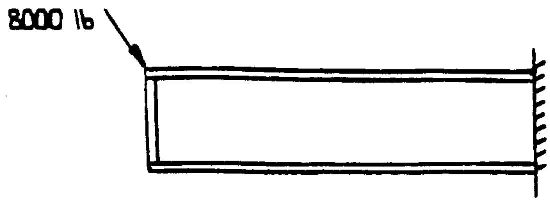
Since this beam is rectangular, the shear flow is the same on all edges.

$$q = \frac{8000 \text{ lb}}{5 \text{ in}} = 1600 \text{ lb/in}$$

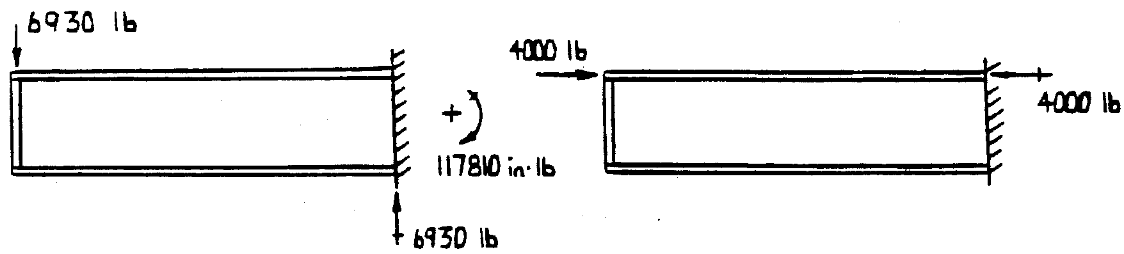
$$R_2 = (1600 \text{ lb/in})(17 \text{ in}) = 27{,}200 \text{ lb}$$

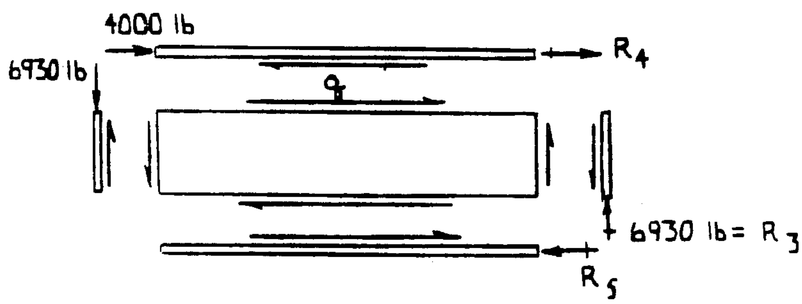
$$R_3 = 8000 \text{ lb}$$

11.2 b)



This problem can be solved by the method of superposition.





Balancing the left cap:

$$q(5 \text{ in}) = 6930 \text{ lb}$$

$$q = 1386 \text{ lb/in}$$

Balancing the upper cap:

$$R_4 = (1386 \text{ lb/in})(17 \text{ in}) - 4000 \text{ lb}$$

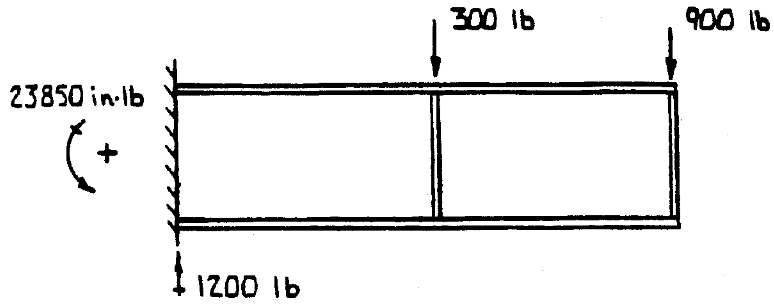
$$R_4 = 19{,}562 \text{ lb}$$

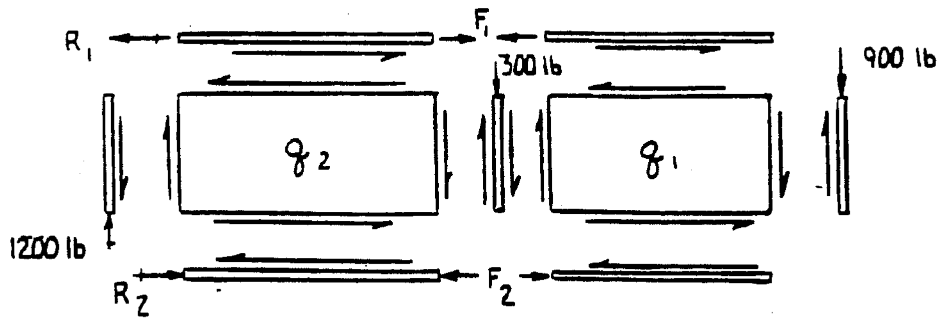
Balancing the lower cap:

$$R_5 = (1386 \text{ lb/in})(17.0 \text{ in})$$

$$R_5 = 23{,}562 \text{ lb}$$

11.2 c)





Balancing the rightmost cap

$900\ lb = (q_1)(6\ in)$

$q_1 = 150\ lb/in$

Balancing the upper and lower caps:

$F_1 = F_2 = (150\ lb/in)(10.5\ in)$

$F_1 = 1575\ lb$

$F_2 = 1575\ lb$

Balancing the middle cap:

$300\ lb + (150\ lb/in)(6\ in) = (q_2)(6\ in)$

$q_2 = 200\ lb/in$

Balancing the upper cap:

$R_1 = q_2(12.0\ in) + F_1 = (200\ lb/in)(12.0\ in) + 1575\ lb$

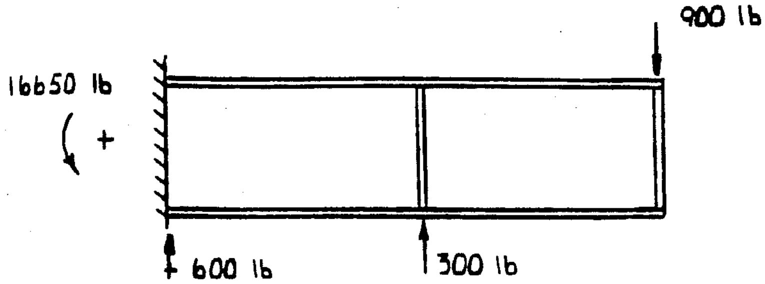
$R_1 = \underline{3975\ lb}$

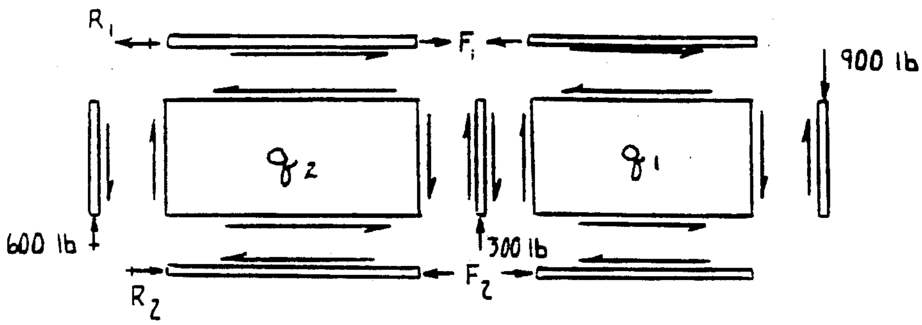
Balancing the Lower Cap:

$R_2 = (q_2)(12.0\ in) + F_2 = (200\ lb/in)(12.0\ in) + 1575\ lb$

$R_2 = \underline{3975\ lb}$

11.2 d)





Balancing the rightmost cap:

$900 \text{ lb} = q_1 \text{ (6 in)}$

$q_1 = 150 \text{ lb/in}$

Balancing the upper and lower caps:

$F_1 = (150 \text{ lb/in})(10.5 \text{ in})$

$F_2 = F_1 = 1575 \text{ lb}$

Balancing the middle cap:

$(150 \text{ lb/in})(6 \text{ in}) = 300 \text{ lb} + q_2 \text{ (6 in)}$

$q_2 = 100 \text{ lb/in}$

Balancing the lower cap:

$R_2 = F_2 + (q_2)(12.0 \text{ in}) = 1575 \text{ lb} + (100 \text{ lb/in})(12.0 \text{ in})$

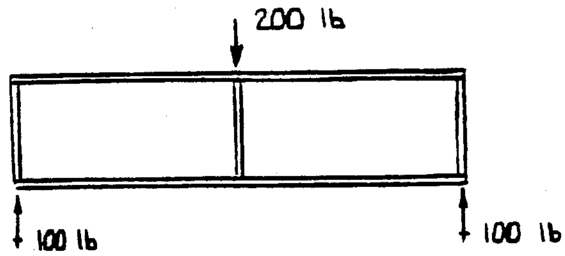
$R_2 = \underline{2775 \text{ lb}}$

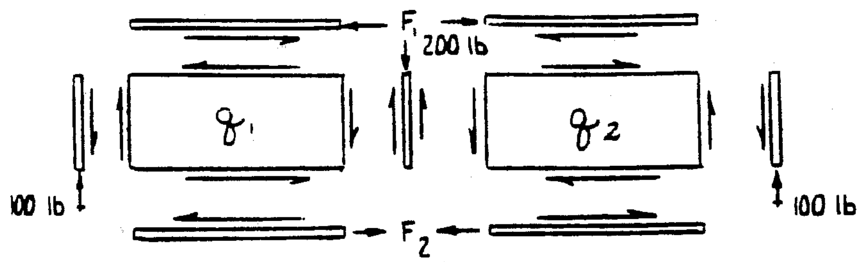
Balancing the upper cap:

$R_1 = F_1 + (q_2)(12.0 \text{ in}) = 1575 \text{ lb} + (100 \text{ lb/in})(12.0 \text{ in})$

$R_1 = \underline{2775 \text{ lb}}$

11.3 a)





Balancing the end caps:

q_1 (5 in) = 100 lb

q_1 = 20 lb/in

q_2 (5 in) = 100 lb

q_2 = 20 lb/in

Balancing the upper and lower caps

F_1 = (10 in)(20 lb/in) = 200 lb

F_2 = (10 in)(20 lb/in) = 200 lb

F_1 = 200 lb

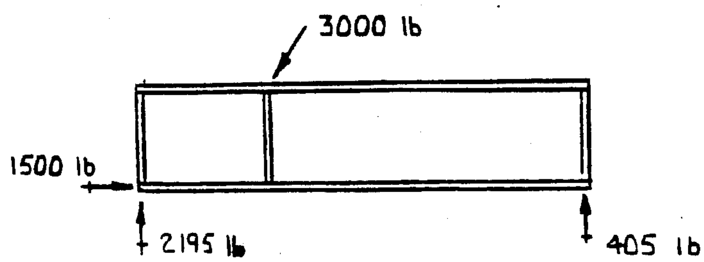
F_2 = 200 lb

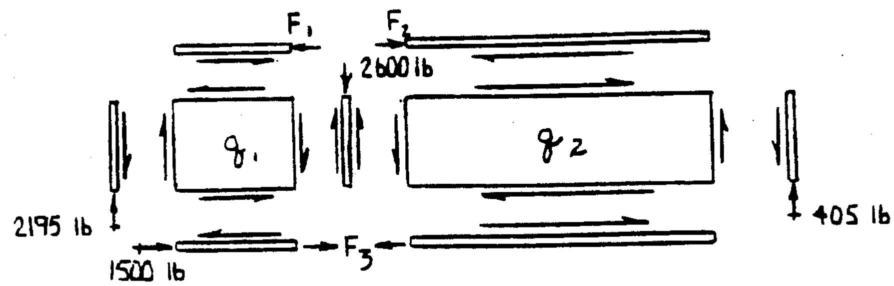
Checking the center vertical member:

q_1 (5.0 in) + q_2 (5.0 in) $\stackrel{?}{=}$ 200 lb

(20 lb/in)(5.0 in) + (20 lb/in)(5.0 in) = 200 lb

11.3 b)





Balancing the Caps:

Left Cap:

$(5\text{ in})q_1 = 2195\text{ lb}$

$q_1 = 439\text{ lb/in}$

Right Cap:

$q_2(5\text{ in}) = 405\text{ lb}$

$q_2 = 81\text{ lb/in}$

Upper Cap Above Left Hand Panel:

$(439\text{ lb/in})(6\text{ in}) = F_1$

$F_1 = 2634\text{ lb}$

Upper Cap Above Right Hand Panel:

$q_2\ (14\text{ in}) = F_2$

$(81\text{ lb/in})(14\text{ in}) = F_2 = 1134\text{ lb}$

Balance Joint above Center Stiffener

Lower Cap Below Right Hand Panel:

$(q_2)(14\text{ in}) = F_3$

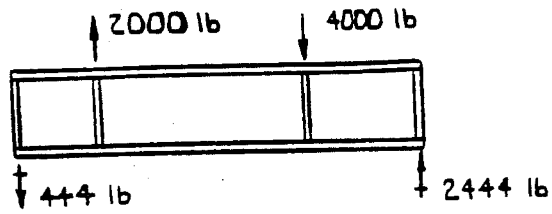
$(81\text{ lb/in})(14\text{ in}) = F_3 = 1134\text{ lb}$

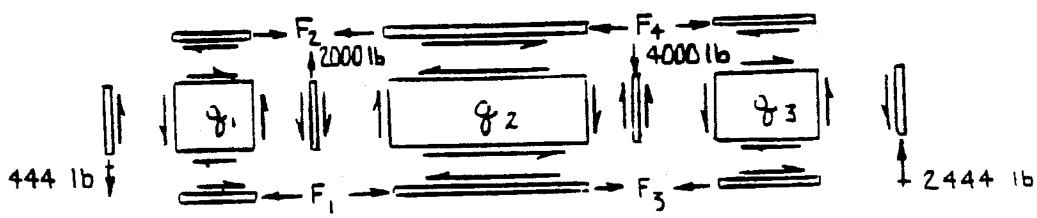
Checking the Lower Cap Below the Left Hand Panel:

$(q_1)(6.0\text{ in}) - 1500\text{ lb} \stackrel{?}{=} F_3$

$(439\text{ lb/in})(6.0\text{ in}) - 1500\text{ lb} = 1134\text{ lb} = F_3$

11.3 c)





Balance the Left Cap:

444 lb = (q_1)(4 in)

q_1 = 111 lb/in

Balance the Right Cap:

2444 lb = q_3(4 in)

q_3 = 611 lb/in

Balance the left center cap:

$(q_1 + q_2)$(4 in) = 2000 lb

q_2 = 389 lb/in

Balance the lower caps:

(111 lb/in)(4 in) = F_1 = 444 lb

By Symmetry:

$F_2 = F_1$ = 444 lb

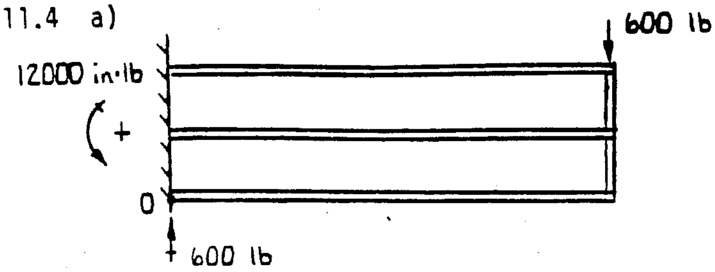
Determine F_3:

(389 lb/in)(9 in) - 444 lb = F_3 = 3057 lb

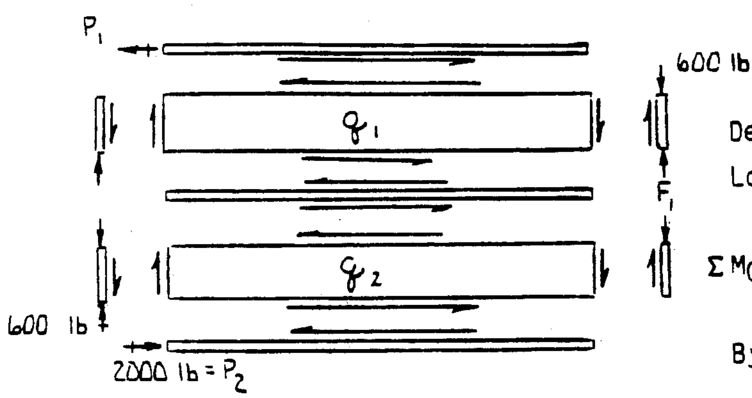
Again, by Symmetry:

$F_3 = F_4$ = 3057 lb

11.4 a)



Cap Areas = .080 in^2



Determine the Upper and Lower Cap Loads at Wall:

$$\Sigma M_0 = (600\ lb)(20\ in) - P_1(6\ in) = 0$$

$$P_1 = 2000\ lb$$

By Symmetry:

$$P_2 = 2000\ lb$$

Since the middle cap is at the neutral axis, it cannot pick up load.

So balancing the lower cap:

$$2000\ lb = (q_2)(20\ in)$$

$$q_2 = 100\ lb/in$$

Balancing the upper cap:

$$2000\ lb = q_1(20\ in)$$

$$q_1 = 100\ lb/in$$

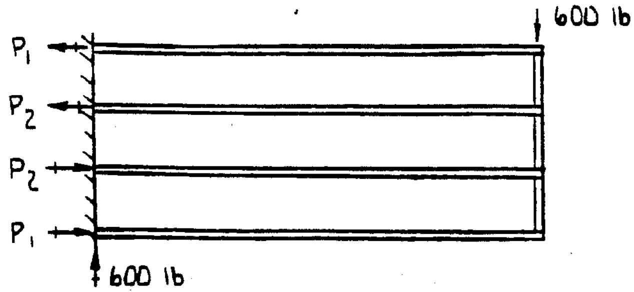
Balancing the right caps :

$$600\ lb - q_1\ (3\ in) = F_1$$

$$F_1 = 300\ lb$$

Note that the shear flow would have been the same if the center stiffener had not been there. This example illustrates that having a stiffening member at the neutral axis is useless, unless an external load is to be "dumped in" at its end. or if it is to be used as a panel breaker for diagonal tension webs (See Lesson 13).

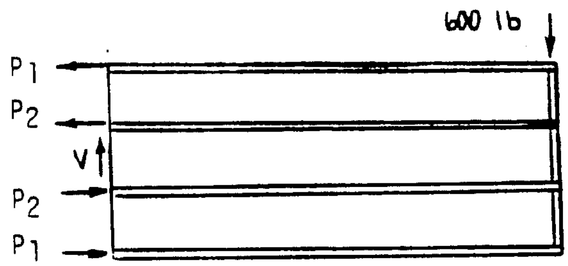
11.4 b)



Moment of Inertia:

$$I = 2(A_1 y_1^2 + A_2 y_2^2)$$
$$= 2[(.080\ \text{in}^2)(4.50\ \text{in})^2 + (.050\ \text{in}^2)(1.50\ \text{in})^2]$$
$$= 3.47\ \text{in}^4$$

An alternate method from the one presented in the text can be used to determine the panel shear flows which has the advantage of knowing the total cap loads initially:



By Geometrical Considerations:

$V = 600$ lb

Load in Upper and Lower Caps:

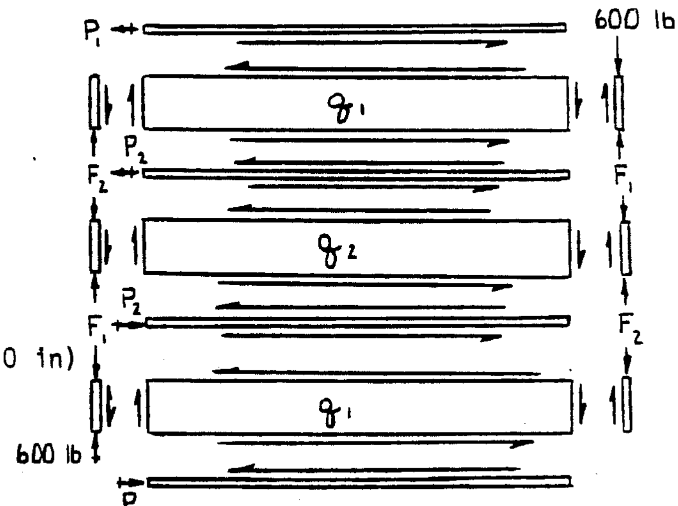
$$\sigma_1 = \frac{Mc}{I} = \frac{(12{,}000\ \text{in lb})(4.50\ \text{in})}{3.47\ \text{in}^2}$$

$$P_1 = (15560\ \text{psi})(.080\ \text{in}^2) = 1245\ \text{lb}$$

Load in the Middle Caps:

$$\sigma_2 = \frac{Mc}{I} = \frac{(12{,}000\ \text{in lb})(1.50\ \text{in})}{3.47\ \text{in}^2}$$

$$P_2 = (5190\ \text{psi})(0.50\ \text{in}^2) = 260\ \text{lb}$$



Determine Shear Flows:

Balancing the Upper Cap:

$$(q_1)(20.0\ \text{in}) = P_1 = 1245\ \text{lb}$$
$$q_1 = 62.3\ \text{lb/in}$$

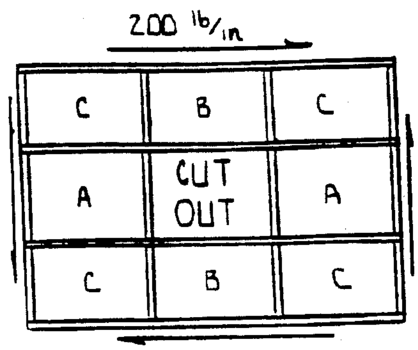
Balancing a Middle Cap:

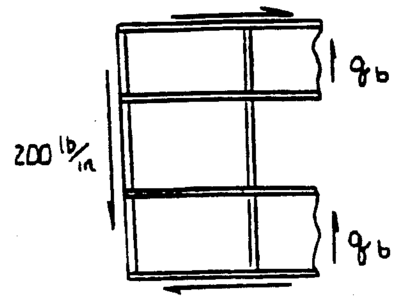
$$(q_1)(20.0\ \text{in}) + P_2 = (q_2)(20.0\ \text{in})$$
$$(62.3\ \text{lb/in})(20.0\ \text{in}) + 260\ \text{lb} = (q_2)(20.0\ \text{in})$$
$$q_2 = 75.3\ \text{lb/in}$$

Balancing the End Vertical Members:

$$(q_1)(3.0\ \text{in}) + F_1 = 600\ \text{lb}$$
$$F_1 = 600\ \text{lb} - (62.3\ \text{lb/in})(3.0\ \text{in}) = 413\ \text{lb}$$
$$(q_2)(3.0\ \text{in}) + F_2 = F_1$$
$$F_2 = 413\ \text{lb} - (75.3\ \text{lb/in})(3.0\ \text{in}) = 187\ \text{lb}$$

11.5 a)



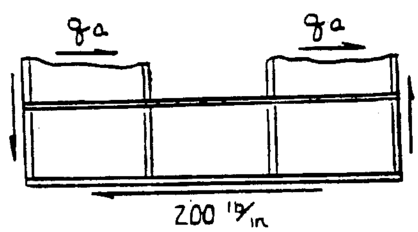


I. First determine the internal shear flow on panels adjacent to cutout:

From the sketch above:

$(q_b)(10\text{ in}) = (200\text{ lb/in})(15\text{ in})$

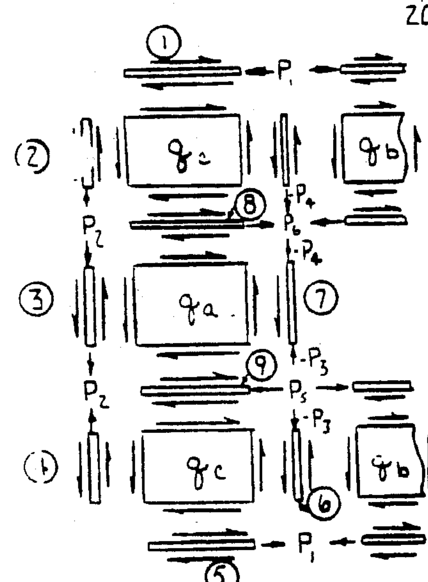
$q_b = 300\text{ lb/in}$



From the sketch at left:

$(q_a)(14\text{ in}) = (200\text{ lb/in})(21\text{ in})$

$q_a = 300\text{ lb/in}$



II. Set up free body statics equations:

** 1: $P_1 + (q_c)(7\text{ in}) = (200\text{ lb/in})(7\text{ in})$

* 2: $P_2 + (q_c)(5\text{ in}) = (200\text{ lb/in})(5\text{ in})$

3: $2P_2 + (200\text{ lb/in})(5\text{ in}) = (300\text{ lb/in})(5\text{ in})$

* 4: $P_2 + (q_c)(5\text{ in}) = (200\text{ lb/in})(5\text{ in})$

** 5: $P_1 + (q_c)(7\text{ in}) = (200\text{ lb/in})(7\text{ in})$

6: $P_3 + (q_c)(5\text{ in}) = (300\text{ lb/in})(5\text{ in})$

7: $P_3 + P_4 = (300\text{ lb/in})(5\text{ in})$

*** 8: $P_6 + (q_c)(7\text{ in}) = (300\text{ lb/in})(7\text{ in})$

*** 9: $P_5 + (q_c)(7\text{ in}) = (300\text{ lb/in})(7\text{ in})$

Unknowns: P_1, P_2, P_3, P_4, P_5, P_6, q_c

Note equations 1 and 5, 2 and 4, 8 and 9 are equivalent

Solve for unknowns:

P_2: $2P_2 + (200 \text{ lb/in})(5 \text{ in}) = (300 \text{ lb/in})(5 \text{ in})$ (3)

$$2P_2 = 500 \text{ lb}$$
$$P_2 = 250 \text{ lb}$$

q_c: $P_2 + (q_c)(5 \text{ in}) = (200 \text{ lb/in})(5 \text{ in})$ (2)

$$q_c = \frac{750 \text{ lb}}{5 \text{ in}}$$
$$q_c = 150 \text{ lb/in}$$

P_1: $P_1 + (q_c)(7 \text{ in}) = (200 \text{ lb/in})(7 \text{ in})$ (1)

$$P_1 + (150 \text{ lb/in})(7 \text{ in}) = (200 \text{ lb/in})(7 \text{ in})$$
$$P_1 = 350 \text{ lb}$$

P_3: $P_3 + (q_c)(5 \text{ in}) = (300 \text{ lb/in})(5 \text{ in})$ (6)

$$P_3 + (150 \text{ lb/in})(5 \text{ in}) = (300 \text{ lb/in})(5 \text{ in})$$
$$P_3 = 750 \text{ lb}$$

P_4: $P_3 + P_4 = (300 \text{ lb/in})(5 \text{ in})$ (7)

$$P_4 = 750 \text{ lb}$$

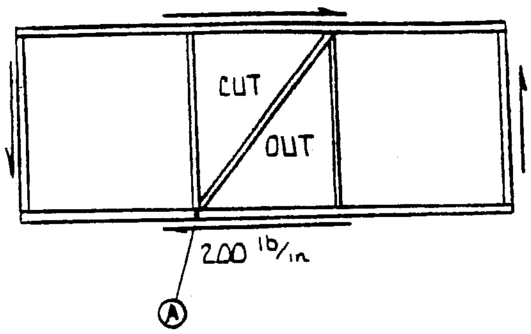
P_5: $P_5 + (q_c)(7 \text{ in}) = (300 \text{ lb/in})(7 \text{ in})$ (9)

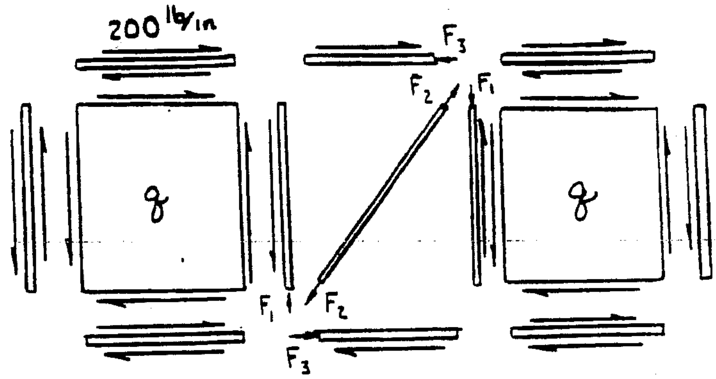
$$P_5 = 1050 \text{ lb}$$

P_6: By subtracting equation (9) from (8):

$$P_6 = P_5$$
$$P_6 = 1050 \text{ lb}$$

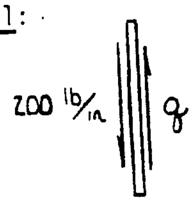
11.5 b)





First solve for the shear flow in either shear panel:

Cap 1:



$$(q)(11\ \text{in}) = (200\ \text{lb/in})(11\ \text{in})$$
$$q = 200\ \text{lb/in}$$

Next find the cap loads:

At joint A: $\Sigma F_y = 0 = F_1 - F_2 \text{Cos}\left[\tan^{-1}\left(\frac{8\ \text{in}}{11\ \text{in}}\right)\right]$ (1)

$$F_1 = .809\ F_2$$

$\Sigma F_x = 0 = F_3 - F_2 \sin\left[\tan^{-1}\left(\frac{8\ \text{in}}{11\ \text{in}}\right)\right]$ (2)

$$F_3 = .588\ F_2$$

$F_1 = (200\ \text{lb/in})(11\ \text{in})$ (3)

$F_3 = (200\ \text{lb/in})(8\ \text{in})$ (3)

Solving: $F_1 = 2200$ lb

$F_3 = 1600$ lb

$F_1 = .809\ F_2$ (1)

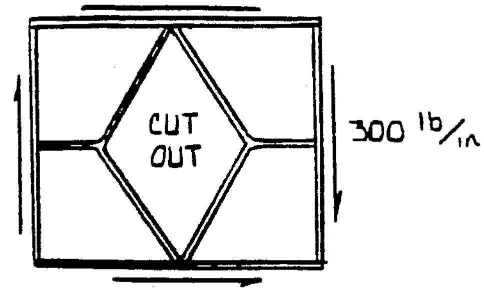
$F_2 = 2720$ lb

Check: $F_3 = .588\ F_2$

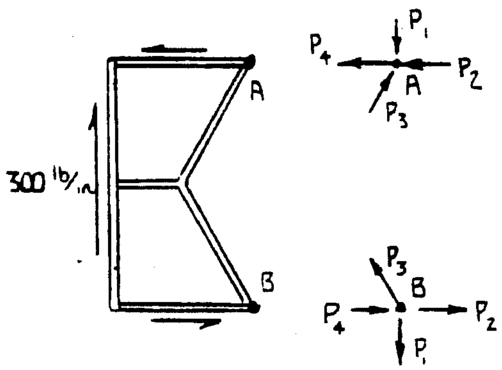
1600 lb $\stackrel{?}{=}$ (.588)(2720 lb)

1600 lb = 1600 lb ✓

11.5 c)



1) Cut Figure at A-A and balance left hand portion at point A and B



The loads at points A and B will be equal but act in opposite directions

a) Balance the structure externally determining the loads at A and B:

$$P_1 = \frac{300\ (14)}{2} = 2100 \text{ lb}$$

$$\Sigma M_B = P_2(14) + 300\ (8)(14) - (300)(8)(14) = 0$$

$$P_2 = 0$$

b) Balance the external loads P_1, and P_2 acting at point A with internal loads P_3 and P_4. The balance is the same at point B.

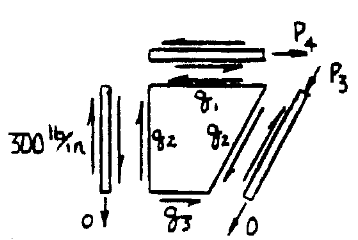
$$P_3 \cos\left(\tan^{-1} \frac{4}{7}\right) = P_1 = 2100 \text{ lb}$$

$$P_3 = 2419 \text{ lb}$$

$$P_4 = P_3 \sin\left(\tan^{-1} \frac{4}{7}\right)$$

$$P_4 = 1200 \text{ lb}$$

2) Determine the internal balance of one quadrant. The remaining quadrants have an identical balance



$$q_1(8 \text{ in}) = 300(8 \text{ in}) - 1200 \text{ lb}$$

$$q_1 = 150 \text{ lb/in}$$

$$q_2 = q_1\left(\frac{8 \text{ in}}{4 \text{ in}}\right) = 300 \text{ lb/in}$$

$$q_3 = q_2\left(\frac{8 \text{ in}}{4 \text{ in}}\right) = 600 \text{ lb/in}$$

SOLUTIONS TO PROBLEMS

LESSON 12 BEAMS AND OTHER BENDING MEMBERS

12.1 The moment of inertia of the section is:

$$I_{x-x} = 2\,A_{cap}\,\bar{y}^2 + \frac{t(h_{web})^3}{12}$$

$$= 2\left[2.5(.20)\left(\frac{4.0}{2} - \frac{.20}{2}\right)^2\right] + \frac{.10\,[4.0 - 2(.20)]^3}{12}$$

$$= 3.610 + .388 = 3.998 \text{ IN}^4$$

The extreme fiber stress is:

$$f_b = \frac{M(\pm c)}{I} = \frac{80000(\pm 2.0)}{3.998} = \pm\ 40020 \text{ psi}$$

The static moment of the area above the neutral axis is:

$$Q = A_1\,\bar{y}_1 + A_2\,\bar{y}_2 = (2.5)(.20)\left(\frac{4.0}{2} - \frac{.20}{2}\right) + (2.0 - .20)(.10)\left(\frac{2.0 - .20}{2}\right)$$

$$= .950 + .162 = 1.112 \text{ IN}^3$$

The shear stress at the neutral axis is:

$$f_s = \frac{VQ}{It} = \frac{11000(1.112)}{3.998(.10)} = 30600 \text{ psi}$$

Material allowables - Ref. Lesson 6, Page 6-4

$F_{TU} = 61$ KSI

$F_{SU} = 31$ KSI

The minimum margin of safety is:

$$M.S. = \frac{F_{SU}}{f_s} - 1 = \frac{31.0}{30.6} - 1 = \underline{+.013}$$

12.2 The allowable plastic moment is:

$M_{ALL} = (Q_1 + Q_2)F_{RB}$ where Q_1 and Q_2 are the static moments about the neutral axis.

$Q_1 = Q_2 = 1.112 \text{ IN}^3$ (REF. PROBLEM 12.1)

$f_o/F_{TU} = .84$ for 2024-T4 extrusion (MAC 339, Page 14.02)

$$K = \frac{Q_1 + Q_2}{I/C}$$

$I = 3.998$ IN $C = 2.0$ IN (Ref. Problem 12.1)

$$K = \frac{(1.112 + 1.112)}{3.998/2.0} = 1.11$$

Find F_{RB} from page 14.01 of MAC. 339

$$\frac{F_{RB}}{F_{TU}} = .985$$

$F_{RB} = .985\ (61000) = 60100$ psi

By the procedures of Lesson 9 the allowable crippling stress of the compression flange is found to be:

$F_{cc} = 45560$ psi

Calculate M_{ALL}

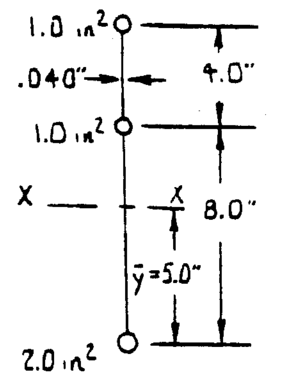
$M_{ALL} = (2 \times 1.112)(45560) = 101330$ "#

$M = 80000$"#

$$M.S. = \frac{M_{ALL}}{M} - 1 = \frac{101330}{80000} - 1 = \underline{+\ .266}$$

12.3 For convenience, a section one inch from the free end of the beam is used.

Find the neutral axis



$$\bar{y} = \frac{\Sigma Ay}{A}$$

$$= \frac{(1.0)(12.0) + 1.0(8.0)}{1.0 + 1.0 + 2.0} = 5.0 \text{ IN}$$

$$I = \Sigma Ay^2$$

$$= 1.0(7.0)^2 + 1.0(3.0)^2 + 2.0(-5.0)^2$$

$$= 108 \text{ IN}^4$$

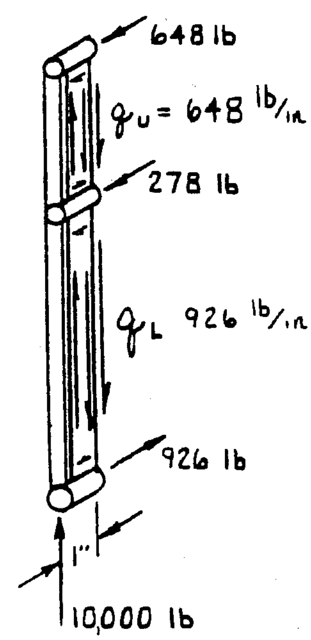
Find cap loads and shear flow on 1.0 in width:

$M = 10000(1.0) = 10000$ "#

UPPER CAP:

$$P_1 = \frac{MyA}{I} = \frac{10000(7.0)(1.0)}{108}$$

$= 648$#



INTERMEDIATE CAP:

$$P_2 = \frac{MyA}{I} = \frac{10000(3.0)(1.0)}{108}$$

$= 278$#

LOWER CAP:

$$P_3 = \frac{10000(-5.0)(2.0)}{108} = 926\text{\#}$$

Find the panel shear stress, upper panel:

$$q_u = \frac{P_1}{1.0} = \frac{648}{1.0} = 648 \ \#/\text{IN}$$

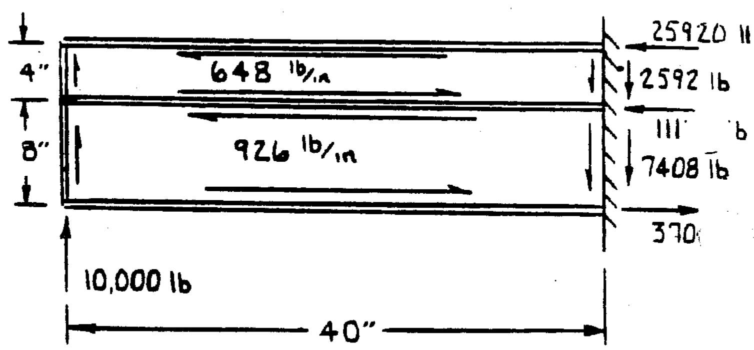
$$f_s = \frac{648}{.040} = 16200 \text{ psi}$$

LOWER PANEL

$$q_L = \frac{926}{1.0} = 926 \#/\text{IN}$$

$$f_s = \frac{926}{.040} = 23150 \text{ psi}$$

12.4 The support reactions are found using the information from problem 12.3 :



Upper Axial Reaction

$R_u = 648(40) = 25920\#$

Intermediate Axial Reaction

$R_I = 278(40) = 11120\#$

Lower Axial Reaction

$R_L = 926(40) = 37040\#$

$$\Sigma F_H = 0 = 25920 + 11120 - 37040$$

Upper Shear Reaction

$V_U = 648(4.0) = 2592\#$

Lower Shear Reaction

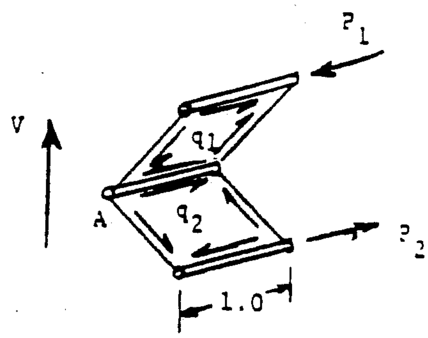
$V_L = 926(8) = 7408\#$

$$\Sigma F_V = 0 = 10000 - 2592 - 7408$$

12.5

Figure 1

$h = 2(6.0) \sin 45° = 8.485$ in.



$$P_1 = P_2 = \frac{M}{h} = \frac{10{,}000 \times 1.0}{8.485} = \underline{1178\#}$$

(Note: $\frac{Mc}{I}$ would give the same cap load.)

By considering the loads on the upper and lower cap, it can be seen that:

$$q_1 = q_2 = \frac{\Delta P}{L} = \frac{1178}{1.00} = \underline{1178\#/''}$$

$\Sigma M_A = Ve - 2Aq = 0$ [Reference Lesson 12, p.11]

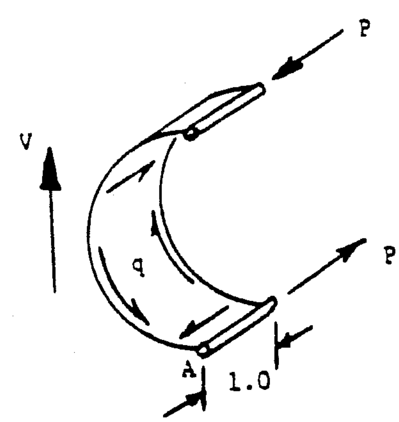
$\Sigma M_A = 10{,}000\, e - 2(0)q = 0$

$\underline{e = 0}$

Figure 2

$$P = \frac{M}{h} = \frac{10{,}000 \times 1.0}{6.0} = \underline{1667\#}$$

$$q = \frac{\Delta P}{L} = \frac{1667}{1.0} = \underline{1667\#/''}$$



$\Sigma M_A = Ve - 2aq = 0$

$$= 10{,}000\, e - 2\left(\frac{\pi \times 3.0^2}{2}\right)1667$$

$\underline{e = 4.71 \text{ in.}}$

Note: For a section with a constant shear flow, the shear flow can also be found from the equation:

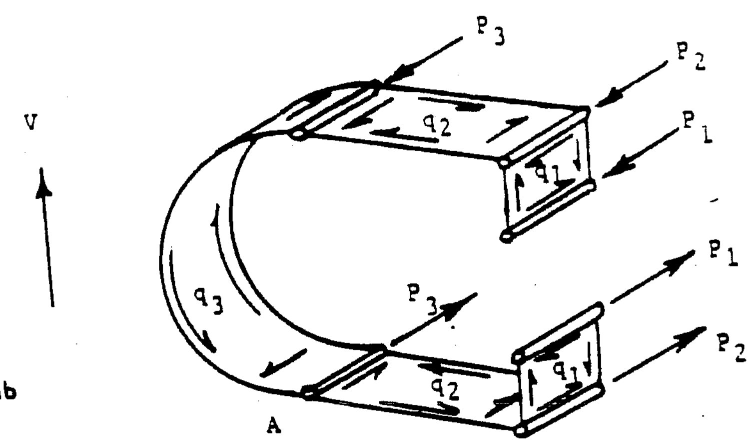
$$q = \frac{V}{h} = \frac{10{,}000}{6.0} = 1667\#/"$$

and the shear center location can be found directly from the equation:

$$e = \frac{2A}{h} = \frac{2}{h}\left[\frac{\pi (3)^2}{2}\right] = 4.71 \text{ in.}$$

Figure 3

Because the caps are not all the same distance from the neutral axis, the cap loads must be found from the equation:



$$P = f_b A = \frac{Mc}{I} A$$

$$I = 2[(.50)(2.0)^2 + 2\,(.50)(3.0)^2]$$

$$I = 22 \text{ in}^4$$

$$M = 20{,}000 \times 1.0 = 20{,}000 \text{ in}\cdot\text{lb}$$

$$P_1 = \frac{Mc_1}{I} A = \frac{20{,}000(2.0)(.5)}{22} = \underline{909\#}$$

$$P_2 = P_3 = \frac{Mc_2}{I} A = \frac{20{,}000(3.0)(.5)}{22} = \underline{1364} \text{ lb}$$

The shear flows are determined by considering the loads acting on the caps, starting with the caps at the free edges (cap #1).

At cap #1:

$\Sigma F_H = P_1 - Lq_1 = 0$

$= 909 - 1.0\ q_1 = 0$

$q_1 = 909\ \#/"$

At cap #2:

$\Sigma F_h = P_2 + Lq_1 - Lq_2 = 0$

$= 1364 + 1.0 \times 909 - 1.0\ q_2 = 0$

$q_2 = 2273\#/"$

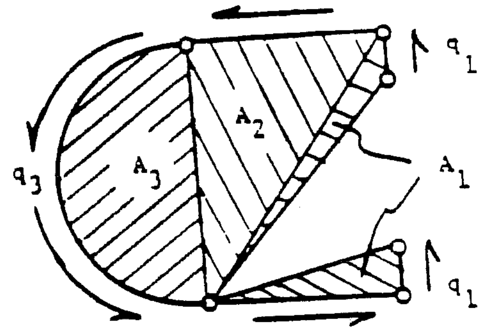
At cap #3:

$\Sigma F_h = P_3 + Lq_2 - Lq_3 = 0$

$= 1364 + 1.0 \times 2273 - 1.0\ q_3 = 0$

$q_3 = 3637\#/"$

The shear center is found by summing moments of the shear flows about any point on the cross-section.



$$\Sigma M_A = Ve - 2A_1q_1 - 2A_2q_2 - 2A_3q_3 = 0$$

$$\Sigma M_A = 20{,}000\ e - 2(4.0)\ 909 - 2\left(\frac{4.0 \times 6.0}{2}\right)2273 - 2\left[\frac{\pi\ (3.0)^2}{2}\right]3637 = 0$$

$e = 8.23"$

12.6 Because of symmetry, 1/2 of the 10000 lb. load will be reacted on each side of the center line.

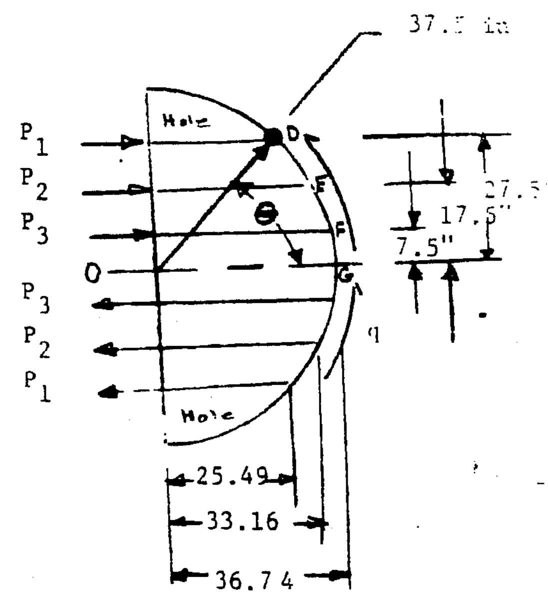
Find the reacting shear flows from the equation:

$$q = \frac{V}{h} = \frac{5000}{55.0} = 90.9 \text{ \#/" per side}$$

The loads in the horizontal members are found by treating the bulkhead as a beam. Find the bending moment at the center by taking the moment of the shear flow about any point on the vertical centerline of the bulkhead using the equation:

$$M = 2Aq$$

The area (A) is a function of angle θ.



$$\theta_{DG} = \arcsin\left(\frac{27.5}{37.5}\right) = 47.17°$$

$$\theta_{FG} = \arcsin\left(\frac{7.5}{37.5}\right) = 11.54°$$

$$\theta_{EG} = \arcsin\left(\frac{17.5}{37.5}\right) = 27.82°$$

$$\theta_{DE} = \theta_{DG} - \theta_{EG} = 47.17 - 27.82 = 19.35°$$

$$\theta_{EF} = \theta_{EG} - \theta_{FG} = 27.82 - 11.54 = 16.28°$$

$$A_{DEO} = (37.5)^2 \sin\frac{19.35}{2}\cos\frac{19.35}{2} = 232.97 \text{ in}^2$$

$$A_{EFO} = (37.5)^2 \sin\frac{16.28}{2}\cos\frac{16.28}{2} = 197.108 \text{ in}^2$$

$$A_{FGO} = (37.5)^2 \sin\frac{11.54}{2}\cos\frac{11.54}{2} = 140.66 \text{ in } 2$$

For moments about point O, the area is:

$$A_O = 2\,(A_{DEO} + A_{EFO} + A_{FGO}) = 1141.5 \text{ in}^2$$

$$M_O = 2\,A_O q = 2\,(1141.5)\,(90.9) = 207,525 \text{ in lbs}$$

$$I = 2A(7.5^2 + 17.5^2 + 27.5^2) = 2237.5A$$

$$P_1 = \frac{207.525\,(27.5A)}{2237.5A} = 2550.6 \text{ lb.}$$

$$P_2 = \frac{207,525\,(17.5A)}{2237.5A} = 1623.1 \text{ lb.}$$

$$P_3 = \frac{207,525\,(7.5A)}{2237.5A} = 695.6 \text{ lb.}$$

Find the panel shear flows by putting the horizontal stiffeners on one side of the vertical centerline in static equilibrium (starting with the top or bottom stiffener) and by balancing the shear panels as trapezoidal panels, neglecting the ML curvature.

Upper member:

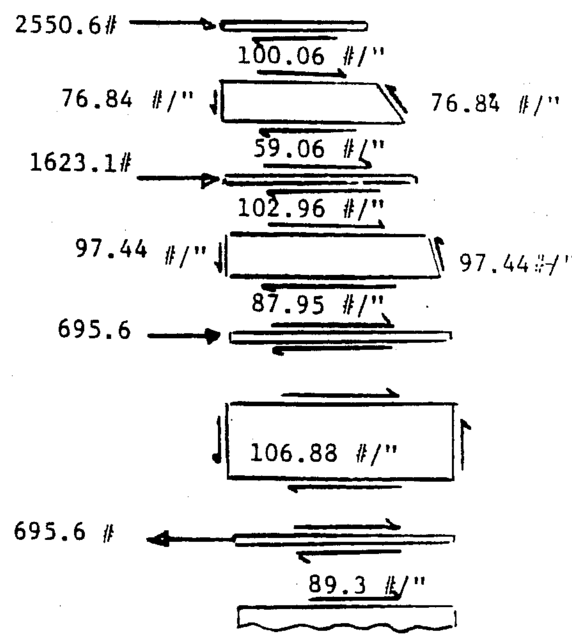
$F_h = 2550.6 - 25.49q = 0$

$q = 100.06$ #/"

Upper panel:

end $q_e = q_u \frac{b}{a} = 100.06 \frac{25.49}{33.19} = 76.84$ #/"

lower $q_1 = q_u \frac{b}{a}^2 = 100.06 \left(\frac{25.49}{33.19}\right)^2 = 59.01$ #/"



Next member:

$F_h = 1623.1 + 33.16 \times 59.01 - 33.16 = 0$

$q = 107.96$

Next panel:

end $q_e = 107.96 \left(\frac{33.16}{36.74}\right) = 97.44$ #/"

lower $q_e = 107.96 \left(\frac{33.16}{36.74}\right)^2 = 87.95$ #/"

Last member:

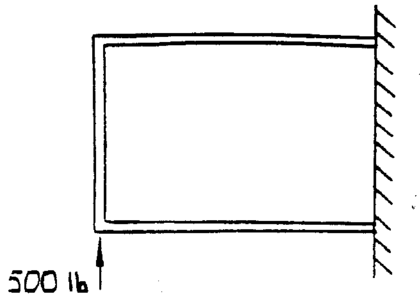
$F_h = 695.6 + 36.74 \times 87.95 - 36.74q =$

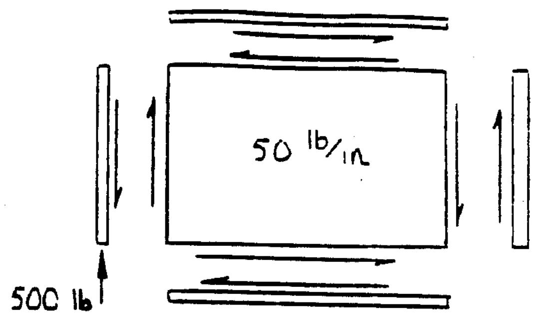
$q = 106.88$ #/"

Center panel: This panel is rectangular so the skin is uniform with Q=106.88 lb/in on all four sides.

ABDR - ALLOWABLE SHEAR HOMEWORK SOLUTIONS

1a)





Solution:

$$\tau = \frac{q}{t} = \frac{50.0 \text{ lb/in}}{.040 \text{ in}} = 1250 \text{ psi}$$

Find $\frac{Is}{het^3}$: $\frac{Is}{het^3} = \frac{.0016 \text{ in}^4}{(15.00 \text{ in})(.040 \text{ in})^3} = 1.67$

$\frac{he}{d}$: $\frac{he}{d} = \frac{15.00 \text{ in}}{10.00 \text{ in}} = 1.500$

From Figure 1:

$K_s = 6.1$

Now:

$$\tau_{cr} = K_S E \frac{t^2}{d}$$

$$\tau_{cr} = (6.1)(10.3 \times 10^6 \text{ psi})\left(\frac{.040 \text{ in}}{10.00 \text{ in}}\right)$$

$$\tau_{cr} = 1000 \text{ psi}$$

$$\frac{\tau}{\tau_{cR}} = \frac{1250 \text{ psi}}{1000 \text{ psi}} = 1.25$$

Find τ_{all}:

From Fig. 2.5:

$\tau_{all} = 31.5$ KSI for $\frac{\tau}{\tau_{cR}} = 2.0$

1b) for 2500# Loading:

$$\tau = \frac{2500\ \text{lb}}{(10.00\ \text{in})(.040\ \text{in})} = 6250\ \text{psi}$$

$$\text{Find } \frac{qd^2}{t^3} = \frac{(250.0\ \text{lb/in})(10.00\ \text{in})^2}{(.040\ \text{in})^3} = 3.91 \times 10^8\ \text{psi}$$

For Figure 2.5:

$$\frac{\tau}{\tau_{CR}} = 5.8$$

or computing directly:

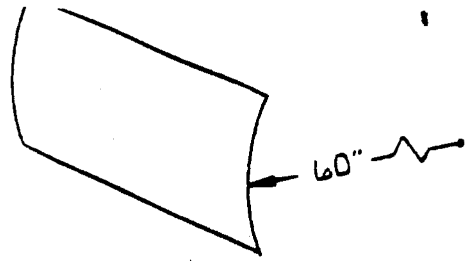
$$\frac{\tau}{\tau_{CR}} = \frac{6250\ \text{psi}}{1000\ \text{psi}} = 6.25$$

Reading off the figure gives:

$\tau_{all} = 28.5$ KSI | $\tau_{all} = 28.4$ KSI

Note, that the allowable shear is relatively insensitive to changes in the ratio of τ/τ_{CR} for 7075-T6 for $\tau/\tau_{CR} > 5.0$.

1c)



a) Find τ_{cr}:

$$\frac{a}{b} = \frac{15.00 \text{ in}}{10.00 \text{ in}} = 1.500$$

$$\frac{b^2}{rt} = \frac{(10.00 \text{ in})^2}{(60.00 \text{ in})(.040 \text{ in})} = 41.7$$

From Fig. 17:

$K_S = 14.5$

$$F_{cr} = \frac{K_S E}{\left(\frac{b}{t}\right)^2} = \frac{(14.5)(10^3 \times 10^6 \text{ psi})}{\left(\frac{10.00 \text{ in}}{.040 \text{ in}}\right)^2} = 2390 \text{ psi}$$

b) Find τ : $q = \frac{2500 \text{ lb}}{10 \text{ in}} = 250 \text{ lb/in}$

$$\tau = \frac{250 \text{ lb/in}}{.040 \text{ in}} = 6250 \text{ psi}$$

c) Loading Ratio:

$$\frac{\tau}{\tau_{CR}} = \frac{6250 \text{ psi}}{2390 \text{ psi}} = 2.62$$

d) Diagonal Tension Factor:

$$\frac{300 \, td}{Rh} = \frac{(300)(.040 \text{ in})(15.00 \text{ in})}{(60.00 \text{ in})(10.00 \text{ in})} = .300$$

From Figure 19:

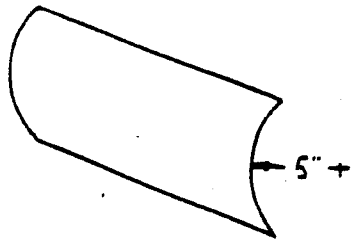
$K = .32$

or using the equation for these curves:

$$K = \tanh\left(\left(.05 + 300\frac{td}{Rh}\right)\log\frac{\tau}{\tau_{CR}}\right) = .322$$

e) Allowable shear:
From Figure 20, for K = .32, 2024-T3 AL:
$\tau_{all} = 23.3$ KSI

1d)



a) Find τ_{cr}:

$$\frac{a}{b} = \frac{15.00 \text{ in}}{10.00 \text{ in}} = 1.500$$

$$\frac{b^2}{rt} = \frac{(10.00 \text{ in})^2}{(5.00 \text{ in})(.040 \text{ in})} = 500$$

From Fig. 17:

$K_S = 70.0$

$$F_{cr} = \frac{K_S E}{\left(\frac{b}{t}\right)^2} = \frac{(70.0)(10^3 \times 10^6 \text{ psi})}{\left(\frac{10.00 \text{ in}}{.040 \text{ in}}\right)^2} = 11{,}540 \text{ psi}$$

Note the huge increase in stiffness gained from the smaller radius on the same panel.

b) Find τ :

$$q = \frac{2500 \text{ lb}}{10 \text{ in}} = 250 \text{ lb/in}$$

$$\tau = \frac{250 \text{ lb/in}}{.040 \text{ in}} = 6250 \text{ psi}$$

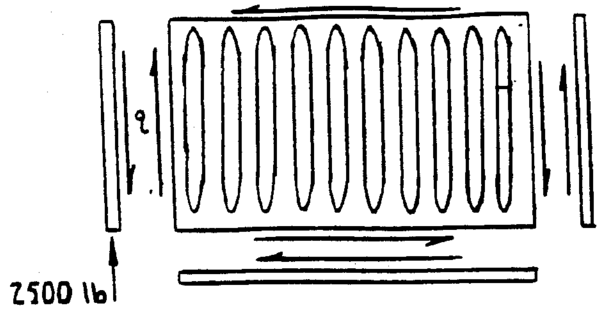
c) Loading Ratio:

$$\frac{\tau}{\tau_{cR}} = \frac{6250 \text{ psi}}{11540 \text{ psi}} = .542 < 1$$

e) Allowable shear:
From Figure 20, for .04" 7075-T6 and K = 0

$\tau_{all} = 35.0$ KSI

1e)



$$q = \frac{2500\ lb}{10\ in} = 250\ lb/in$$

From Figure 6:

Critical Shear:

i) $q_{cr} = 325$ lb/in $\qquad \tau_{cr} = \frac{325\ lb/in}{.040\ in} = 8125$ psi

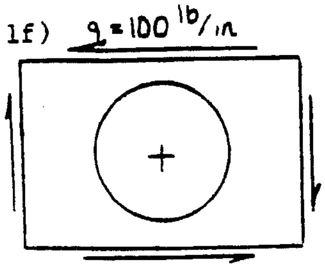
Allowable Shear:

ii) $q_{fail} = 460$ lb/in $\qquad \tau_{all} = \frac{460\ lb/in}{.040\ in} = 11.5$ KSI

Applied Shear:

iii) $\tau = \frac{250\ lb/in}{.040\ in} = 6250$ psi

1f)



Try D ≈ .8h or D = 7.71" (Closest value to .8h)

$c = b - D = 15.00" - 7.71" = 7.29$ in

from Table 1:

$B = .32$

$C' = 7.29 \text{ in} - 2(.32 \text{ in}) = 6.65 \text{ in}$

for t = .040 in:

$$\frac{h}{t} = \frac{10.00 \text{ in}}{.040 \text{ in}} = 250$$

$$K = .85 - .0006\frac{h}{t}$$

$$K = .85 - (.0006)(250)$$

$$K = .70$$

From Figure 4:

for $\frac{h}{t} = 250$

$$f_{sh} = 4700 \text{ PSI}$$

for $\frac{c}{t} = \frac{6.65 \text{ in}}{.040 \text{ in}} = 166$

$$f_{sc} = 7200 \text{ PSI}$$

Substitute these values into eqn (3):

$$q_{all} = Kt\left[f_{sh}\left(1 - \left(\frac{D}{h}\right)^2\right) + f_{sc}\sqrt{\frac{D}{h}}\right]\frac{C'}{b}$$

$$q_{all} = (.70)(.040 \text{ in})\left[(4700 \text{ PSI})\left(1 - \left(\frac{7.71 \text{ in}}{10.00 \text{ in}}\right)^2\right) + (7200 \text{ PSI})\sqrt{\frac{7.71 \text{ in}}{10.00 \text{ in}}}\right]\frac{6.65 \text{ in}}{15.00 \text{ in}}$$

$$q_{all} = 102 \text{ lb/in}$$

$$\text{M.S.} = \frac{102 \text{ lb/in}}{100 \text{ lb/in}} - 1 = .02$$

Net Section Shear:

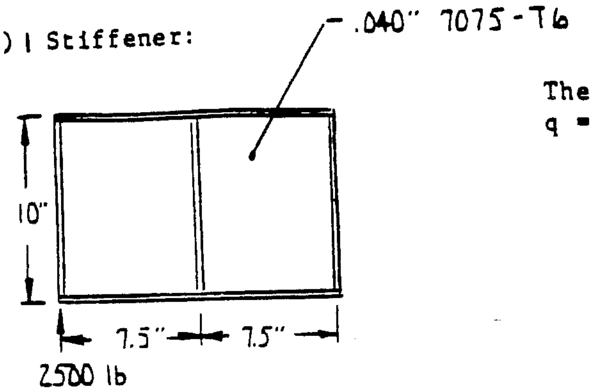
$$\tau_{net} = \frac{2500 \text{ lb}}{(10.00" - 7.71")(.040")} = 27.3 \text{ KSI}$$

From MIL-5D:

$$F_{su} = 44 \text{ KSI}$$

$$\text{M.S.} = \frac{44.0 \text{ KSI}}{27.3 \text{ KSI}} - 1 = .61$$

2a) 1 Stiffener:



The applied shear flow is:

$$q = \frac{2500 \text{ lb}}{10 \text{ in}} = 250 \text{ lb/in}$$

The applied shear stress is:

$$\tau = \frac{q}{t} = \frac{250 \text{ lb/in}}{.040 \text{ in}} = 6250 \text{ psi}$$

Find: $\frac{Is}{het^3}$: $\frac{Is}{het^3} = \frac{.0016 \text{ in}^4}{(10.00\text{in})(.040 \text{ in})^3} = 2.5$

$\frac{he}{d}$: $\frac{he}{d} = \frac{10.00 \text{ in}}{7.50 \text{ in}} = 1.33$

From figure 1:

$K_s = 7.8$

$$\tau_{cr} = K_s E \left(\frac{t}{d}\right)^2$$

$$\tau_{cr} = (7.8)(10.3 \times 10^6 \text{ psi}) \left(\frac{.040 \text{ in}}{7.50 \text{ in}}\right)^2$$

$\tau_{cr} = 2285$ psi ← Note 2 fold increase over no stiffener

Loading Ratio:

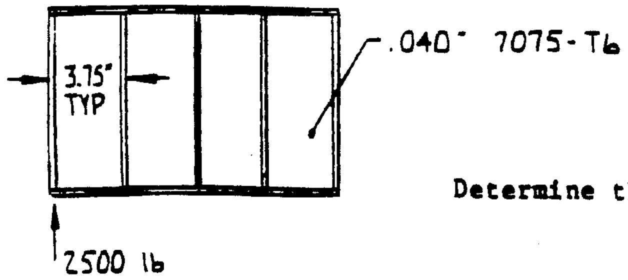
$$\frac{\tau}{\tau_{cr}} = \frac{6250 \text{ psi}}{2285 \text{ psi}} = 2.74$$

Determine the Allowable Shear Flow:

from figure 2.5:

$\tau_{all} = 30.0$ KSI

2b)



Determine the applied shear flow:

$$q = \frac{2500 \text{ lb}}{10 \text{ in}} = 250 \text{ lb/in}$$

The applied shear stress is:

$$\tau = \frac{q}{t} = \frac{250 \text{ lb/in}}{.040 \text{ in}} = 6250 \text{ psi}$$

To determine the critical shear stress, determine:

$$\frac{I_s}{h_e t^3} = \frac{.0016 \text{ in}^4}{(10.00\text{in})(.040 \text{ in})^3} = 2.50$$

$$\frac{h_e}{d} = \frac{10.00 \text{ in}}{3.75 \text{ in}} = 2.67$$

Then from figure 1:

$K_S = 4.71$ (by interpolation)

Since: $\tau_{cr} = K_S E \left(\frac{t}{d}\right)^2$

Substitute the known values:

$$\tau_{cr} = (4.71)(10.3 \times 10^6 \text{ psi}) \left(\frac{.040 \text{ in}}{3.75 \text{ in}}\right)^2$$

$\tau_{cr} = 5520$ psi ← Note 2 fold increase over 1 stiffener

Loading Ratio:

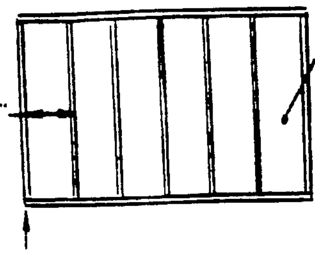
$$\frac{\tau}{\tau_{cr}} = \frac{6250 \text{ psi}}{5520 \text{ psi}} = 1.13$$

Determine the allowable shear stress:

from figure 2.5:

$\tau_{all} = 31.5$ KSI

2c)



Determine the applied shear flow:

$$q = \frac{2500\ \text{lb}}{10\ \text{in}} = 250\ \text{lb/in}$$

The applied shear stress is:

$$\tau = \frac{q}{t} = \frac{250\ \text{lb/in}}{.040\ \text{in}} = 6250\ \text{psi}$$

To determine the critical shear stress, determine:

$$\frac{I_s}{h_e t^3} : \frac{I_s}{h_e t^3} = \frac{.0016\ \text{in}^4}{(10.00\text{in})(.040\ \text{in})^3} = 2.5$$

$$\frac{h_e}{d} : \frac{h_e}{d} = \frac{10.00\ \text{in}}{2.50\ \text{in}} = 4.00$$

From figure 1:

$K_S = 3.73$ (by interpolation)

$$\tau_{cr} = KE\left(\frac{t}{d}\right)^2$$

$$\tau_{cr} = (3.73)(10.3 \times 10^6\ \text{psi})\left(\frac{.040\ \text{in}}{2.50\ \text{in}}\right)^2$$

$\tau_{cr} = 9840$ psi ⇐ Not quite double - this illustrates the law of diminishing returns

Determine the Loading Ratio:

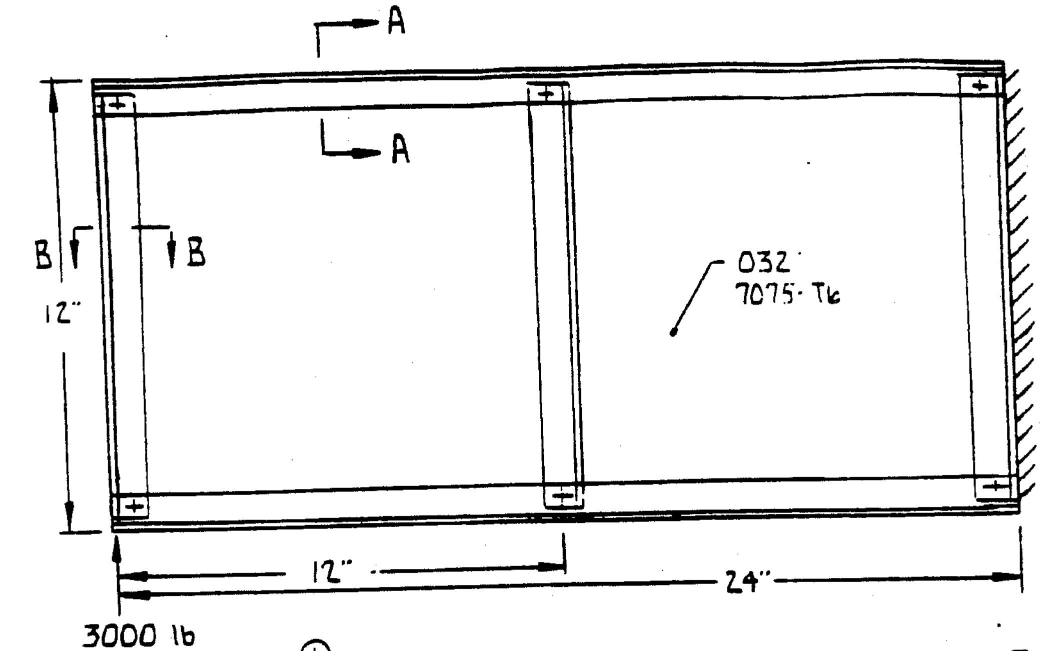
$$\frac{\tau}{\tau_{cr}} = \frac{6250\ \text{psi}}{9840\ \text{psi}} = .635$$

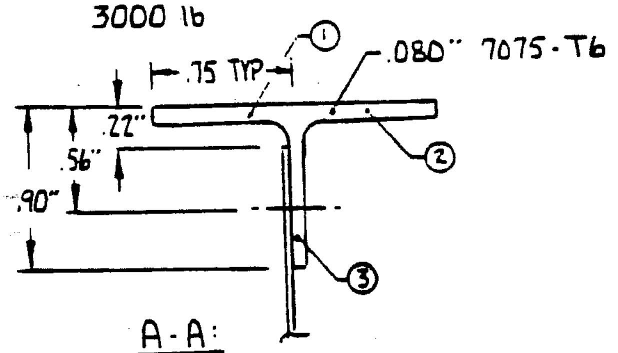
Determine the allowable shear stress:

from figure 2.5:

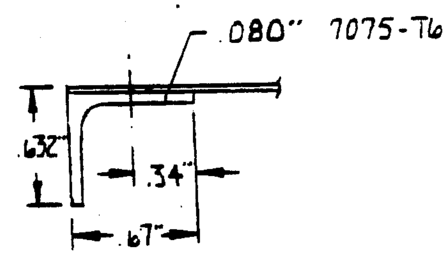
$\tau_{all} = 31.5$ KSI

3.





A-A:



B-B:

Use Crippling Analysis to determine effective sheet for use in computing section properties.

Ele	b	t	b/t	Edge Condition	bt	F_{cc}	$F_{cc}bt$
1	.75	.080	9.4	One Edge Free	.060	49500	2970
2	.75	.080	9.4	One Edge Free	.060	49500	2970
3	.86	.080	10.75	One Edge Free	.069	44000	3036
					.189		8976

$$F_{cc} = \frac{\Sigma F_{cc} bt}{\Sigma bt} = \frac{8976 \text{ lb}}{.189 \text{ in}^2} = 47490 \text{ psi}$$

Since the F_{cc} valve is below F_{cy}, $\eta = 1.0$

and $\frac{(E)_{skin}}{\sqrt{(E)_{stiff}}} \approx 100$

Using figure on p. 16.13

$$\frac{2W_e}{t} = 35.5$$

for a one-edge free skin element:

$$W_e = .35\left(\frac{2W_e}{t}\right) t = .35\,(35.5)(.032\text{ in}) = .398\text{ in}$$

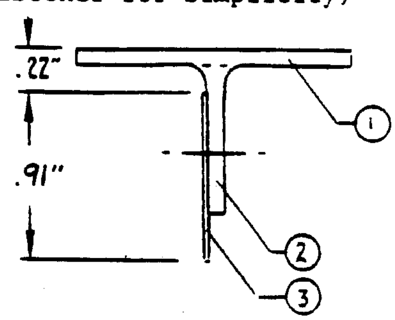
but for this problem, the one-edge-free element is limited by the edge distance, .34 in.

for a no-edge free skin element:

$$W_e = .50\left(\frac{2W_e}{t}\right) t = (.50)(35.5)(.032\text{ in}) = .57\text{ in.}$$

Section Properties of Tee Section: (Ignores effect of fastener for simplicity)

Ele	b	t	A	y	Ay	Ay^2	I
1	1.5	.080	.120	.04	.0048	.0002	.0001
2	.82	.080	.066	.49	.0323	.0158	.0037
3	.91	.032	.029	.68	.0197	.0134	.0020
			.215		.0568	.0294	.0058



$$\bar{y} = \frac{.0568\text{ in}^3}{.215\text{ in}^2} = .264\text{ in.}$$

$$I = .0294\text{ in}^4 + .0058\text{ in}^4 - (.264\text{ in})^2(.215\text{ in}^2) = 0.0202\text{ in}^4$$

$$\rho = \sqrt{\frac{.0202\text{ in}^4}{.215\text{ in}^2}} = .307\text{ in.}$$

1. Web Analysis

$$q = \frac{v}{h_e}$$

The effective height is the distance between flange centroids: for the tee section

$b = 1.50$ in

$t = .080$ in $\qquad \frac{b}{t} = 18.8 \qquad \frac{d}{b} = .6$

$d = .90$ in

using Figure 14:

$$\frac{x}{d} = .242$$

$$x = (.242)(.90 \text{ in}) = .218 \text{ in}$$

so the effective height is:

$$h_e = 12.00 \text{ in} - 2(.218\text{in}) = 11.56 \text{ in}$$

a) the shear flow:

$$q = \frac{v}{h_e} = \frac{3000 \text{ lb}}{11.56 \text{ in}} = 260 \text{ lb/in}$$

b) Shear stress:

$$\tau = \frac{q}{t} = \frac{260 \text{ lb/in}}{.032 \text{ in}} = 8125 \text{ psi}$$

c) Critical shear stress:

$$\left(\frac{qd^2}{t^3}\right)(10^{-8}) = \left[\frac{(260 \text{ lb/in})(12. \text{ in})^2}{(.032 \text{ in})^3}\right](10^{-8}) = 11.4$$

from figure 2.5

$$\frac{\tau}{\tau_{cr}} = 13.3$$

$$\tau_{cr} = \frac{8125 \text{ psi}}{13.3} = 611 \text{ psi}$$

d) Allowable Shear Stress and Diagonal Tension Factor:

Also from fig. 2.5:

$\tau_{all} = 27.8$ KSI $\qquad K = .50$

e) Web Margin of Safety

$$M.S. = \frac{28\ KSI}{8.1\ KSI} - 1 = 2.46$$

f) Net Section Shear
With four fastener diameter spacing:

$$\frac{Area_{net}}{Area_{TOT}} = .75$$

$$\text{Net } f_s = \tau \frac{Area_{tot}}{Area_{net}} = \frac{(8.1\ KSI)}{.75} = 10.8\ KSI$$

from MIL-HDBK-5D:

$$F_{su} = 44\ KSI$$

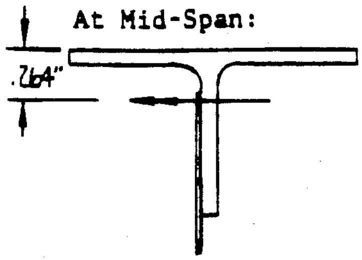
$$M.S. = \frac{44\ KSI}{10.8\ KSI} - 1 = 3.07$$

2. **Flange (Cap) Analysis**

a) **Cap Bending Moment due to Diagonal Tension**

$$M_{ST} = \frac{qkd^2}{16} = \frac{(260\ lb/in)(.50)(12\ in)^2}{16} = 1170\ in.lb$$

At Mid-Span:



$$\sigma_{ten} = \frac{(1170\ in.lb)(.866\ in)}{.0202\ in^4} = 50.2\ KSI$$

$$\sigma_{comp} = \frac{(1170\ in.lb)(.264\ in)}{.0202\ in^4} = 15.3\ KSI$$

Tension Margin of Safety:

$$M.S. = \frac{78.0\ KSI}{50.2\ KSI} - 1 = .55$$

Crippling Check:

Using only the material in compression:

Ele	b	t	b/t	Edge Condition	bt	F_{cc}	$F_{cc}bt$
1	.75	.08	9.4	One Edge Free	.060	49500	2970
2	.75	.08	9.4	One Edge Free	.060	49500	2970
3	.144	.08	1.8	No Edge Free	.012	70000	840
					.132		6780

$$F_{cc} = \frac{6780\ lb}{.132\ in^2} = 51.4\ KSI$$

Crippling Margin of Safety:

$$M.S. = \frac{51.4\ KSI}{15.3\ KSI} - 1 = 2.36$$

b) **Cap Analysis Using Primary Loads:**

At the end of the first bay, the equations below are applicable for determining the cap loads:

$$P_T = \frac{M}{h_e} - 0.5\ KV$$

$$P_c = \frac{M}{h_e} + 0.5\ KV$$

Substitution of parameters gives:

$$P_T = \frac{(3000\ lb)(12\ in)}{11.56\ in} - (0.5)(0.5)(3000\ lb) = 2364\ lb$$

$$P_c = \frac{(3000\ lb)(12\ in)}{11.56\ in} - (0.5)(0.5)(3000\ lb) = 3864\ lb$$

At the reactions, the cap loads are:

$$P_T = -P_c = \frac{(3000\ lb)(24\ in)}{12\ in} = 6000\ lb$$

To check for the tension margin of safety, divide the cap load by the cap area, and then compare to F_{TU}:

$$\sigma_T = \frac{6000\ lb}{.189\ in^2} = 31750\ psi$$

$$M.S. = \frac{78.0\ KSI}{31.75\ KSI} - 1 = 1.46$$

To check for the compression margin of safety, use the figure on p. 16.38 of MAC 339 to determine an equivalent column load. Then check this load against the allowable load.

$$\frac{P_1}{P_2} = \frac{3864\ lb}{6000\ lb} = .644$$

Using figure on p. 16.38 of MAC 339:

$$\frac{P_2}{P_E} = 1.25$$

$$P_E = \frac{P_2}{1.25} = \frac{6000\ lb}{1.25} = 4800\ lb$$

The allowable column stress is found using the figure on p. 16.31 of MAC 339:

with $F_{cc} = 47.5$ KSI

and $\frac{L'}{\rho} = \frac{(0.7)(12.00\ in)}{.307\ in} = 27.4$

gives $F_c = 43.5$ KSI

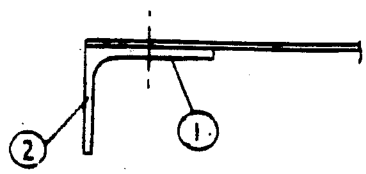
$$P_{cr} = (43.5\ KSI)(.215\ in^2) = 9353\ lb$$

Computing the Margin of Safety:

$$M.S. = \frac{9353\ lb}{4800\ lb} - 1 = .95$$

3. Stiffener Analysis

a) To determine the stiffener stress, first determine its section properties:



First, use crippling analysis to determine an effective width of skin to use:

Ele	b	t	b/t	Edge Condition	bt	F_{cc}	$F_{cc}bt$
1	.63	.08	7.9	One Edge Free	.050	57000	2850
2	.56	.08	7.0	One Edge Free	.045	62000	2790
					.095		5640 lb

$$F_{cc} = \frac{5640\ lb}{.095\ in^2} = 59,370\ psi$$

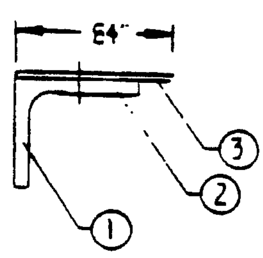
for the one edge free sheet:

$$W_e = .35\ \frac{(2We)t}{t} = .35(32)(.032\ in) = .358\ in$$

the actual effective width is limited by the edge distance = .33 in for the no edge free sheet:

$$W_e = .50\ \frac{(2We)t}{t} = .5\ (32)(.032\ in) = .512\ in$$

So for compression analysis, the section is:



Ele	b	t	A	y	Ay	Ay^2	I
1	.60	.08	.048	.332	.016	.0053	.0014
2	.59	.08	.047	.072	.003	.0002	.0000
3	.84	.032	.027	.016	.000	.0000	.0000
			.122		.019	.0055	.0014

$$\bar{y} = \frac{.019 \text{ in}^3}{.122 \text{ in}^2} = .156 \text{ in}$$

$$I = .0014 \text{ in}^4 + .0055 \text{ in}^4 - (.122 \text{ in}^2)(.156 \text{ in})^2 = .0039 \text{ in}^4$$

$$\rho = \sqrt{\frac{.0039 \text{ in}^4}{.122 \text{ in}^2}} = .179 \text{ in}$$

To determine e, the distance from the median plane of the web to the stiffener centroid, use the figure on p. 2.30 of MAC 339 to get the location of the stiffener centeroid, then add one-half the web thickness.

$$\frac{b}{t} = \frac{.67 \text{ in}}{.08 \text{ in}} = 8.4$$

$$\frac{d}{b} = \frac{.60 \text{ in}}{.67 \text{ in}} = .90$$

from p. 2.30 of MAC 339 (Figure 14)

$$\frac{x}{b} = .2875$$

$$x = .193 \text{ in}$$
$$e = .193 \text{ in} + .016 \text{ in} = .209 \text{ in}$$

Now to compute the stiffener stress, use figure 12, computing the necessary terms along the way:

$$\frac{e}{\rho} = \frac{.209 \text{ in}}{.179 \text{ in}} = 1.17$$

Using the chart gives: $\frac{A_{se}}{A_s} = .43$

$$\frac{A_s}{dt} = \frac{.095 \text{ in}^2}{(12.0")(.032")} = .247$$

The chart gives $\frac{A_{se}}{dt} = .105$

Recall $\frac{\tau}{\tau_{cr}} = 13.3$

The chart gives: $\frac{\sigma_s}{\tau} = 1.38$

The applied stiffener stress due to tension field effects is:

$$\sigma_s = \left(\frac{\sigma_s}{\tau}\right)\tau = (1.38)(8125\ \text{psi}) = 11213\ \text{psi}$$

This applied stress should not exceed the stiffener column yield stress, F_{cy}. For 7075-T6 extrusion, $F_{cy} = 70$ KSI

$$M.S. = \frac{70\ \text{KSI}}{11.2\ \text{KSI}} - 1 = \text{large}$$

The maximum stiffener stress, σ_s, max occurs at the midpoint of the stiffener, and is obtained from figure 13:

since $\frac{d}{h_s} = 1.0$ and $\frac{\tau}{\tau_{cr}} = 13.3$

figure 13 gives: $\frac{\sigma_{s,max}}{\sigma_s} = 1.07$

$$\sigma_{s,max} = (1.07)(11.2\ \text{KSI}) = 12\ \text{KSI}$$

b) Forced Crippling Check of Stiffeners

To prevent forced crippling of the stiffener, the maximum stress should not exceed σ_o, where σ_o is determined from equation:

$$\frac{\sigma_o}{\tau} = \sqrt[3]{K^2\left(\frac{ts}{t}\right)} = \sqrt[3]{(.50)^2\ \frac{(.080\ \text{in})}{(.032\ \text{in})}} = .85$$

$$\sigma_o = (32500\ \text{psi})(.85) = 27625\ \text{psi}$$

$$M.S. = \frac{27.6\ \text{KSI}}{12\ \text{KSI}} - 1 = 1.3$$

c) Column Strength of Stiffeners

The average stress over the cross-section of the stiffener shall not exceed the allowable stress as a column. To determine the allowable column stress, use the figure on p. 16.31 of MAC 339.

$$F_{cc} = 59.4\ \text{KSI}$$

$$\frac{L'}{\rho} = \frac{(0.5)(12.0\ \text{in})}{.179\ \text{in}} = 33.5$$

The figure on p. 16.31 gives $F_c = 49$ KSI

The average stress is

$$\sigma_{s,av} = \sigma_s \left(\frac{A_{se}}{A_s}\right) = (11.2 \text{ KSI})(.43) = 4.8 \text{ KSI}$$

Computing the margin of safety:

$$\text{M.S.} = \frac{49 \text{ KSI}}{4.8 \text{ KSI}} - 1 = 9.2$$

4. Attachment Analysis

a) Web to Cap Connection:

Compute the running load:

$h_r = 12.00 \text{ in} - 2\,(.56 \text{ in}) = 10.88 \text{ in}$

$$R = \frac{S(1 + .414\ K)}{h_r} = \frac{(3000 \text{ lb})[1 + .414\ (.50)]}{10.88 \text{ in}} = 333 \text{ lb/in}$$

To avoid inter-rivet buckling, a spacing of four diameters will be used. For a #5 rivet, 4D = .625 in. From p. 1.13 of MAC 339:
MS20470-AD5 Rivet: $P_{ALL} = 551$ lb

Maximum spacing: $$\frac{P_{ALL}}{R} = \frac{551 \text{ lb}}{333 \text{ lb/in}} = 1.65 \text{ in}$$

Computing the Margin of Safety:

$$M.S. = \frac{1.65 \text{ in}}{.625 \text{ in}} - 1 = 1.64$$

b) Stiffener to Cap Connection

$$P_{DT} = \sigma_s A_{se} = \sigma_s \left(\frac{A_{se}}{A_s}\right) A_s$$

$$P_{DT} = (11.2 \text{ KSI})(.43)(.095 \text{ in}^2) = 458 \text{ lb}$$

From the analysis above, use a MS20470AD5 rivet.

Compute the Margin of Safety:

$$M.S. = \frac{551 \text{ lb}}{458 \text{ lb}} - 1 = .20$$

c) Stiffener to Web Connection:

Required tension strength $\geq .15\ \sigma_{ULT} t$

$$P_{t,req} \geq (.15)(72 \text{ KSI})(.032 \text{ in})$$

$$P_{t,req} \geq 346 \text{ lb/in}$$

from p. 1.11 of MAC 339, for a MS20470-AD-5 rivet

$P_t = 301$ lb

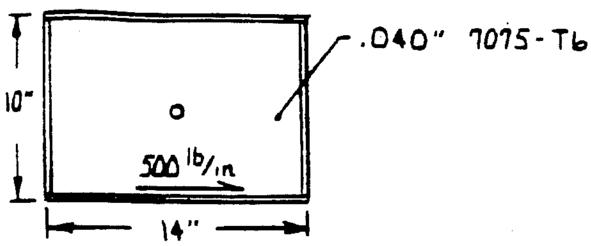
The maximum spacing would be:

$$\frac{P_t}{P_{t,req}} = \frac{301 \text{ lb}}{346 \text{ lb/in}} = .87 \text{ in}$$

If the four diameter spacing is used in the stiffener, the margin of safety is:

$$M.S. = \frac{.87 \text{ in}}{.625 \text{ in}} - 1 = .39$$

4.



Solution:

a) Determine Cross-Sectional Area of Doubler

$$A_{DBLR} \geq (1.5)(A_{rem})$$
$$\geq (1.5)(1.00")(.040")$$
$$\geq .060 \text{ in}^2$$

b) Doubler gage

Choose 1 gage thicker than web material, that is .050". This gives a minimum circular diameter:

$$CD_{min} = \frac{.060 \text{in}^2}{.050 \text{ in}} + 1.00 \text{ in} = 2.20 \text{ in}$$

c) Rivets between hole tangents

$$P = 2\ D_h q = 2(1.00 \text{ in})(500 \text{ lb/in}) = 1000 \text{ lb}$$

from MAC 339:
Two MS20470AD5 Rivets (BJ|5 's) in .040" (575 lbs/rivet):

$$P_{All} = 2(575) = 1150 \text{ lb}; \quad M.S. = \frac{1150 \text{ lb}}{1000 \text{ lb}} - 1 = .15$$

d) Spacing:

Inner Circle Circumference = $(2)(\pi)(.50" + .37")$ = 5.47 in (Δ)

$$\frac{\text{Circumference}}{\text{4D spacing}} = 8 \text{ rivets}$$

Outer Circle Circumference = $(2)(\pi)(.87" + .625")$ = 9.39 in

$$\frac{\text{Circumference}}{\text{4D spacing}} = 15 \text{ rivets}$$

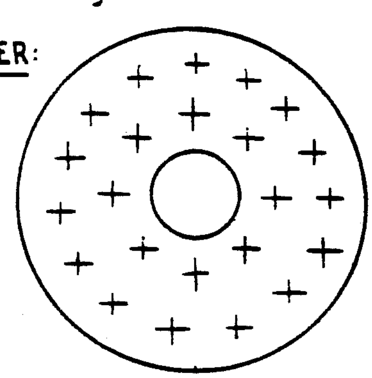
Outer Radius = .50" + .37" + .625" + .37" = 1.87"

Check for cross sectional area:

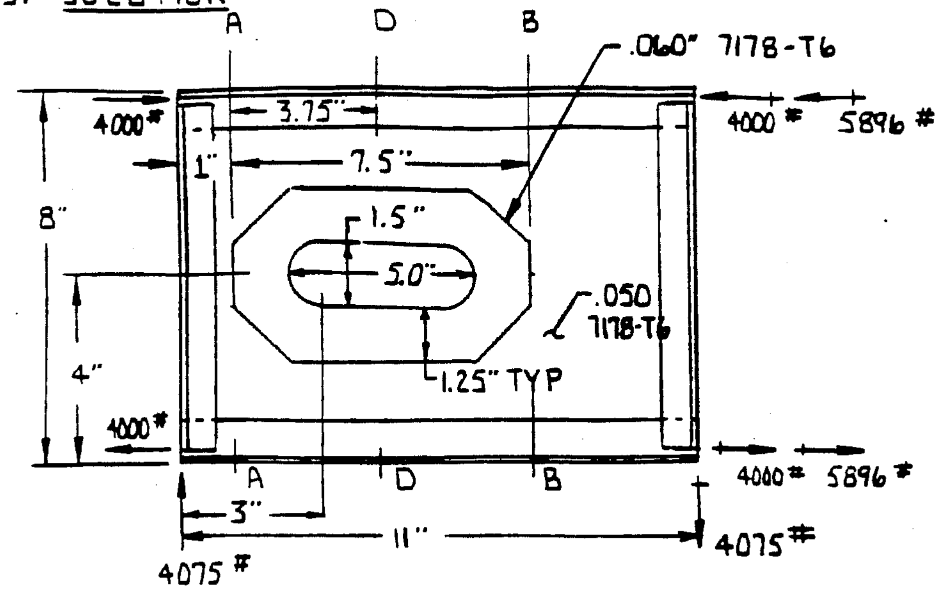
$$A = (.05")\left[3.74" - 1" - 4(.156")\right] = .106 \text{ in}^2 > .06 \text{ in}^2 \ \checkmark \text{OK}$$

(Δ) .37" = 2d + .06"

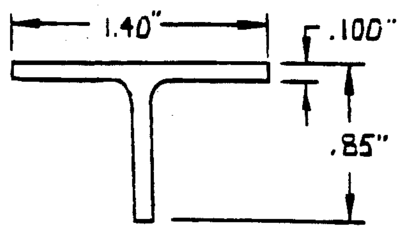
REQD. DOUBLER:



5. SOLUTION:



Determine Shear Flow:



TYPICAL CAP

a) Effective Height

$$\frac{b}{t} = \frac{1.40"}{.100"} = 14$$

$$\frac{d}{b} = \frac{.85"}{1.40"} = .61$$

From Fig. 14:

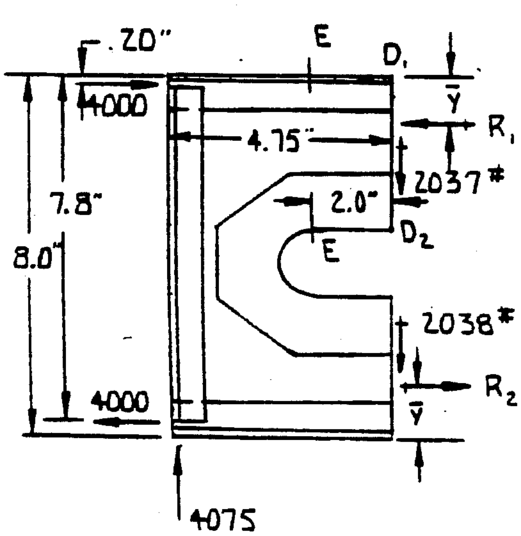
$$\frac{x}{d} = .233$$

$$x = .198"$$

Effective Height = 8" - 2(.198") = 7.6"

$$q = \frac{4075\#}{7.6"} = 536\ \#/\text{in}$$

Free body of web from edge to D-D:



$$\Sigma F_x = 0:\ 4000^\# - 4000^\# - R_1 + R_2 = 0 \quad (1)$$

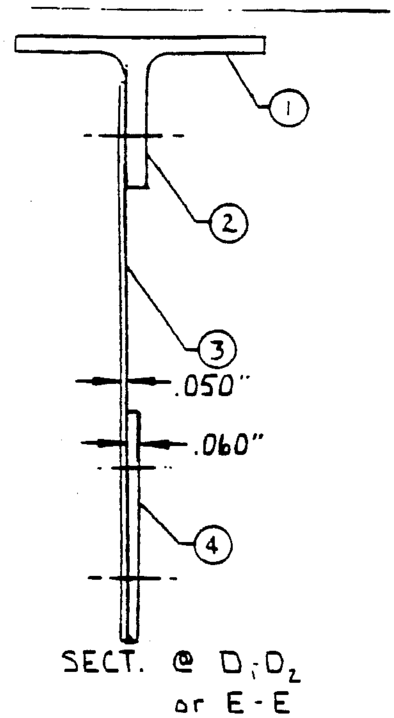
$$+\Sigma M_{D_1} = 0: -(4075^\#)(4.75") + (4000^\#)(.20") - (4000^\#)(7.8") + R_2(8" - \bar{y}) - R_1\bar{y} = 0 \quad (2)$$

FROM EQN. 1:

$$R_1 = R_2$$

SUBSTITUTING INTO EQN. 2:

$$R_2(8" - 2\bar{y}) = 49756 \text{ in-lb}$$

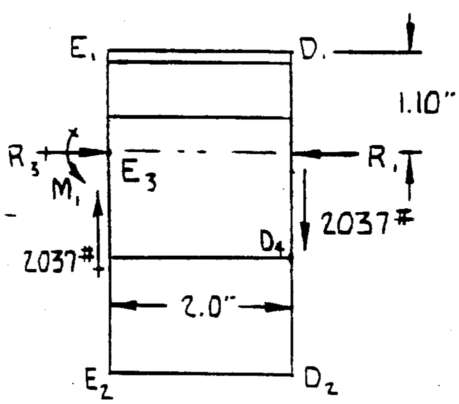


ELE	b	h	A	y	Ay	Ay^2	I
1	1.40	0.10	.140	.050	.007	.0004	.0001
2	0.10	0.75	.075	.475	.036	.0171	.0035
3	0.05	3.00	.150	1.75	.263	.4603	.1125
4	0.06	1.25	.075	2.625	.197	.5171	.0098
			.453		.503	.9949	.1259

$\bar{y} = \frac{Ay}{A} = \frac{.503}{.453} = 1.10"$

$I = .1259 + .9949 - (1.10")^2 (.503) = .512 \text{ in}^4$

So: $R_2 = R_1 = \frac{49756 \text{ in·lb}}{(8" - 2(1.10"))} = 8579\#$



$\Sigma F_x = 0: \quad R_3 - 8579\# = 0$

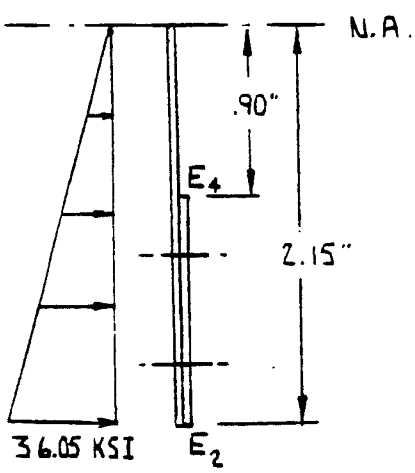
$R_3 = 8579\#$

$\circlearrowright \Sigma M_{E_3} = 0: \quad M_1 - (2037\#)(2") = 0$

$M_1 = 4075 \text{ in·lb}$

Stress @ E_2

$\sigma_{comp} = \frac{(4075 \text{ in lb})(2.15")}{.512 \text{ in}^4} + \frac{8579\#}{.453 \text{ in}^2} = 36050 \text{ psi}$



Considering just the web & doubler combination as column:

$\sigma_{avg} = \frac{\frac{(.90")}{(2.15")}(36.05 \text{ KSI}) + 36.05 \text{ KSI}}{2} = 25.57 \text{ KSI}$

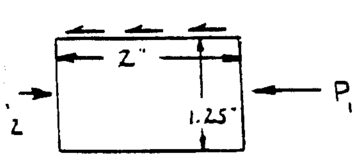
AVERAGE LOAD FROM E_2 TO E_4:

$P_2 = (25.57 \text{ KSI})(1.25")(.050" + .060") = 3516\#$

AVERAGE LOAD FROM D_2 TO D_4:

$P_1 = \frac{8579\#}{.453 \text{ in}^2}(1.25")(.05" + .060") = 2604\#$

Using a Column with Distributed Axial Loading:
From MAC 339, p. 16.38:



$\frac{P_1}{P_2} = \frac{2604}{3516} = .74$

$\frac{P_2}{P_E} = 1.17 \qquad P_E = \frac{3516\#}{1.17} = 3005\#$

Column Allowable:

Conservatively assume column pinned @ both ends:

$L' = L = 2.0"$

Determine least radius of gyration, ρ : $\left(\rho = \sqrt{\frac{I}{A}} \right)$

$I = \frac{(1.25")(.11")^3}{12} = .000139 \text{ in}^4$ $\quad A = (1.25")(.050" + .060") = .138 \text{ in}^2$

$\rho = \sqrt{\frac{.000139}{.138}} = .032 \text{ in}$

Slenderness ratio:

$\frac{L'}{\rho} = \frac{2.0"}{.032} = 62.5$

Euler Column Allowable:

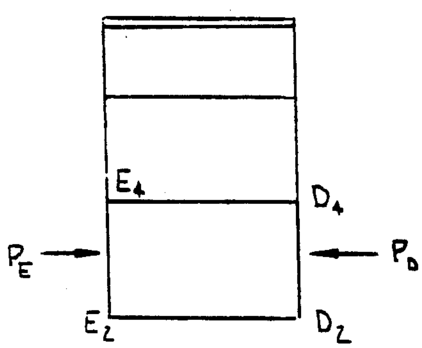
$F_C = \frac{\pi^2 E}{(L'/\rho)^2} = 26020 \text{ psi} \qquad (E = 10.3 \times 10^6 \text{ psi})$

$P_{CR} = F_C A = (26020 \text{ psi})(.138 \text{ in}^2) = 3590 \text{ lb}$

Column Margin of Safety:

$M.S. = \frac{3590 \#}{3005 \#} - 1 = \underline{\underline{0.19}}$

Fastener Check:



Load in Doubler:

At E_2-E_4:

$P_E = (25.57 \text{ KSI})(1.25")(.06") = 1918 \text{ lb}$

At D:

$P_D = (18.94 \text{ KSI})(1.25")(.06") = 1421 \text{ lb}$

Load Transfer = 497 #

$q = \frac{497 \#}{2.0"} = 249 \#/\text{in}$

From MAC 339, p. 1.13:

Shear Strength of BJ | 5 rivet in .050" 7178-T6 Alum Sheet:

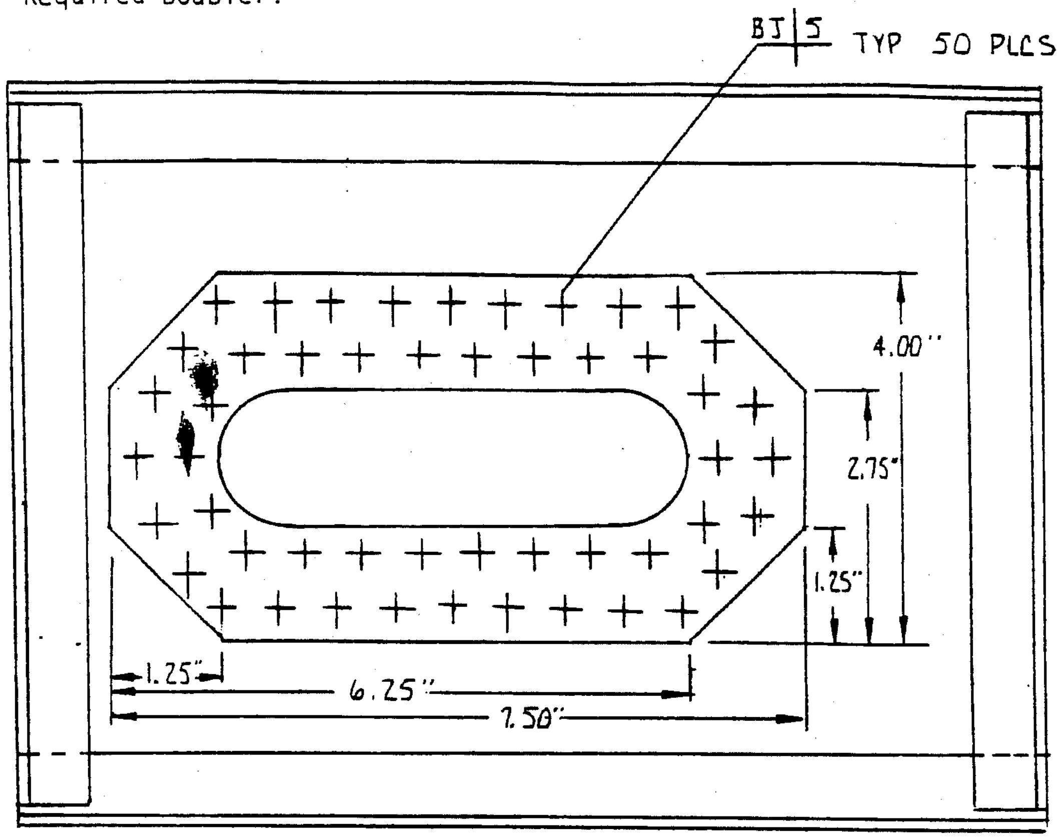
$P = 594 \#$

Assuming 4D fastener spacing: $q_{allow} = \frac{594 \#}{.625"} = 950 \#/\text{in}$

Margin of Safety:

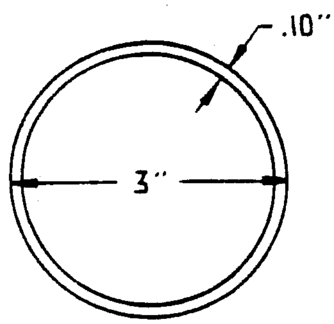
$M.S. = \frac{950 \#/\text{in}}{249 \#/\text{in}} - 1 = \underline{\underline{2.82}}$

Required Doubler:



Solutions: Torsion

1.



Determine the Polar Moment of Inertia:

$$J = \frac{\pi}{32}\left[(3\text{ in})^4 - (2.8\text{ in})^4\right]$$

$$J = 1.92\text{ in}^4$$

For 6061-T6 tubing, WW-T-700/6, F_{TY} = 35 KSI, F_{TU} = 42 KSI

Using the assumption, that $F_{SY} = (.6)(F_{TY})$, F_{SY} = 21 KSI

So the maximum torque which can be applied while maintaining an elastic distribution is:

$$\tau_{All} = F_{sy} = 21\text{ KSI}$$

$$T_{All} = \frac{J\tau}{\rho} = \frac{(1.92\text{ in}^4)(21\text{ KSI})}{(1.5\text{ in})}$$

$$\underline{T_{All} = 26880\text{ in}\cdot\text{lb}}$$

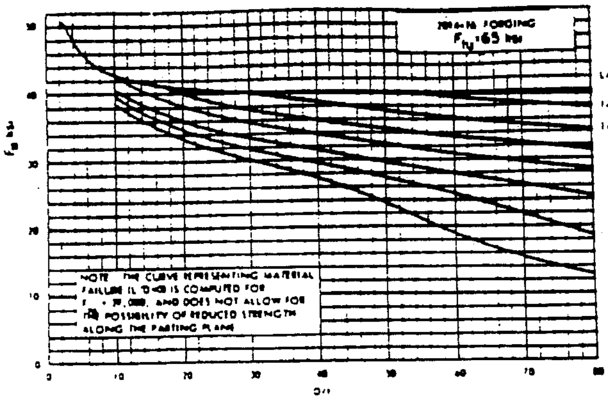


Fig. 18 Torsional modulus of rupture - 2014-T6 aluminum alloy forging.

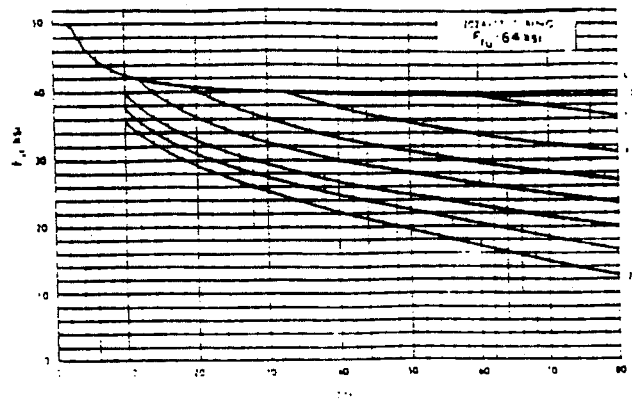


Fig. 19 Torsional modulus of rupture - 2024-T3 aluminum alloy tubing.

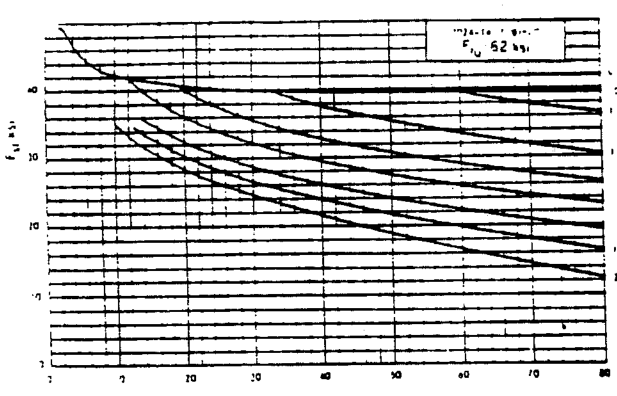


Fig. 20 Torsional modulus of rupture - 2024-T4 aluminum alloy tubing.

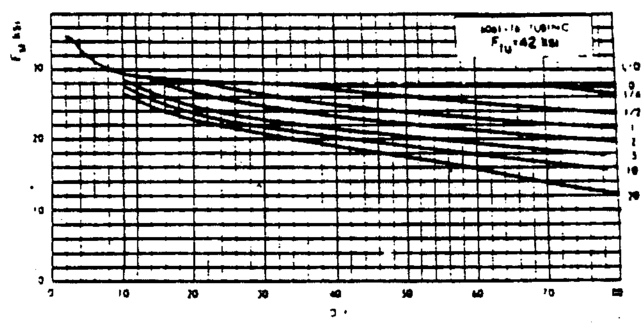


Fig. 21 Torsional modulus of rupture - 6061-T6 aluminum alloy tubing.

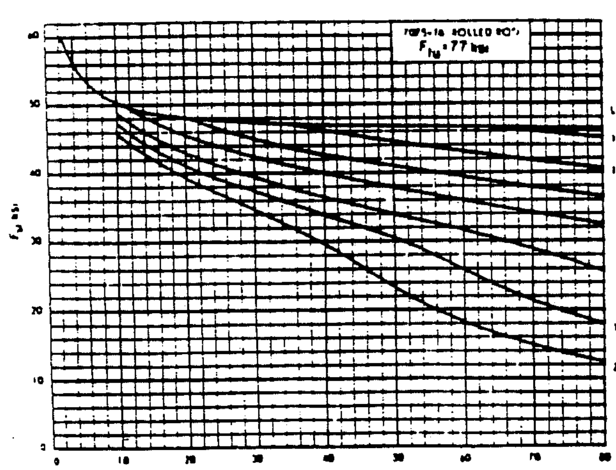


Fig. 22 Torsional modulus of rupture - 7075-T6 aluminum alloy rolled rod.

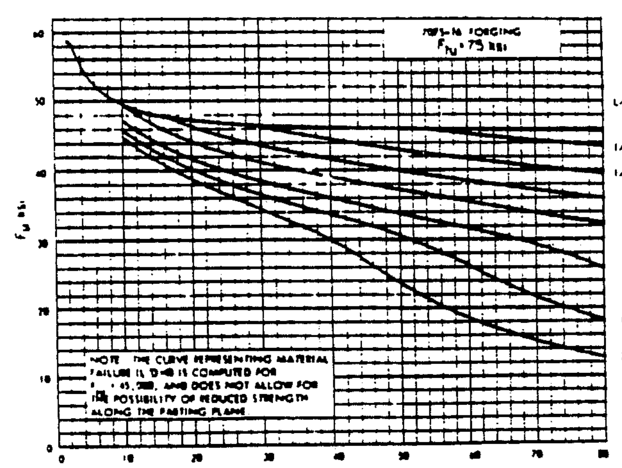


Fig. 23 Torsional modulus of rupture - 7075-T6 aluminum alloy forging.

2. The maximum allowable torsion will be based on plastic analysis.

$$\frac{D}{t} = \frac{3.0''}{.10''} = 30 \quad , \quad \frac{L}{D} = \frac{10''}{3''} = 3.33 \quad , \quad F_{TU} = 42{,}000 \text{ psi}$$

From figure 21:

$$F_{ST} = 24 \text{ KSI}$$

Thus the maximum torsion is:

$$T_{All} = \frac{J\,F_{st}}{r}$$

$$T_{All} = \frac{(1.92 \text{ in}^4)(24 \text{ KSI})}{(1.5 \text{ in})}$$

$$\underline{T_{All} = 30720 \text{ in} \cdot \text{lb}}$$

3. Determine the polar moment of inertia:

$$J = \frac{\pi}{32} d^4 = \left(\frac{\pi}{32}\right)(3 \text{ in})^4 = 7.95 \text{ in}^4$$

Using F_{sy} = 21 KSI from problem 1

$$T = \frac{J\tau}{\rho} = \frac{(7.95 \text{ in}^4)(21 \text{ KSI})}{(1.5 \text{ in})} = 111.3 \text{ in-kips}$$

4. The maximum allowable torsion is found using the technique of problem 2:

for: $\frac{D}{t} = \frac{3''}{1.5''} = 2$ and: $\frac{L}{D} = \frac{10''}{3''} = 3.3$

from figure 21:

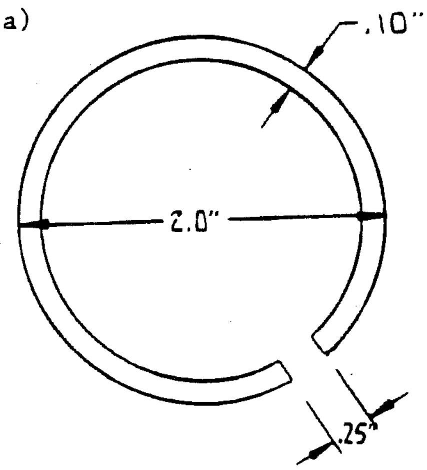
$$F_{st} = 34.5 \text{ KSI}$$

$$T_{All} = \frac{F_{st} J}{\rho}$$

$$T_{All} = \frac{(34.5\ KSI)(7.95\ in^4)}{(1.5\ in)} = 183\ in\cdot kips$$

From the previous four problems, note that depending on the thickness and type of stress distribution assumed, the allowable torsion can change significantly. (6.7 times for these cases.)

5. a)



Determine the parameter, α :

for: $\frac{b}{t}_{av} = \frac{5.72"}{.10"} = 57.2"$

from the table on p.4 of the lesson

$\alpha = .333$

substitute α into equation for τ,

$$\tau = \frac{T}{\alpha b t^2}$$

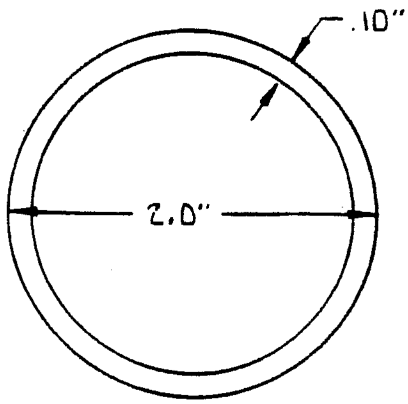
or

$$T = \tau \alpha bt^2$$

$$T = (21\ KSI)(.333)(5.72")(.10")^2$$

$$T = 4000\ in\cdot lb$$

b) for a section without a slit:



Determine the polar moment of inertia, J

$$J = \frac{\pi}{32} [(2 \text{ in})^4 - (1.8 \text{ in})^4]$$

$$J = .54 \text{ in}^4$$

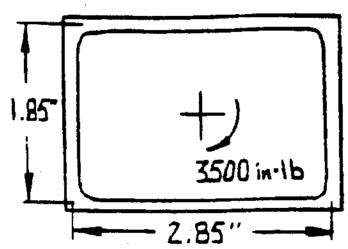
Determine the maximum allowable elastic torque.

$$T = \frac{J \ \tau max}{r_c} = \frac{(.54 \text{ in}^4)(21 \text{ KSI})}{(1 \text{ in})}$$

$$T = 11340 \text{ in.lb}$$

Note the three fold increase in capability to carry torque that the continuous section exhibits over the slit section. For this tube then, the design loading should be based on the torque carrying capability at the slit section.

6. This problem is similar to the example in the text without the applied shear load. (See figure 7)



The enclosed area for this section is taken as the product of the average length and average width, where average is defined as the average between the internal and external dimension.

$$A = (1.85\ in)(2.85\ in) = 5.27\ in^2$$

The shear flow is defined by:

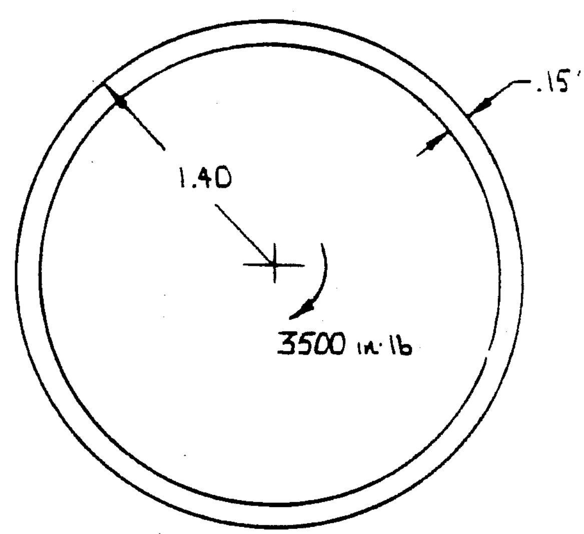
$$q = \frac{T}{2A}$$

Substituting the given values:

$$q = \frac{3500\ in \cdot lb}{2(5.27\ in^2)} = 332\ lb/in$$

7. For this cross-section, the average of the two areas enclosed by the inside and the outside surfaces of the tube is:

$$A_{AVG} = \frac{\pi r_o^2 + \pi r_i^2}{2}$$



$$A_{AVG} = \frac{\pi[(1.4\ in)^2 + (1.25\ in)^2]}{2}$$

$$A_{AVG} = 5.53\ in^2$$

Then the shear flow is

$$q = \frac{T}{2A} = \frac{3500 \text{ in·lb}}{2\,(5.53 \text{ in}^2)} = 316 \text{ lb/in}$$

Note that the shear flow is approximately the same as in the rectangular torque box of problem 6. This is so because its enclosed area is about the same. The rectangular tube is less preferred, however, because local stress concentrations exist at the corners.

8. To solve for the shear flow, first determine the area enclosed by the torque box. Since this is a complex shape, it will be assumed that it is composed of a semicircle and a rectangle.

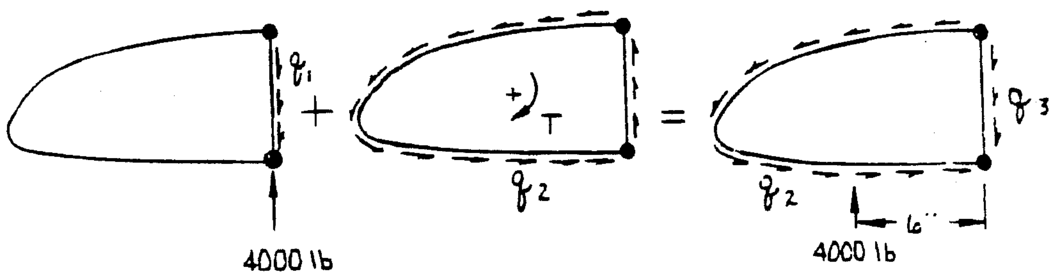
$$A = 1/2\,\pi r^2 + \ell w$$

$$A = 1/2\,(3.14)\left[\frac{2.00 + 1.85}{2}\right]^2 + \left(2.5'' - \frac{.15''}{2}\right)(1.85 \text{ in}) = 10.31 \text{ in}^2$$

So the shear flow is:

$$q = \frac{T}{2A} = \frac{3500 \text{ in·lb}}{2\,(10.31 \text{ in}^2)} = 170 \text{ lb/in}$$

9. To determine the shear flow, use the principle of superposition which was introduced in the text.



$q_1 = \frac{V}{h} = \frac{4000\ \text{lb}}{6\ \text{in}}$	$T = (6")(4000\ \text{lb})$	$q_2 = 239\ \text{lb/in}$
	$T = 24000\ \text{in}\cdot\text{lb}$	
$q_1 = 667\ \text{lb/in}$		$q_3 = q_1 - q_2$
	$A=(6")(6")+0.5\pi(3")^2$	
	$A=50.14\text{in}^2$	$q_3 = (667 - 239)\ \text{lb/in}$
	$q_2 = \frac{T}{2A} = \frac{24000\ \text{in}\cdot\text{lb}}{2(50.14\ \text{in}^2)}$	$q_3 = 428\ \text{lb/in}$
	$q_2 = 239\ \text{lb/in}$	

Problem Solutions

Lesson 15 - Pressure

1. The stress level in a sphere is uniform, and is found by passing a section through the center of the sphere. From section 15.1:

$$\sigma = \frac{Pr}{2t}$$

The pressure will be taken as the burst pressure*:

$$P_B = 4(30 \text{ psi}) = 120 \text{ psi}$$

* The burst pressure factor is the multiplier applied to the operating pressure. The values range between two and four and depend on the usage of the compartment being designed.

The stress can be computed:

$$\sigma = \frac{(120 \text{ psi})(10.0'')}{2(.050'')} = 12 \text{ KSI}$$

From MIL-HDBK-5D for 2024-T3 sheet:

$$F_{TU} = 63 \text{ KSI (L-T)}$$

So the margin at safety is:

$$M.S. = \frac{\sigma_{ULT}}{\sigma_{B.P.}} - 1 = \frac{63 \text{ KSI}}{12 \text{ KSI}} - 1 = 4.25$$

2. The maximum stress in the cylinder will be the hoop stress:

$$\sigma_{hoop} = \frac{Pr}{t}$$

Applying the burst pressure factor to the operating pressure:

$$p_b = (30 \text{ psi})(4) = 120 \text{ psi}$$

So the hoop stress is:

$$\sigma_{hoop} = \frac{(120 \text{ psi})(10.0")}{(.050")} = 24 \text{ KSI}$$

The axial stress is:

$$\sigma_{hoop} = \frac{Pr}{2t} = \frac{(120 \text{ psi})(10.0")}{2\ (.050")} = 12 \text{ KSI}$$

The margin of safety is:

$$M.S. = \frac{63 \text{ KSI}}{24 \text{ KSI}} - 1 = 1.625$$

3. The method of solving this problem is the same as the method presented in the lesson, section 15.3.

An equilibrium balance could be made to determine the reactions, which would then be divided by the cross-sectional area to determine the stress.

Alternatively, one could use the hoop stress equation to determine the stress directly:

$$f_h = \frac{Pr}{t} = \frac{(25 \text{ psi})(20")}{(.032")} = 15625 \text{ psi}$$

4.a) Assume that 1 psi causes less than .020" deflection (1/2t). Then use the equations for a thick plate:

$$\frac{w}{b} = \frac{14.2\left(\frac{10^7}{E}\right)P}{\left(\frac{1000t}{b}\right)^3} = \frac{(14.2)\left(\frac{10^7}{10.3\times10^6\ \text{psi}}\right)(1\ \text{psi})}{\left(\frac{(1000)(.040")}{(4")}\right)^3}$$

$w = (4\ \text{in})(.014\ \text{in}) = \underline{.055\ \text{in}}$

Since the edges were assumed to be built in:

deflection = 1/5 w = <u>.011 in</u> (assumption: OK)

The stress:

$$\sigma_b = \frac{750{,}000P}{\left(\frac{1000t}{b}\right)^2} = \frac{(750{,}000)(1\ \text{psi})}{\left(\frac{(1000)(.040")}{(4")}\right)^2} = 7500\ \text{psi}$$

Using the assumption of built in edges:

$\sigma = 2/3\ \sigma_b = 2/3\ (7500\ \text{psi}) = \underline{5000\ \text{psi}}$

b) Assume that 50 psi causes the panel to act as a membrane:

(i.e. w > 5t)

<u>Deflection</u>:

$$\frac{w}{b} = 0.162\left(\frac{\left(\frac{10^7}{E}\right)P}{\frac{1000t}{b}}\right)^{1/3} = 0.162\left(\frac{\left(\frac{10^7}{10.3\times10^6\text{psi}}\right)(50\ \text{psi})}{\frac{(1000)(.040")}{(4")}}\right)^{1/3}$$

$w = (4\ \text{in})(.274) = \underline{1.10\ \text{in}}$ (which is greater than .40 in)

The tensile stress:

$$\sigma_T = 7700\left(\frac{\frac{\sqrt{10E}}{10^4}P}{\frac{1000t}{b}}\right)^{2/3} = 7700\left(\frac{\frac{\sqrt{10.3\times10^7}}{10^4}(50\ \text{psi})}{\frac{(1000)(.040")}{(4")}}\right)^{2/3}$$

$\sigma_T = (7700)(1.60) = \underline{22740\ \text{psi}}$

c) From the preceeding analysis, it will be assumed that 10 psi causes a condition which is between a thick plate and a membrane.

Use the figures in MAC 339 for thin plates with built-in edges:

Parameters:

$$\frac{10^7 p}{E} = \frac{10^8 \text{ psi}}{10.3 \times 10^6 \text{ psi}} = 9.7$$

$$\frac{1000t}{b} = \frac{(1000)(.040 \text{ in})}{(4 \text{ in})} = 10$$

Compute the maximum stress:

from p. 17.40:

$$\frac{10^4 \sigma_T}{E} = 4.35$$

$$\sigma_T = 4481 \text{ psi}$$

from p. 17.41:

$$\frac{10^4 \sigma_b}{E} = 30$$

$$\sigma_b = 30900 \text{ psi}$$

$$\sigma = \sigma_t + \sigma_b = 35{,}400 \text{ psi}$$

or from p. 17.42:

$$\frac{10^4 \sigma max}{E} = 34.5$$

$$\underline{\sigma max = 35{,}500 \text{ psi}}$$

This shows that both methods agree in the final analysis.

Compute the deflection:

$$\frac{Wmax}{b} = .0126$$

$$Wmax = (.0126)(4 \text{ in}) = \underline{.050 \text{ in}}$$